A1 Calculate the thermal energy of 1 mol of Cu at T = θ_D using the classical theory. (for Cu, θ_D = 340 K)

The thermal energy for each degree of freedom is

$$u(T) = \frac{1}{2} k_B T$$

A solid has 6 degrees of freedom (3 × translation, 3 × rotation) so the thermal energy is

$$u(T) = 3 N k_B T = 3 n N_A k_B T = 3 n R T$$

⇒ for 1 mol Cu ($n = 1$ mol) and $T = \theta_D = 340$ K we find for the energy:

$$u(T) = 3RT = 8480.80 \ \frac{J}{mol}$$

Note: 3kT comes from the three **vibrational** degrees of freedom, Cu atoms **don't rotate**.

A2. A hot wire at 3200 K 80 μm across and 5 cm generates how much heat (write this as power dissipation):

Assuming is a black body

$$S = \sigma T^4$$

$$P = SA = \sigma T^4 (2\pi r l)$$

$$= (2\pi)\left(5.6703\ x\ 10^{-8} \frac{Watts}{m^2 \cdot K^4}\right)(3200K)^4\left(\frac{0.080x\ 10^{-3} m}{2}\right)(5.0x\ 10^{-2} m)$$

$$= 74.7\ Watts$$

A3. 300 g of aluminum block at 100 °C is place in a calorimeter cup with 400 g of water. The mass of the copper calorimeter cup is 80 g. The initial temperature of the water and the cup is 22 °C. What's the final temperature?

Solution: The heat input of the water and cup is the same as the heat output of the aluminum block. Assuming the final temperature is T:

(90-T) C_{Al} =(T-22) (C_{water}+C_{copper})
C_{Al} = 0.3*0.904=0.2712 kJ/K
C_{water} = 0.4*4.184=1.674 kJ/K

C_{copper} = 0.08*0.385= 0.0308 kJ/K
Hence: T=32.7 °C

A4. In a vacuum tube of pressure 1.333×10^{-3} Pa, at 27 °C, calculate 1) number of gas particles per m^3, 2) volume occupied per particle, 3) average distance between particles.

Solution:
1) Using PV=nRT or PV = Nk_BT, one can calculate the particle density N/V = P/k_BT = 3.22×10^{17}/ m^3.
2) Volume occupied per particle is: V/N=3.11×10^{-18} / m^3
3) Average distance is $(V/N)^{1/3}$=1.46×10^{-6} m

B1. An object with constant heat capacity C is initially at temperature T_1. It is brought into contact with a heat reservoir at temperature T_R, where $T_R < T_1$.
a) Find the entropy change of both the object and the reservoir.
b) Show that the total change in entropy is consistent with the second law of thermodynamics.

ΔSbody=\int C dT/T= C ln(T_R/T_1) (<0)
ΔSres=C(T_1-T_R)/T_R (>0)
ΔStotal= C ln(T_R/T_1)+C(T_1/T_R-1)

x=T_1/T_R>0   x-1-ln x>0, therefore ΔStotal>0.

B2. One mole of diatomic ideal gas (C_V = 2.5 nR) performs a transformation from an initial state for which temperature and volume are, 291 K and 21,000 ml to a final state in which temperature and volume are 305 K and 12,700 ml. The transformation is represented on the (V, P) diagram by a straight line. Find the work performed and the heat absorbed by the system.

Solution:

	Initial	**Final**	**Change**
P (Pa)	1.15E5	2E5	0.85E5
V (m^3)	**21E-3**	**12.7E-3**	-8.3E-3
T (K)	**291**	**305**	14

A straight line in (P,V) diagram: $P = P_1 + \frac{\Delta P}{\Delta V}(V - V_1)$,
$W = \int PdV = \int \left[P_1 + \frac{\Delta P}{\Delta V}(V - V_1)\right] dV = \left(P_1 - \frac{\Delta P}{\Delta V}V_1\right)\Delta V + \frac{1}{2}\Delta P(V_1 + V_2)$ =-1307 J

$$\Delta U = CV\Delta T = 290\,J$$
$$Q = \Delta U + W = -1017\,J$$

B3. Show that, for ideal gas, if the heat capacity of a process is constant, then the process is polytropic $PV^l = C$. Assuming that Cp and Cv are constant.

Solution: $dU = CndT-PdV$
$(Cv-Cn)dT = -PdV=-nRTdV/V$
$(Cv-Cn)\ dT/T = -\ nR\ dV/V$
$(Cv-Cn)\ dlnT = nR\ dlnV$
$dlnT^{Cv-Cn} + dlnV^{nR} = 0$
$d\ (lnT^{Cv-Cn} + dlnV^{nR}) = 0$
$lnT^{Cv-Cn} + dlnV^{nR} = constant$
$T^{Cv-Cn}\ V^{nR} = constant$
$(PV)^{Cv-Cn}\ V^{nR} = constant$
$P^{Cv-Cn}\ V^{Cv-Cn+nR} = constant$
$P^{Cv-Cn}\ V^{Cp-Cn} = constant$
$P\ V^{l} = constant$
$l=(Cp-Cn)/(\ Cv-Cn)$

B4. A diatomic gas ($C_V = 2.5\ nR$) expands adiabatically to a volume 1.35 times larger than the initial volume. The initial temperature is 18 °C. Find the final temperature.

Solution:
$CvdT = -PdV=-nRTdV/V$
$Cv\ dT/T = -\ nR\ dV/V$
$Cv\ dlnT = nR\ dlnV$
$dlnT^{Cv} + dlnV^{nR} = 0$
$d\ (lnT^{Cv} + dlnV^{nR}) = 0$
$lnT^{Cv} + dlnV^{nR} = constant$
$T^{Cv}\ V^{nR} = constant$

$(T2/T1)^{Cv}\ (V2/V1)^{nR}=1$
$(T2/T1)^{Cv/nR}\ (V2/V1) =1$

V2/V1 = 1.35, T2/T1 = 0.887.
T1= 18+273.16=291.16 K, T2=258.22 K or -14.94 celsius.

Problem B.4

Saturday, May 8, 2021 08:47

(a) $\tau = I\alpha \rightarrow \alpha = \frac{\tau}{I}$

$$\omega(t) = \omega_0 + \alpha t = \left(\frac{\tau}{I}\right)t$$

$$I = \frac{\tau \cdot t}{\omega} = \frac{(50\,N\cdot m)(20\,s)}{\left(600\,\frac{rev}{min}\right)\left(\frac{1\,min}{60\,s}\right)\left(\frac{2\pi\,rad}{1\,rev}\right)}$$

$$\boxed{I = \frac{25}{\pi}\,kg\cdot m^2}$$

(b) $$\tau_f = \frac{\omega \cdot I}{t} = \frac{\left(\frac{25}{\pi}\,kg\cdot m^2\right)\left(600\,\frac{rev}{min}\right)\left(\frac{1\,min}{60\,s}\right)\left(\frac{2\pi}{1\,rev}\right)}{120\,s}$$

$$= \frac{25}{6}\,N\cdot m$$

Problem A.1

Saturday, May 8, 2021 8:04 AM

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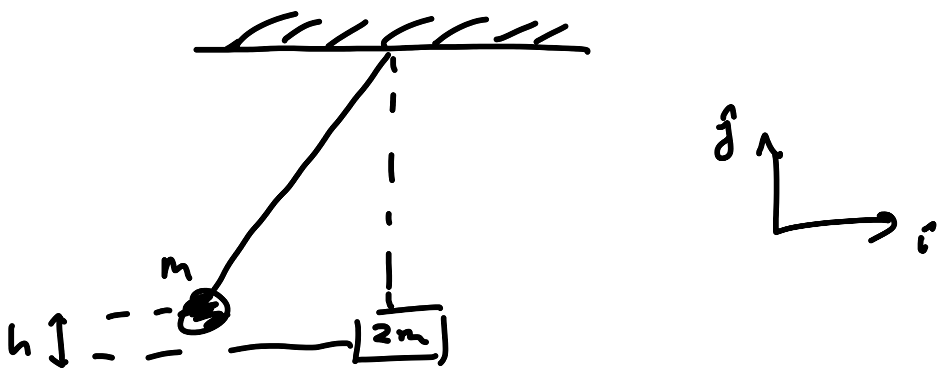


step 1 : apply conservation of energy to mass m

$E_0 = mgh$ (at release)

$E_f = \frac{1}{2} m v_0^2$ (at collision)

$v_0 = \sqrt{2gh}$

step 2 : apply conservation of momentum to collision

$\Delta \vec{p} = 0$

$\vec{p}_0 = m v_0 \, \hat{i}$

step 3: apply conservation of energy to combined mass m+2m

$$E_0 = \frac{1}{2}(m+3m)v_f^2$$

$$E_f = (m+3m)gh'$$

$$E_0 = E_f$$

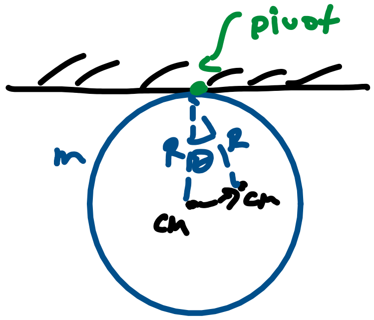
$$\Rightarrow \quad h' = \frac{v_f^2}{2g} = \frac{1}{2g}\left(\frac{v_0}{3}\right)^2$$

$$= \frac{1}{18g}\left(\sqrt{2gh}\right)^2$$

$$\boxed{h' = \frac{1}{9}h}$$

Problem A.2

Saturday, May 8, 2021 08:11



sum torques about pivot

$$\sum \tau_{pivot} = -mg\,(R\sin\theta) = I\alpha = I\ddot{\theta}$$

For small oscillations, $\sin\theta \simeq \theta$.

$$\ddot{\theta} + \omega^2\theta = 0, \quad \omega^2 = \sqrt{\frac{mgR}{I}}$$

For a hoop, $I = mR^2 \rightarrow \omega = \sqrt{g/R}$

$$\boxed{T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{R}{g}}}$$

Problem A.3

Saturday, May 8, 2021 08:11

$$v(x) = \beta x^{-n}$$

$$a = \frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = v(x)\frac{dv}{dx}$$

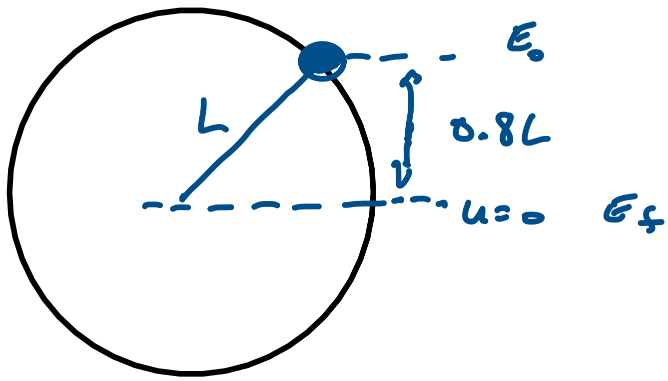
$$\frac{dv}{dx} = -\beta n\, x^{-n-1}$$

$$a(x) = v(x)\frac{dv}{dx}$$

$$\boxed{a(x) = -\beta^2 n\, x^{-2n-1}}$$

Problem A.4

Saturday, May 8, 2021 08:11



Conservation of energy: $\Delta E = W_{\text{non-cons}} = 0$

$E_0 = mgh = mg(0.8L)$

$E_f = \frac{1}{2}mv^2$

$E_0 = E_f \Rightarrow v^2 = 1.6\,gL$

At horizontal, $\sum F_r = -T = -\frac{mv^2}{L}$

$$\boxed{T = 1.6\,mg}$$

Problem B.1

Saturday, May 8, 2021 08:46

(a) $T = \frac{1}{2} m v^2$

(b) $T = \frac{1}{2} m (u+v)^2$

(c) work-energy theorem

$$W_{net} = W_{woman} = \Delta T = \frac{1}{2} m v^2$$

(d) Applying work-energy theorem from perspective of an observer along the track,

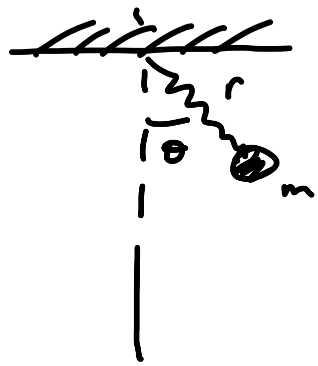
$$W_{net} = W_{woman} + W_{train} = \Delta T$$

$$= \frac{1}{2} m (u+v)^2 - \frac{1}{2} m u^2$$

$$= \underbrace{\frac{1}{2} m v^2}_{W_{woman}} + \underbrace{m u v}_{W_{train}}$$

Problem B.2

Saturday, May 8, 2021 08:46



$$L = T - U$$

$$= \left(\frac{1}{2}m\dot{r}^2 + \frac{1}{2}mr^2\dot{\theta}^2\right) - \left(\frac{1}{2}k(b-r)^2 - mgr\cos\theta\right)$$

Euler-Lagrange: $\frac{\partial L}{\partial q} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) = 0$

$\ddot{r}$: $\frac{\partial L}{\partial r} = mr\dot{\theta}^2 + k(b-r) + mg\cos\theta$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{r}}\right) = \frac{d}{dt}\left(m\dot{r}\right) = m\ddot{r}$$

$\ddot{\theta}$: $\frac{\partial L}{\partial \theta} = -mgr\sin\theta$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) = \frac{d}{dt}\left(mr^2\dot{\theta}\right) = mr^2\ddot{\theta} + 2mr\dot{r}\dot{\theta}$$

$$r^2\ddot{\theta} + 2r\dot{r}\dot{\theta} + g r\sin\theta = 0$$

Problem B.3

Saturday, May 8, 2021 08:47

(a) 5 equilibrium points

x_0, x_2, x_4 are stable

x_1, x_3 are unstable

(b) Need large enough ΔT so that

$E_2 + \Delta T > E_3$

work done on mass by work-energy theorem

$\Rightarrow \Delta T > E_3 - E_2$

(c) Let total energy of particle be E

then it follows from conservation of energy that

$E = E_0 + \frac{1}{2}mv^2$

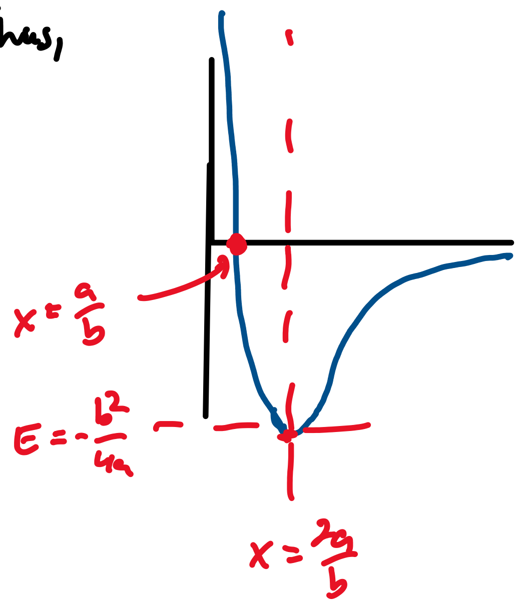
$v = \sqrt{\frac{2}{m}(E - E_0)}$

(d) $U(x) = ax^{-2} - bx^{-1}$

$$= -b^2/4a$$

$$U(x) = 0 \Rightarrow x = \frac{a}{b}$$

Thus,



- motion is periodic for $E < 0$

- for $E \geq 0$, the motion is unbounded as $x \to \infty$