QM

A1

2 $(\Delta x)^2 = \langle x^2 \rangle - \langle x \rangle^2$

2 $\langle x \rangle = 0$ because of the symmetry of the harmonic potential, therefor

$$(\Delta x)^2 = \langle x^2 \rangle$$

1 Similar $(\Delta p)^2 = \langle p^2 \rangle$

1 $\langle H \rangle = \frac{\langle p^2 \rangle}{2m} + \frac{k}{2} \langle x^2 \rangle$

From the uncertainty principle $(\Delta p)^2 (\Delta x)^2 \geq \frac{\hbar^2}{4}$

4 or $\langle p^2 \rangle \langle x^2 \rangle \geq \frac{\hbar^2}{4}$

2 $$\frac{\langle p^2 \rangle}{2m} + \frac{k}{2} \langle x^2 \rangle \geq \frac{\hbar^2}{8m \langle x^2 \rangle} + \frac{k}{2} \langle x^2 \rangle$$

minimize this function in $y = \langle x^2 \rangle$

4 $$-\frac{\hbar^2}{8my^2} + \frac{k}{2} = 0 \rightarrow y = \frac{\hbar}{2\sqrt{mk}}$$

6 $$\langle H \rangle \geq \frac{\hbar^2 \, 2\sqrt{mk}}{8m \, \hbar} + \frac{k}{2} \frac{\hbar}{2\sqrt{mk}} = \frac{\hbar}{2}\sqrt{\frac{k}{m}} = \frac{\hbar\omega}{2}$$

QM A2.

(a) $[A,B] = AB - BA$

5 $[i(AB-BA)]^\dagger = -i(BA-AB) = i[A,B]$ ↑ hermitian

5 (b) $[i(\vec{A}\cdot\vec{B} - \vec{B}\cdot\vec{A})]^\dagger = -i(\vec{B}\cdot\vec{A} - \vec{A}\cdot\vec{B}) = i(\vec{A}\cdot\vec{B} - \vec{B}\cdot\vec{A})$ hermitian

4 since for each component

$$(A_i B_i)^\dagger = B_i A_i$$

6 (c) $i(\vec{A}\times\vec{B} - \vec{B}\times\vec{A}) = -i(\vec{A}\times\vec{B} - \vec{B}\times\vec{A})$ not hermitian

since, for example, for x component

5 $C_x = (A_y B_z - A_z B_y)^\dagger = (B_z A_y - B_y A_z) = -(\vec{B}\times\vec{A})_z$

and generally $(\vec{A}\times\vec{B})^\dagger = -(\vec{B}\times\vec{A})$

QM

A3

6 (a) $E + \frac{Ry}{n^2} = \frac{hc}{\lambda}$, $n=2$

4 $E_{ph} = \frac{hc}{\lambda} = 5eV + \frac{13.6eV}{4} = 8.4eV$

4 $\lambda = \frac{hc}{E_{ph}} = \frac{1240\ eV\cdot nm}{8.4eV} = 148nm$

6 (b) $p = \sqrt{2m_e E} = \sqrt{2\times 511 \frac{keV}{c^2} \cdot 5eV} = 2.26\times 10^3 \frac{eV}{c}$

$= 2.26\times 10^3 \frac{eV}{\cancel{c}} \cdot \frac{1.60\times 10^{-19}\ J/eV}{3\cdot 10^8 \frac{m}{s}} = 1.21\times 10^{-24} \frac{kg\cdot m}{s}$

5 (c) no, because of the uncertainty principle

$\vec{L}$ does not commute with $\vec{p}$

QM

A4

$n = n_r + l + 1 \qquad n_r = 0, 1, \ldots$

9 For the lowest n $n_r = 0$ $n = l+1 = 6$

7 $E = -\frac{13.6eV}{36} = 0.378eV$

9 Since $n_r = 0$ the radial wavefunction has no nodes

QM

B1.

$$\psi(x) = A\sin kx + B\cos kx, \quad |x| < \frac{L}{2}$$
$$0 \qquad |x| > \frac{L}{2}$$

3

$$k = \frac{\sqrt{2mE}}{\hbar}$$

$$\psi\left(-\frac{L}{2}\right) = \psi\left(\frac{L}{2}\right) = 0$$

$$-A\sin\frac{kL}{2} + B\cos\frac{kL}{2} = 0$$

$$A\sin\frac{kL}{2} + B\cos\frac{kL}{2} = 0$$

alternatively, they can start with $\psi = B\cos kx$ since the ground state is symmetric (even)

5 From here $A = 0$ $\quad \frac{kL}{2} = \frac{\pi}{2} + \pi n, \; n = 0, 1, \ldots$

ground state: $n = 0 \quad k = \frac{\pi}{L}, \quad E = \frac{\hbar^2}{2m}\left(\frac{\pi}{L}\right)^2$

normalisation

2 $$B^2 \int_{-\frac{L}{2}}^{L/2} \cos^2 kx \, dx = \frac{B^2}{2} \int_{-L/2}^{L/2} (1 + \cos 2kx)\, dx \qquad k = \frac{\pi}{L}$$

6 $$= \frac{B^2}{2}\left(x + \frac{L}{2\pi}\sin\frac{2\pi}{L}x\right)\Big|_{-L/2}^{L/2} = B^2\frac{L}{2} = 1$$

$$B = \sqrt{\frac{2}{L}}$$

2 (6) $$P = 2\int_{L/4}^{L/2} \psi^2 dx = 2\cdot\frac{2}{L}\int_{L/4}^{L/2} \cos^2\frac{\pi}{L}x\, dx =$$

7 $$= \frac{4}{L}\cdot\frac{1}{2}\left(x + \frac{L}{2\pi}\sin\frac{2\pi}{L}x\right)\Big|_{L/4}^{L/2} = \frac{2}{L}\left(\frac{L}{4} - \frac{L}{2\pi}\sin\frac{\pi}{2}\right) = \frac{1}{2} - \frac{1}{\pi} = 0.182$$

(B2.)

Secular ~~matrix~~ determinant

2 $$\begin{vmatrix} -E & A & 0 \\ A & -E & 0 \\ 0 & 0 & A-E \end{vmatrix} = 0$$

$$(A-E)(E^2 - A) = 0$$

5 $E_1 = A \qquad E_{2,3} = \pm A$

eigenvector $\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$

$$-E v_1 + A v_2 = 0 \qquad (A-E) v_3 = 0$$

2 For the first state $v_3 = 1, \; v_1 = v_2 = 0 \qquad v^{(1)} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

2 the second state $v_2 = v_1 \qquad v^{(2)} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$

2 the third state $v_2 = -v_1 \qquad v^{(3)} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$

(b) the first eigenstate of B is $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

5 probability of the $E_1 = A$ $= \left[(1\,0\,0) \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right]^2 = 0$

5 probability of the $E_2 = A$ $= \left[(1\,0\,0) \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} \right]^2 = \frac{1}{2}$

$E_3 = -A = \left[(1\,0\,0) \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \right] = \frac{1}{2}$

2 therefore $E = A$ with 50%
and $E = -A$ with 50%

B3 QM

1 a)

2 Region I $(-\infty \leq x \leq 0)$: $\Psi_I(x) = A\exp(ikx) + B\exp(-ikx)$ where $k = \frac{\sqrt{2mE}}{\hbar}$

2 Region II $(0 < x < a)$: $\Psi_{II}(x) = C\exp(ik_0x) + \mathrm{D}'\exp(-ik_0x)$ where $k_0 = \frac{\sqrt{2m(E-V_0)}}{\hbar}$

2 Region III $(a \leq x \leq \infty)$: $\Psi_{III}(x) = \overset{F}{\cancel{E}}\exp(ik_1x)$ where $k_1 = \frac{\sqrt{2m(E-V_1)}}{\hbar}$

b)

$$
\begin{aligned}
2 \quad & A + B = C + D \\
3 \quad & k(A - B) = k_0(C - D) \\
2 \quad & C\exp(ik_0a) + D\exp(-ik_0a) = \overset{F}{\cancel{E}}\exp(ik_1a) \\
3 \quad & k_0C\exp(ik_0a) - k_0D\exp(-ik_0a) = k_1\underset{F}{\cancel{E}}\exp(ik_1a)
\end{aligned}
$$

c)

5
$$j_I(x) = \frac{\hbar k}{m}\left(|A|^2 - |B|^2\right),\ j_{II}(x) = \frac{\hbar k_0}{m}\left(|C|^2 - |D|^2\right),\ j_{III}(x) = \frac{\hbar k_1}{m}|\overset{F}{\cancel{E}}|^2$$

and therefore, using the equality between $j_I(x)$ and $j_{III}(x)$:

2
$$k\left(|A|^2 - |B|^2\right) = k_1|\overset{F}{\cancel{E}}|^2$$

. Dividing both sides by $k|A|^2$ we get:

2
$$1 - \left|\frac{B}{A}\right|^2 = \frac{k_1}{k}\left|\frac{\overset{F}{\cancel{E}}}{A}\right|^2$$

That allows for the identifications of $T = \frac{k_1}{k}\left|\frac{\overset{F}{\cancel{E}}}{A}\right|^2$ and $R = \left|\frac{B}{A}\right|^2$, as transmission and reflection probabilities, respectively.

QM B4

(a) at $t=0$ the state vector is ~~the~~ eigenstate

of $\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$ with eigenvalue $\lambda = +1$

secular matrix

6 $\begin{pmatrix} -\lambda & -i \\ i & -\lambda \end{pmatrix}$ gives the equation

$-\lambda a_1 - i a_2 = 0 \quad a_2 = i\lambda a_1$ with $\lambda = 1$

3 and $\psi(0) = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix}$

(b) $i\hbar \frac{\partial \psi}{\partial t} = \gamma B S_z \psi$ where $S_z = \frac{\hbar}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

5 the eigenstates of S_z are $|\uparrow\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $|\downarrow\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$\psi(0) = \frac{1}{\sqrt{2}} (|\uparrow\rangle + i|\uparrow\rangle)$ and eigenvalues of H $\pm \gamma B \frac{\hbar}{2}$

therefore for $t > 0$

5 $\psi(t) = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{-i\gamma B \frac{\hbar}{2} t} + \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ i \end{pmatrix} e^{i\gamma B \frac{\hbar}{2} t}$

$= \frac{1}{\sqrt{2}} \begin{pmatrix} e^{-it\omega/2} \\ i\, e^{it\omega/2} \end{pmatrix}$ $\quad \omega = \gamma B$ Larmor frequency

(c) denoting $z = e^{it\omega/2}$, we have $\psi = \frac{1}{\sqrt{2}} \begin{pmatrix} z^* \\ iz \end{pmatrix}$

6 $\langle S_x \rangle = \frac{\hbar}{2} \cdot \frac{1}{2} (z, -iz^*) \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} z^* \\ iz \end{pmatrix} = \frac{\hbar}{4} (z, -iz^*) \begin{pmatrix} iz \\ z^* \end{pmatrix}$

$= \frac{\hbar}{4} (iz^2 - iz^{*2}) = \frac{i\hbar}{4} \left(e^{i\omega t} - e^{-i\omega t}\right) = -\frac{\hbar}{2} \sin \omega t$

EM problems

1. An infinite plane of a uniform surface charge density σ_0 is placed at distance $z = h$ above the surface of a half-space grounded metal.

(1) Find the potential and the electric field in all space.

(2) Find the induced surface charge density on a metal surface.

Solution:

(1) The solution can be obtained using a method of images. An image charge plane is located at $z = -h$ below the surface of a metal and has charge $-\sigma_0$ per unit area. The electric file produced by the two planes of charge is uniform between the planes and is equal to $\mathbf{E} = -\frac{\sigma_0}{\varepsilon_0}\hat{\mathbf{z}}$. Above the

5 plane of charge, $z > h$, the electric field is zero. Since $\mathbf{E} = -\nabla\Phi$, we find that between the planes the potential is $\Phi(z) = \frac{\sigma_0}{\varepsilon_0} z + C$, where constant C must be equal to zero in order to have zero potential on the grounded metal surface. Above the plane of charge the potential is constant and is equal to $\Phi(z) = \frac{\sigma_0}{\varepsilon_0} h$. Thus,

$$\Phi(z) = 0; \quad \mathbf{E} = 0; \quad z < 0$$

$$\Phi(z) = \frac{\sigma_0}{\varepsilon_0} z; \quad \mathbf{E} = -\frac{\sigma_0}{\varepsilon_0}\hat{\mathbf{z}}; \quad 0 < z < h, \qquad 5+5+5 \tag{1}$$

$$\Phi(z) = \frac{\sigma_0}{\varepsilon_0} h; \quad \mathbf{E} = 0; \quad z > h$$

at $z = 0$ and $z = h$ the potential is continuous, while the electric field is discontinuous.

(2) The induced surface charge density σ on a metal surface is given by

$$\mathbf{E}_{above} - \mathbf{E}_{below} = \frac{\sigma}{\varepsilon_0}\hat{\mathbf{n}}, \qquad 5 \tag{2}$$

$\mathbf{E}_{above}$ is the electric field above the metal surface, $\mathbf{E}_{below}$ is the electric field below the metal surface, and $\hat{\mathbf{n}} = \hat{\mathbf{z}}$ is normal to the surface. From Eqs. (1) and (2), we find that $\sigma = -\sigma_0$.

2. A flat surface $z = 0$ of a semiinfinite linear dielectric material of uniform dielectric permittivity ε is affected by an external non-uniform electric field whose magnitude and direction in the absence of the dielectric is $\mathbf{E}^{ext}(\mathbf{r})$. There are no free charges in the dielectric. Find the induced surface polarization charge density $\sigma_P(\mathbf{r})$. For this purpose:

(1) show that the normal component of the polarization-induced electric field $E_z^P(\mathbf{r})$ near the surface inside the dielectric is given by $E_z^P(\mathbf{r}) = -\dfrac{\sigma_P(\mathbf{r})}{2\varepsilon_0}$;

(2) express $\sigma_P(\mathbf{r})$ is terms of the total electric field at the surface;

(3) solve the problem using (1) and (2).

Solution:

(1) Electrostatic boundary conditions state that the normal component of the electric field experiences a step of $\dfrac{\sigma(\mathbf{r})}{\varepsilon_0}$ when crossing the surface. Since there are no free charges, the surface charge density $\sigma(\mathbf{r})$ is equal to the surface polarization charge density $\sigma_P(\mathbf{r})$. Since the surface is a plane, by symmetry and according to Gauss's law, the normal component of the polarization-induced electric field above and below the surface must by equal in magnitude but pointing in the opposite directions. Therefore $E_z^P(\mathbf{r}) = -\dfrac{\sigma_P(\mathbf{r})}{2\varepsilon_0}$. 5

(2) The surface polarization charge density is given by

$$\sigma_P(\mathbf{r}) = \mathbf{P} \cdot \mathbf{n} \quad 5$$

where **n** is the normal to the surface. Polarization **P** is a linear function of electric field so that

$$\mathbf{P} = (\varepsilon - \varepsilon_0)\mathbf{E}.$$

Therefore we find:

$$\sigma_P(\mathbf{r}) = (\varepsilon - \varepsilon_0)E_z \ , \quad 5$$

where E_z is the normal component of the *total* electric field inside the dielectric near its surface.

(3) The total electric field is the sum of the external field and the field produced by polarization charge $\mathbf{E} = \mathbf{E}^P + \mathbf{E}^{ext}$. Therefore, we have:

$$\sigma_P = (\varepsilon - \varepsilon_0)E_z = (\varepsilon - \varepsilon_0)\left(E_z^P + E_z^{ext}\right) = (\varepsilon - \varepsilon_0)\left(-\frac{\sigma_P}{2\varepsilon_0} + E_z^{ext}\right) \quad 7$$

Solving this equation with respect to σ_P we find:

$$\sigma_P(\mathbf{r}) = 2\varepsilon_0 \frac{\varepsilon - \varepsilon_0}{\varepsilon + \varepsilon_0} E_z^{ext}(\mathbf{r}) \quad 3$$

3. An electric circuit represents two concentric metal spheres of radii a and b, with $b > a$, serving as electrodes, and a homogeneous material of conductivity σ between the spherical electrodes. Find the resistance R of this material.

Solution:

Under the application of bias voltage V between the electrodes, an electric current I will flow between them. According to Ohm's law current density is given by $\mathbf{J}(\mathbf{r}) = \sigma \mathbf{E}(\mathbf{r})$. 5By symmetry both $\mathbf{J}$ and $\mathbf{E}$ are pointing along $\hat{\mathbf{r}}$ and can depend only on r,2 and hence the total electric current across a surface of radius r, such that $b > r > a$, is given by

$$I = J(r)4\pi r^2 = 4\pi\sigma r^2 E(r) \quad 5$$

It is independent of r by current continuity (charge conservation) condition.

We therefore have for the electric field:

$$E(r) = \frac{I}{4\pi\sigma r^2}. \quad 2$$

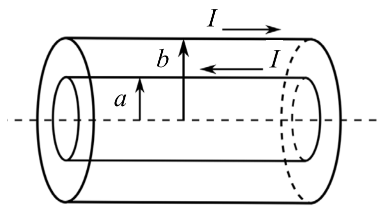
On the other hand, the potential and the field are related as follows:

$$V = \int_a^b E(r)dr = \int_a^b \frac{I}{4\pi\sigma r^2} dr = \frac{I}{4\pi\sigma}\left(\frac{1}{a} - \frac{1}{b}\right) \quad 8$$

The resistance is therefore:

$$R = \frac{V}{I} = \frac{1}{4\pi\sigma}\frac{b-a}{ab}. \quad 3$$

4. Find the self-inductance L per unit length for an infinitely long coaxial cable of radii a, b with $a < b$, carrying a current I, as shown in in the figure.



Solution:

The self-inductance L is related to the energy W as follows: $W = \frac{LI^2}{2}$ 3

The magnetostatic energy is given by

$$W = \frac{1}{2\mu_0}\int B^2 d^3 r$$ 3

The magnetic field can be found from Ampère's law. Integrating over circle of radius s ($b > s > a$), we obtain:

$$\oint \mathbf{B}\cdot d\mathbf{l} = 2\pi s B = \mu_0 I\,,$$

and hence $\mathbf{B} = \frac{\mu_0 I}{2\pi s}\hat{\boldsymbol{\phi}}$. 8

The energy of the system of length l is then:

$$W = l\frac{1}{2\mu_0}\int_a^b \left(\frac{\mu_0 I}{2\pi s}\right)^2 2\pi s ds = l\frac{\mu_0 I^2}{4\pi}\int_a^b \frac{1}{s}ds = l\frac{\mu_0 I^2}{4\pi}\ln\frac{b}{a}.$$ 8

The self-inductance per unit length is then:

$$\frac{L}{l} = \frac{2W}{lI^2} = \frac{\mu_0}{2\pi}\ln\frac{b}{a}.$$ 3

5. An electric field has a wave form $\mathbf{E}(z,t) = E_0\hat{\mathbf{x}}\cos(kz)\cos(\omega t)$. (1) Using Maxwell's equations, find the magnetic field $\mathbf{B}(z,t)$. Then, find (2) Poynting's vector $\mathbf{S}$ and (3) time-averaged Poynting's vector $\langle\mathbf{S}\rangle$. (4) What conclusion about the wave intensity can be made from the latter result? What kind of wave does the given electric field represent?

Solution: (1) The relevant Maxwell's equation are $\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t}$ 2 and $\nabla\times\mathbf{B} = \frac{1}{c^2}\frac{\partial\mathbf{E}}{\partial t}$.2 From the first equation, we have:

$$\nabla\times\mathbf{E} = \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & 0 & 0 \end{vmatrix} = \hat{\mathbf{y}}\frac{\partial E_x}{\partial z} - \hat{\mathbf{z}}\frac{\partial E_x}{\partial y} = -E_0 k\hat{\mathbf{y}}\sin(kz)\cos(\omega t) = -\frac{\partial\mathbf{B}}{\partial t}, \quad 4$$

which implies that only $B_y(z,t)$ component is non-zero. From the second equation, we obtain:

$$\nabla\times\mathbf{B} = \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & B_y & 0 \end{vmatrix} = -\hat{\mathbf{x}}\frac{\partial B_y}{\partial z} = \frac{1}{c^2}\frac{\partial\mathbf{E}}{\partial t} = -E_0\frac{\omega}{c^2}\hat{\mathbf{x}}\cos(kz)\sin(\omega t). \quad 4$$

Taking into account $\omega = kc$, we therefore have:

$$\frac{\partial B_y}{\partial t} = E_0\frac{\omega}{c}\sin(kz)\cos(\omega t), \quad 2$$

$$\frac{\partial B_y}{\partial z} = E_0\frac{k}{c}\cos(kz)\sin(\omega t).$$

These two equations are consistent when

$$\mathbf{B} = \frac{E_0}{c}\hat{\mathbf{y}}\sin(kz)\sin(\omega t). \quad 2$$

(2) Poynting's vector is

$$\mathbf{S} = \frac{1}{\mu_0}\mathbf{E}\times\mathbf{B} = \frac{1}{\mu_0}E_0\cos(kz)\cos(\omega t)\frac{E_0}{c}\sin(kz)\sin(\omega t)\left(\hat{\mathbf{x}}\times\hat{\mathbf{y}}\right) = \frac{E_0^2}{4\mu_0 c}\sin(2kz)\sin(2\omega t)\hat{\mathbf{z}}. \quad 5$$

(3) Time-averaged Poynting's vector is $\langle\mathbf{S}\rangle = 0$, because the average of sine over an entire cycle is zero. 2

(4) Since time-averaged Poynting's vector determines intensity of an electromagnetic wave, we can conclude that no intensity is transmitted along the z direction is zero. Hence, the given electric field represents a stationary (standing) wave. 2

6. Two plane electromagnetic waves propagate in z direction and have the form

$$\mathbf{E}_1(\mathbf{r},t) = \hat{\mathbf{x}}E_0 e^{ikz-i\omega t+i\varphi_1},$$
$$\mathbf{E}_2(\mathbf{r},t) = \hat{\mathbf{y}}E_0 e^{ikz-i\omega t+i\varphi_2},$$

where $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$ are unit vectors in x and y directions respectively and E_0 is a real amplitude.

1. What relationship should φ_1 and φ_2 obey to make the superposition of these two waves, $\mathbf{E}(\mathbf{r},t) = \mathbf{E}_1(\mathbf{r},t) + \mathbf{E}_2(\mathbf{r},t)$, a linearly polarized wave? What is the angle of the polarization plane of this superposed wave with respect to the x axis? Write down the x- and y-components of the resulting electric field in form of the *real* part of $\mathbf{E}$.
2. What relationship should φ_1 and φ_2 obey to make this wave circularly polarized with positive (negative) helicity? Write down the x- and y-components of the resulting electric field in form of the *real* part of $\mathbf{E}$.

Solution:

(1) The wave is linearly polarized if it can be represented as $\mathbf{E}(\mathbf{r},t) = E_0 \mathbf{e} e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t+\varphi)}$, where $\mathbf{e}$ is the (real) polarization vector. In our case,

$$\mathbf{E}(\mathbf{x},t) = \mathbf{E}_1 + \mathbf{E}_2 = E_0\left[\hat{\mathbf{x}} + \hat{\mathbf{y}}e^{i(\varphi_2-\varphi_1)}\right]e^{ikz-i\omega t+\varphi_1}. \quad 8$$

Therefore, we have $\mathbf{e} = \hat{\mathbf{x}} + \hat{\mathbf{y}}e^{i(\varphi_2-\varphi_1)}$. For the vector $\mathbf{e}$ to be real, we need to have the same phase, i.e. $\varphi_1 = \varphi_2$. The polarization vector in this case is $\mathbf{e} = \hat{\mathbf{x}} + \hat{\mathbf{y}}$, and the angle of the polarization plane of this superposed wave with respect to the x axis is given by

$$\theta = \tan^{-1} 1 = \frac{\pi}{4}. \quad 5$$

The x- and y-components of the resulting *real* electric field are given by

$$E_x(\mathbf{r},t) = E_0\cos(kz - i\omega t + \varphi_1).$$

$$E_y(\mathbf{r},t) = E_0\cos(kz - i\omega t + \varphi_1) \qquad 6 \text{ ignore i}$$

(2) The condition for circular polarization is $\mathbf{e}_\pm = \mathbf{e}_x \pm i\mathbf{e}_y$, for positive and negative helicity respectively. In our case, $\mathbf{e} = \hat{\mathbf{x}} + \hat{\mathbf{y}}e^{i(\varphi_2-\varphi_1)}$, and therefore the superposed wave must have $\varphi_2 - \varphi_1 = \pm\frac{\pi}{2}$. The x- and y-components of the resulting electric field are therefore

$$E_x(\mathbf{r},t) = E_0\cos(kz - \omega t + \varphi_1),$$

$$E_y(\mathbf{r},t) = \mp E_0\sin(kz - \omega t + \varphi_1), \quad 6$$

for the wave of positive and negative helicity, respectively.

EM A4

5 (a) $F_{el} = F_{magn}$

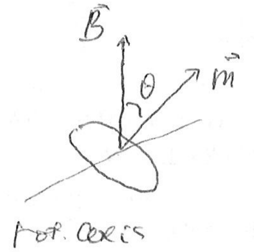
5 $eE = evB$

5 $v = \frac{E}{B} = \frac{12.4 \times 10^{2} \frac{V}{m}}{0.1\,T} = 12.4 \times 10^{3} \frac{m}{s}$

5 (b) $\frac{mv^2}{r} = evB$

5 $r = \frac{mv}{eB} = \frac{1.67 \times 10^{-27}\,kg \cdot 1.24 \times 10^{4} \frac{m}{s}}{1.6 \times 10^{-19}\,C \cdot 0,005\,T} = 0.0258\,m$

EM (B4)



the restoring

torque $\vec{\tau} = \vec{m} \times \vec{B}$

3 $\tau = mB\sin\theta = \pi r^2 I B \sin\theta$

2^{nd} Newton law for rot. motion

3 $$\frac{dL}{dt} = -\pi r^2 I B \sin\theta$$

at small θ $\sin\theta \approx \theta$

$L = I_{inertia}\,\omega = I_{in}\,\dot{\theta}$

2 according to $\perp$ axis theorem $I_{in} = \frac{1}{2} M r^2$

2 $$\frac{1}{2} M r^2 \ddot{\theta} = -\pi r^2 I B \theta$$

4 (1) $$\omega^2 = \frac{2\pi I B}{M} \qquad T = \frac{2\pi}{\omega} = \left(\frac{2\pi M}{IB}\right)^{1/2}$$

(b) Magnetic flux through the loop

3 $$\Phi = B \cdot \pi r^2 \cos\theta \approx \pi r^2 B \left(1 - \frac{\theta^2}{2}\right)$$

2 $$\mathcal{E} = -\frac{d\Phi}{dt} = \pi r^2 B \theta \dot{\theta}$$

for initial displacement θ_0 $\theta = \theta_0 \cos\omega t$

2 $\dot{\theta} = -\omega\theta_0 \sin\omega t$

4 and $\mathcal{E} = -\pi r^2 B \omega \theta_0^2 \cos\omega t \sin\omega t$ where ω is given by (1).