UNL - Department of Physics and Astronomy

Preliminary Examination - Day I
Friday, August 8, 2025

This test covers the topics of *Electrodynamics* (Topic 1) and *Quantum Mechanics* (Topic 2). Each topic has 4 "A" questions and 4 "B" questions. Work two problems from each group. Thus, you will work on a total of 8 questions today, 4 from each topic.

Note: If you do more than two problems in a group, only the first two (in the order they appear in this handout) will be graded. For instance, if you do problems A1, A3, and A4, only A1 and A3 will be graded.

WRITE YOUR ANSWERS ON ONE SIDE OF THE PAPER ONLY

Quantum Mechanics Group A

Answer only two Group A questions

A1. Estimate the quantum mechanical penetration depth of a small particle of dust with a radius r of 10^{-9} m and a density ρ of 10^5 kg/m^3. The dust particle moves at a velocity v of 10 m/s into a potential energy barrier that has 4 times the particle's kinetic energy E.

A2. For a system of two particles with spins $s = 1/2$ find the four eigenvalues (repeating corresponding number of times in case of degeneracy) of the following operators, where a and b are given constants.

a) $\boldsymbol{S}^2$, where $\boldsymbol{S} = \boldsymbol{s}_1 + \boldsymbol{s}_2$.

b) $V_1 = a\hbar^2 + 4bs_1 \cdot s_2$

c) $V_2 = a\hbar(s_{1z} + s_{2z}) + 4b\boldsymbol{s}_1 \cdot \boldsymbol{s}_2$.

A3. The wave function of a particle in the position space is connected with the wave function in the momentum space by the Fourier transform

$$\psi(\mathbf{r}) = \frac{1}{(2\pi\hbar)^{3/2}} \int e^{i\mathbf{p}\cdot\mathbf{r}/\hbar} \phi(\mathbf{p})\ d\mathbf{p}$$

Show that if $\psi(\mathbf{r})$ is normalized to 1, then $\phi(\mathbf{p})$ is also normalized to 1.

A4. For the hydrogen atom in its ground state, calculate the expectation values of the radial coordinate r and the Cartesian coordinate z.

Quantum Mechanics Group B

Answer only two Group B questions

B1. Consider a state |LM> with definite values of the angular momentum and its projection on the z axis $\hbar M$. For such a state determine the expectation values of the following operators.

(a) L_x and L_y.

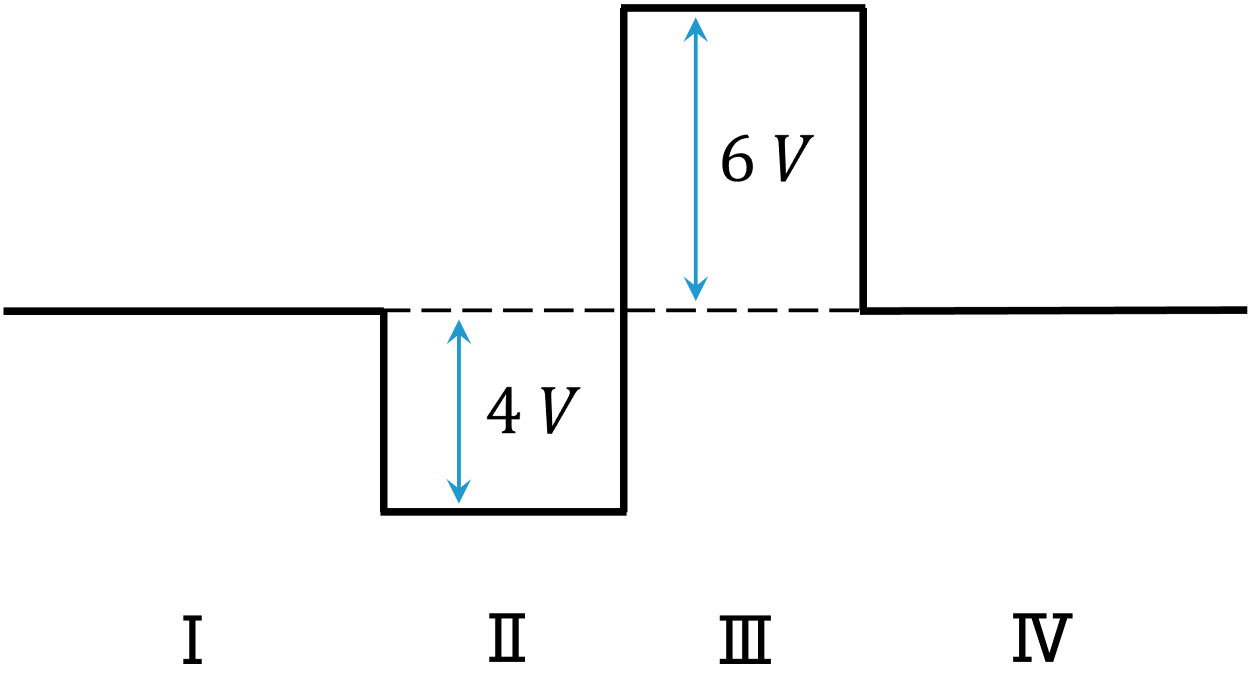
(b) $L_x^2 + L_y^2$.

(c) $L_x^2 - L_y^2$.

(d) $L_{z'}$, which is the projection of L on an axis z' with an angle θ to the z axis.

(e) $L_{z'}^2$

B2. Consider the electric potential shown. Draw in as much detail as you can the wave function of a particle with a charge +*e* at energy +4 eV and explain the main characteristics of your wave function. In particular, explain if the wave function is decaying or growing, or oscillating. In the latter case compare the amplitudes and oscillation frequencies in different regions. Be as quantitative as possible.



B3. A particle of mass m moves in one dimension in the infinite square well

$V=0$ for $-a<x<a$, $V=\infty$ for $|x|>a$.

At $t=0$ the particle's wave function is

$$\psi(x) = A(a^2 - x^2)$$

where A is the normalization constant.

(a) Find A.
(b) At $t=0$ the particle's energy is measured. What are possible outcomes of this measurement?
(c) What is the probability that the result of the measurement will be exactly the energy of the ground state?
(d) Suppose $\psi(x)$ is an approximation for the wave function of the ground state. What does your result in (c) tell you about the quality of the function $\psi(x)$?

B4. A rotational state of a large molecule with the total angular momentum quantum number $j \gg 1$ is represented by

$$|\psi\rangle = \frac{1}{\sqrt{3}}|j, m+1\rangle + \frac{1}{\sqrt{3}}|j, m\rangle + \frac{1}{\sqrt{3}}|j, m-1\rangle$$

with m being the quantum number of z-axis projection of the angular momentum.

(a) What are the probabilities of different outcomes for measurement of J_z in this state?
(b) Find the expectation value of measurement J_z in this state, and uncertainty of its measurement.
(c) Using properties of $J_\pm = J_x \pm iJ_y$ operators show that the expectation value of J_y in this state is zero. Find the expectation value of J_x in this state, simplify it in the case $j \pm m \gg 1$.

Electrodynamics Group A

Answer only two Group A questions

A1. Two parallel electric dipoles $\mathbf{p}_1$ and $\mathbf{p}_2$ are separated by distance a and oriented perpendicular to the line connecting them.

(1) Find the magnitude and direction of the electric field which is produced by dipole $\mathbf{p}_1$ at the position of $\mathbf{p}_2$.

(2) Find the electrostatic energy corresponding to the given dipole configuration.

(3) Calculate the work which must be done to reverse the orientation of one of the dipole moments.

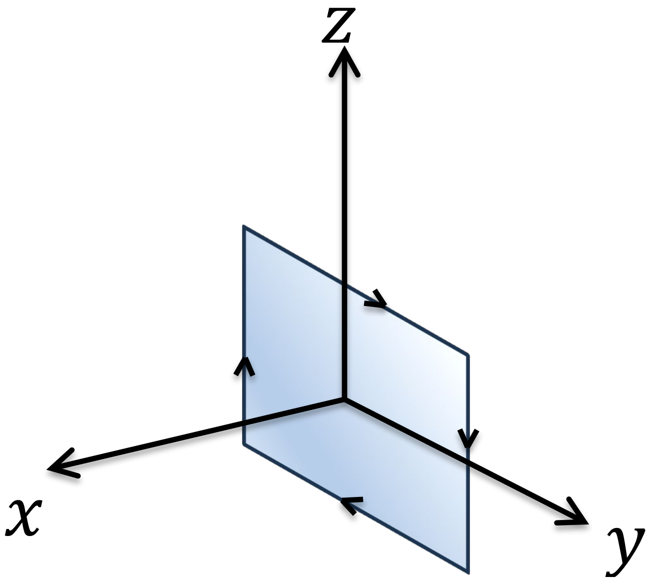
A2. An RC circuit consists of a resistor with resistance R and a charged capacitor with capacitance C and voltage V_0, across the plates. At time $t = 0$ the switch is closed.

(1) Calculate the electric current flowing in the circuit as a function of time.
(2) Show that the initial electrical energy stored in the capacitor is fully dissipated in the resistor at $t = \infty$.

A3. The magnetic field in some region has the form

$$\mathbf{B}=kz\hat{\boldsymbol{x}},$$

where k is a positive constant.



(a) Find the current density which gives rise to this field.

(b) Find the force (magnitude and direction) on a square loop (side *a*) lying in the *yz* plane and centered at the origin of the *xyz* coordinate system, if it carries a current *I*, flowing clockwise as shown in the figure.

A4. An electron beam passes through uniform mutually perpendicular electric and magnetic fields. The electric field is adjusted such that the electrons move along a straight line. Then the electric field is turned off, and the electrons start to move along a curved trajectory with the radius of curvature R=5 cm.

(a) Draw a force diagram showing why the net force on the electron in the first configuration is zero.

(b) Find the charge to mass ratio of electron if the magnetic field is 5x10^{-4} T, and the electric field 2200 V/m.

Electrodynamics Group B

Answer only two Group B questions

B1. A long coaxial cable carries current I such that it flows uniformly down the surface of the inner cylinder of radius a and goes back along the outer cylinder of radius b, as shown in the figure below. Assuming that the two conductors have the potential difference V and the space between the cylinders is filled by a dielectric material of dielectric permittivity ε. Note that the coaxial cable has no net charge

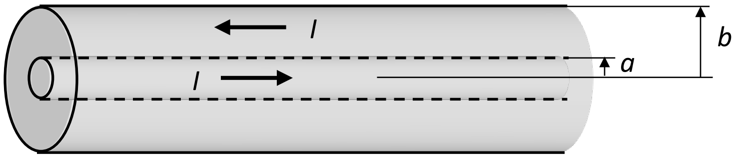
1. find the magnitude of the linear charge density λ on the inner wire;

2. find the electric field **E** everywhere in space;

3. calculate the electric polarization **P** between the two cylinders related to the electric displacement **D** by $\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P}$;

4. calculate the bound charge densities (ρ_B and σ_B) induced by the polarization **P**. Show that the total bound charge is zero;

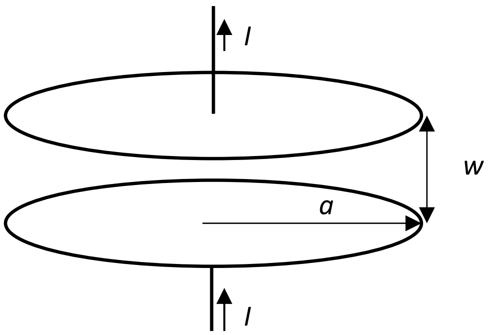
5. find the magnetic field **B** everywhere.



B2. A parallel-plate capacitor (see a figure below) is charging such that the electric current I is constant in time and the surface charge σ is uniform at any given time and is zero at $t = 0$.

1. Find the electric field **E** between the plates.
2. Find the magnetic field **B** between the plates.
3. Find the magnitude and direction of the EM energy flow given by Poynting's vector **S**.

Assume that the separation between the plates w is much less than the radius of the capacitor a ($w << a$) and neglect effects of fringe fields.



B3. A metal hollow sphere of radius R is kept under a constant potential Φ_0. Find the electric field **E** and the electrostatic potential Φ inside and outside the sphere and determine the surface charge density σ

1. by using Gauss's law;
2. by solving Laplace's equation.

Show that the solutions are identical.

Useful formula: $\nabla^2\Phi(r,\theta,\phi) = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\Phi}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\Phi}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2\Phi}{\partial\phi^2}$.

B4. Consider electromagnetic waves in free space in the form:

$$\mathbf{E}(x,y,z,t) = \mathbf{E}_0(x,y)e^{i(kz-\omega t)}$$
$$\mathbf{B}(x,y,z,t) = \mathbf{B}_0(x,y)e^{i(kz-\omega t)}$$

where $\mathbf{E}_0$ and $\mathbf{B}_0$ lie in the xy-plane.

1. Using Maxwell's equations, find the relation between k and ω.
2. Show that $\mathbf{E}_0$, $\mathbf{B}_0$, and $\hat{\mathbf{z}}$ are mutually orthogonal, and find the relation between $|\mathbf{E}_0|$ and $|\mathbf{B}_0|$.

3. Show that $\mathbf{E}_0(x, y)$ and $\mathbf{B}_0(x, y)$ satisfy the equations of electrostatic and magnetostatic in free space.

Physical constants

Speed of light .... $c = 2.998\times10^{8}$ m/s

Planck's constant .... $h = 6.626\times10^{-34}$ J·s

Planck's constant / 2π.... $\hbar = 1.055\times10^{-34}$ J·s

Electron mass .... $m_e = 9.109\times10^{-31}$ kg

Electron's rest energy .... 511.0 keV

Boltzmann constant .... $k_\mathrm{B} = 1.381\times10^{-23}$ J/K

Compton wavelength ..... $\lambda_\mathrm{C} = \dfrac{h}{m_\mathrm{e}c} = 2.426$ pm

Elementary charge .... $e = 1.602\times10^{-19}$ C

Proton mass .... $m_p = 1.673\times10^{-27}\,\mathrm{kg} = 1836\,m_e$

Atomic mass unit .... 1 u=1.66 × 10^{-27} kg

Electric permittivity ...... $\varepsilon_0 = 8.854\times10^{-12}$ F/m

Bohr radius..................... $a_0 = \dfrac{4\pi\varepsilon_0\hbar^2}{e^2 m_\mathrm{e}} = 0.5292$ Å

Magnetic permeability.... $\mu_0 = 1.257\times10^{-6}$ H/m

Rydberg unit of energy ... $Ry = 13.6$ eV

Rydberg constant……..... R=1.097x10^{7} m^{-1}

1 hartree (= 2 Ry) .... $E_h = \dfrac{\hbar^2}{m_e a_0^2} = 27.21$ eV

Molar gas constant.......... $R = 8.314$ J / mol·K

Gravitational constant...... $G = 6.674\times10^{-11}$ m^3 / kg s^2

Avogadro constant .... $N_\mathrm{A} = 6.022\times10^{23}$ mol^{-1}

hc .... $hc = 1240$ eV·nm

Fine structure constant .. $\alpha = \dfrac{1}{4\pi\varepsilon_0}\dfrac{e^2}{\hbar c}$

$E^2 = p^2c^2 + m^2c^4$

TRIGONOMETRY

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$
$$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$
$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$
$$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$
$$\sin(2\theta) = 2\sin\theta\cos\theta$$
$$\cos(2\theta) = \cos^2\theta - \sin^2\theta = 1 - 2\sin^2\theta = 2\cos^2\theta - 1$$
$$\sin\alpha\sin\beta = \tfrac{1}{2}\left[\cos(\alpha-\beta) - \cos(\alpha+\beta)\right]$$
$$\cos\alpha\cos\beta = \tfrac{1}{2}\left[\cos(\alpha-\beta) + \cos(\alpha+\beta)\right]$$
$$\sin\alpha\cos\beta = \tfrac{1}{2}\left[\sin(\alpha+\beta) + \sin(\alpha-\beta)\right]$$
$$\cos\alpha\sin\beta = \tfrac{1}{2}\left[\sin(\alpha+\beta) - \sin(\alpha-\beta)\right]$$

$$\cos(ix) = \cosh(x)$$
$$\sin(ix) = i\sinh(x)$$

For small x:

$$\sin x \approx x - \tfrac{1}{6}x^3$$
$$\cos x \approx 1 - \tfrac{1}{2}x^2$$
$$\tan x \approx x + \tfrac{1}{3}x^3$$

QUANTUM MECHANICS

$$[AB, C] = A[B, C] + [A, C]B$$

Angular momentum: $[L_x, L_y] = i\hbar L_z \quad \textit{et cycl.}$

Ladder operators:

$$L_+ \,|\,\ell, m\rangle = \hbar\sqrt{(\ell+m+1)(\ell-m)}\,|\,\ell, m+1\rangle$$
$$L_- \,|\,\ell, m\rangle = \hbar\sqrt{(\ell+m)(\ell-m+1)}\,|\,\ell, m-1\rangle$$

$$L_\pm = L_x \pm iL_y$$

Wave function of a particle in an infinite square well with the walls at $x=0$ and $x=L$

$$\psi_n(x) = \left(\frac{2}{L}\right)^{1/2} \sin\left(\frac{\pi n}{L}x\right), \quad n = 1, 2, \ldots$$

Table Spherical harmonics and their expressions in Cartesian coordinates.

$Y_{lm}(\theta,\varphi)$	$Y_{lm}(x,y,z)$
$Y_{00}(\theta,\varphi)=\frac{1}{\sqrt{4\pi}}$	$Y_{00}(x,y,z)=\frac{1}{\sqrt{4\pi}}$
$Y_{10}(\theta,\varphi)=\sqrt{\frac{3}{4\pi}}\cos\theta$	$Y_{10}(x,y,z)=\sqrt{\frac{3}{4\pi}}\frac{z}{r}$
$Y_{1,\pm1}(\theta,\varphi)=\mp\sqrt{\frac{3}{8\pi}}e^{\pm i\varphi}\sin\theta$	$Y_{1,\pm1}(x,y,z)=\mp\sqrt{\frac{3}{8\pi}}\frac{x\pm iy}{r}$
$Y_{20}(\theta,\varphi)=\sqrt{\frac{5}{16\pi}}(3\cos^2\theta-1)$	$Y_{20}(x,y,z)=\sqrt{\frac{5}{16\pi}}\frac{3z^2-r^2}{r^2}$
$Y_{2,\pm1}(\theta,\varphi)=\mp\sqrt{\frac{15}{8\pi}}e^{\pm i\varphi}\sin\theta\cos\theta$	$Y_{2,\pm1}(x,y,z)=\mp\sqrt{\frac{15}{8\pi}}\frac{(x\pm iy)z}{r^2}$
$Y_{2,\pm2}(\theta,\varphi)=\sqrt{\frac{15}{32\pi}}e^{\pm2i\varphi}\sin^2\theta$	$Y_{2,\pm2}(x,y,z)=\mp\sqrt{\frac{15}{32\pi}}\frac{x^2-y^2\pm2ixy}{r^2}$

Stationary states of harmonic oscillator for $n = 0$ and $n = 1$:

$$\varphi_0(x)=\left(\frac{\alpha}{\pi^{1/2}}\right)^{1/2}\mathrm{e}^{-\frac{\alpha^2x^2}{2}},$$

$$\varphi_1(x)=\left(\frac{\alpha}{\pi^{1/2}}\right)^{1/2}2ax\,\mathrm{e}^{-\frac{\alpha^2x^2}{2}},$$

where $\alpha=\left(\frac{m\omega}{\hbar}\right)^{1/2}$.

Hydrogen atom: $E_n=-\frac{Ry}{n^2},\ Ry=\frac{me^4}{2(4\pi\varepsilon_0)^2\hbar^2}$

Radial functions for the hydrogen atom $R_{nl}(r)$:

$$R_{10}(r)=\frac{2}{a_0^{3/2}}\exp\left(-\frac{r}{a_0}\right),$$

$$R_{20}(r)=\frac{2}{(2a_0)^{3/2}}\left[1-\frac{r}{2a_0}\right]\exp\left(-\frac{r}{2a_0}\right),$$

$$R_{21}(r)=\frac{r}{24^{1/2}a_0^{5/2}}\exp\left(-\frac{r}{2a_0}\right).$$

ELECTROSTATICS

$$\oiint_S \mathbf{E}\cdot\hat{\mathbf{n}}\,da = \frac{q_{\text{encl}}}{\varepsilon_0}; \quad \mathbf{E} = -\nabla\Phi; \quad \int_{\mathbf{r}_1}^{\mathbf{r}_2} \mathbf{E}\cdot d\boldsymbol{\ell} = \Phi(\mathbf{r}_1) - \Phi(\mathbf{r}_2); \quad \Phi(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\frac{q(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}.$$

Work done: $W = -\int_{\mathbf{a}}^{\mathbf{b}} q\mathbf{E}\cdot d\boldsymbol{\ell} = q\left[\Phi(\mathbf{b}) - \Phi(\mathbf{a})\right]$.

Energy stored in electric field: $W = \frac{1}{2}\varepsilon_0\int_V E^2 d\tau = Q^2/2C$.

Multipole expansion: $\Phi(\mathbf{r}) = \frac{q}{4\pi\varepsilon_0 r} + \frac{1}{4\pi\varepsilon_0}\frac{\mathbf{r}\cdot\mathbf{p}}{r^3} + \frac{1}{4\pi\varepsilon_0}\frac{1}{2}\sum_{ij} Q_{ij}\frac{x_i x_j}{r^5} + \ldots$

Field of electric dipole:

$$\mathbf{E}(\mathbf{r}) = \frac{3\hat{\mathbf{r}}(\mathbf{p}\cdot\hat{\mathbf{r}}) - \mathbf{p}}{4\pi\epsilon_0 r^3}$$

Monopole moment: $q = \int \rho(\mathbf{r}) d^3\mathbf{r}$.

Dipole moment: $\mathbf{p} = \int \rho(\mathbf{r})\mathbf{r} d^3\mathbf{r}$.

Quadrupole moment : $Q_{ij} = \int \rho(\mathbf{r})\left[3r_i r_j - r^2\delta_{ij}\right] d^3\mathbf{r}$ (notation: $r_1 = x,\ r_2 = y,\ r_3 = z$).

Parallel-plate capacitor: $C = \varepsilon_0\frac{A}{d}$.

Spherical capacitor: $C = 4\pi\varepsilon_0\frac{ab}{b-a}$.

Cylindrical capacitor: $C = 2\pi\varepsilon_0\frac{L}{\ln(b/a)}$ (for a length L).

Relative permittivity: $\varepsilon_{\text{r}} = 1 + \chi_{\text{e}}$.

Bound charges: $\rho_{\text{b}} = -\nabla\cdot\mathbf{P}$; $\sigma_{\text{b}} = \mathbf{P}\cdot\hat{\mathbf{n}}$.

MAGNETOSTATICS

Relative permeability: $\mu_{\text{r}} = 1 + \chi_{\text{m}}$.

Lorentz force: $\mathbf{F} = q\mathbf{E} + q(\mathbf{v}\times\mathbf{B})$.

Current densities: $I = \int \mathbf{J}\cdot d\mathbf{A}$, $I = \int \mathbf{K}\cdot d\boldsymbol{\ell}$.

Biot-Savart Law: $\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi}\int \frac{I d\boldsymbol{\ell}\times\hat{\mathbf{R}}}{R^2}$ ($\mathbf{R}$ is vector from source point to field point $\mathbf{r}$).

B-field inside of an infinitely long solenoid: $\mathbf{B} = \mu_0 n I\hat{\boldsymbol{\varphi}}$ (n is the number of turns per unit length).

Ampere's law: $\oint \mathbf{B}\cdot d\boldsymbol{\ell} = \mu_0 I_{\text{encl}}$.

Magnetic dipole moment of a planar current distribution: $\mathbf{m} = I\int d\mathbf{a}$.

Force on a magnetic dipole: $\mathbf{F} = \nabla(\mathbf{m} \cdot \mathbf{B})$.

Torque on a magnetic dipole: $\boldsymbol{\tau} = \mathbf{m} \times \mathbf{B}$.

B-field of magnetic dipole: $\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi} \frac{3\hat{\mathbf{r}}(\mathbf{m} \cdot \hat{\mathbf{r}}) - \mathbf{m}}{r^3}$.

Bound currents: $J_{\mathrm{b}} = \nabla \times \mathbf{M}$; $K_{\mathrm{b}} = \mathbf{M} \times \hat{\mathbf{n}}$.

Maxwell's equations in vacuum

$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$	Gauss' law
$\nabla \cdot \mathbf{B} = 0$	no magnetic charge
$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$	Faraday's law
$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \varepsilon_0 \mu_0 \frac{\partial \mathbf{E}}{\partial t}$	Ampere's law with Maxwell's correction

Maxwell's equations in linear, isotropic, and homogeneous media

$\nabla \cdot \mathbf{D} = \rho_{\mathrm{f}}$	Gauss' law
$\nabla \cdot \mathbf{B} = 0$	no magnetic charge
$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$	Faraday's law
$\nabla \times \mathbf{H} = \mathbf{J}_{\mathrm{f}} + \frac{\partial \mathbf{D}}{\partial t}$	Ampere's law with Maxwell's correction

Alternative way of writing Faraday's law: $\oint \mathbf{E} \cdot d\boldsymbol{\ell} = -\frac{d\mathsf{F}_B}{dt}$.

Mutual and self inductance: $\mathsf{F}_2 = M_{21} I_1$; $\mathsf{F} = LI$.

Energy stored in magnetic field: $W = \frac{1}{2}\mu_0^{-1} \int_V B^2 d\tau = \frac{1}{2} L I^2 = \frac{1}{2} \oint \mathbf{A} \cdot \mathbf{I}\, d\ell$.

Wave equations in a conducting medium:

$$\nabla^2 \mathbf{E} = \mu\sigma \frac{\partial \mathbf{E}}{\partial t} + \mu\varepsilon \frac{\partial^2 \mathbf{E}}{\partial t^2}; \qquad \nabla^2 \mathbf{B} = \mu\sigma \frac{\partial \mathbf{B}}{\partial t} + \mu\varepsilon \frac{\partial^2 \mathbf{B}}{\partial t^2}.$$

VECTOR IDENTITIES

Triple Products

(1) $\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = \mathbf{B} \cdot (\mathbf{C} \times \mathbf{A}) = \mathbf{C} \cdot (\mathbf{A} \times \mathbf{B})$

(2) $\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \mathbf{B}(\mathbf{A} \cdot \mathbf{C}) - \mathbf{C}(\mathbf{A} \cdot \mathbf{B})$

Product Rules

(3) $\nabla(fg) = f(\nabla g) + g(\nabla f)$

(4) $\nabla(\mathbf{A} \cdot \mathbf{B}) = \mathbf{A} \times (\nabla \times \mathbf{B}) + \mathbf{B} \times (\nabla \times \mathbf{A}) + (\mathbf{A} \cdot \nabla)\mathbf{B} + (\mathbf{B} \cdot \nabla)\mathbf{A}$

(5) $\nabla \cdot (f\mathbf{A}) = f(\nabla \cdot \mathbf{A}) + \mathbf{A} \cdot (\nabla f)$

(6) $\nabla \cdot (\mathbf{A} \times \mathbf{B}) = \mathbf{B} \cdot (\nabla \times \mathbf{A}) - \mathbf{A} \cdot (\nabla \times \mathbf{B})$

(7) $\nabla \times (f\mathbf{A}) = f(\nabla \times \mathbf{A}) - \mathbf{A} \times (\nabla f)$

(8) $\nabla \times (\mathbf{A} \times \mathbf{B}) = (\mathbf{B} \cdot \nabla)\mathbf{A} - (\mathbf{A} \cdot \nabla)\mathbf{B} + \mathbf{A}(\nabla \cdot \mathbf{B}) - \mathbf{B}(\nabla \cdot \mathbf{A})$

Second Derivatives

(9) $\nabla \cdot (\nabla \times \mathbf{A}) = 0$

(10) $\nabla \times (\nabla f) = 0$

(11) $\nabla \times (\nabla \times \mathbf{A}) = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2\mathbf{A}$

FUNDAMENTAL THEOREMS

Gradient Theorem: $\int_{\mathbf{a}}^{\mathbf{b}} (\nabla f) \cdot d\mathbf{l} = f(\mathbf{b}) - f(\mathbf{a})$

Divergence Theorem: $\int (\nabla \cdot \mathbf{A})\, d\tau = \oint \mathbf{A} \cdot d\mathbf{a}$

Curl Theorem: $\int (\nabla \times \mathbf{A}) \cdot d\mathbf{a} = \oint \mathbf{A} \cdot d\mathbf{l}$

CARTESIAN AND SPHERICAL UNIT VECTORS

$$\hat{\mathbf{x}} = (\sin\theta\cos\phi)\hat{\mathbf{r}} + (\cos\theta\cos\phi)\hat{\boldsymbol{\theta}} - \sin\phi\,\hat{\boldsymbol{\varphi}}$$

$$\hat{\mathbf{y}} = (\sin\theta\sin\phi)\hat{\mathbf{r}} + (\cos\theta\sin\phi)\hat{\boldsymbol{\theta}} + \cos\phi\,\hat{\boldsymbol{\varphi}}$$

$$\hat{\mathbf{z}} = \cos\theta\,\hat{\mathbf{r}} - \sin\theta\,\hat{\boldsymbol{\theta}}$$

INTEGRALS

$$\int_0^\infty \frac{1}{1+bx^2}\,dx = \frac{\pi}{2b^{1/2}}$$

$$\int_0^\infty x^n e^{-bx}\,dx = \frac{n!}{b^{n+1}}$$

$$\int (x^2+b^2)^{-1/2}\,dx = \ln\left(x+\sqrt{x^2+b^2}\right)$$

$$\int (x^2+b^2)^{-1}\,dx = \frac{1}{b}\arctan\left(\frac{x}{b}\right)$$

$$\int (x^2+b^2)^{-3/2}\,dx = \frac{x}{b^2\sqrt{x^2+b^2}}$$

$$\int (x^2+b^2)^{-2}\,dx = \frac{\dfrac{bx}{x^2+b^2}+\arctan\left(\dfrac{x}{b}\right)}{2b^3}$$

$$\int \frac{x\,dx}{x^2+b^2} = \frac{1}{2}\ln\left(x^2+b^2\right)$$

$$\int \frac{dx}{x(x^2+b^2)} = \frac{1}{2b^2}\ln\left(\frac{x^2}{x^2+b^2}\right)$$

$$\int \frac{dx}{a^2x^2-b^2} = \frac{1}{2ab}\ln\left(\frac{ax-b}{ax+b}\right) =$$

$$= -\frac{1}{ab}\operatorname{artanh}\left(\frac{ax}{b}\right)$$

$$\int x^4 e^{-x}\,dx = -e^{-x}\left(x^4+4x^3+12x^2+24x+24\right)$$

$$\int_0^\infty x^n e^{-x}\,dx = n!$$

$$\int_0^\infty e^{-x^2}\,dx = \frac{\sqrt{\pi}}{2\sqrt{a}}$$

$$\int_0^\infty x e^{-x^2}\,dx = \frac{1}{2a}$$

$$\int_0^\infty x^2 e^{-x^2}\,dx = \frac{\sqrt{\pi}}{2a^{3/2}}$$

$$\int_0^\infty x^3 e^{-x^2}\,dx = \frac{1}{2a^2}$$

$$\int_0^\infty x^4 e^{-x^2}\,dx = \frac{3\sqrt{\pi}}{8a^{5/2}}$$

$$\int_0^\infty x^5 e^{-x^2}\,dx = \frac{1}{a^3}$$

$$\int_0^\infty x^6 e^{-x^2}\,dx = \frac{15\sqrt{\pi}}{16a^{7/2}}$$