UNL - Department of Physics and Astronomy

Preliminary Examination - Day 1
Tuesday, May 19, 2026

This test covers the topics of *Quantum Mechanics* (Topic 1) and *Electrodynamics* (Topic 2). Each topic has 4 "A" questions and 4 "B" questions. Work two problems from each group. Thus, you will work on a total of 8 questions today, 4 from each topic.

Note: If you do more than two problems in a group, only the first two (in the order they appear in this handout) will be graded. For instance, if you do problems A1, A3, and A4, only A1 and A3 will be graded.

WRITE YOUR ANSWERS ON ONE SIDE OF THE PAPER ONLY

Quantum Mechanics Group A

Answer only two Group A questions

A1. By using the conservation of energy and momentum, prove that a stationary free electron cannot absorb a photon.

A2. An absent-minded physicist obtained the following solution of the time-dependent Schrödinger equation for a particle in a one-dimensional real potential

$$\Psi(x,t) = N \exp(ikx - \alpha x^2 - i\omega t)$$

Where N, α, k, ω are real parameters. (Don't worry about normalization).

Calculate the corresponding probability density and current density, and prove that this wave function cannot be a solution to the Schrödinger equation

A3. Electrons in plasma can be treated as a free electron gas. For a given electron number density n and temperature T find the condition for treating plasma electrons as classical particles versus quantum particles (degenerate gas). Apply this condition to the atmospheric plasma (typical n=1.5x10^{21} m^{-3}) and to the electron gas in metals (typical n=6.0x10^{28} m^{-3}). Assume the room temperature. In which case the electron gas should be treated quantum-mechanically?

Hint: start with the de Broglie wavelength.

A4. A particle of mass m and energy E is moving in the 1-dimensional potential

$$V=\infty,\ x<0, \qquad V=0,\ x>0.$$

(a) Obtain the wavefunction which is the energy eigenstate. Don't worry about normalization.

(b) Is this wavefunction also a momentum eigenstate?
If yes, find the corresponding momentum. If not, expand the energy eigenstate found in part (a) in momentum eigenstates.

Quantum Mechanics Group B

Answer only two Group B questions

B1. Consider a hydrogen atom in the 2p state.

(a) What are possible values of the projection of angular momentum m for this state?
(b) What is the parity of this state?
(c) Find the expectation values of $\cos\theta$ for all possible values of m, where θ is the spherical angle.
(d) Find the most probable values of θ for all possible values of m.
(e) Find the radial probability density for this state.
(f) Calculate the expectation value of r for this state.
(g) Calculate the most probable value of r.

B2. The Hamiltonian of a one-dimensional charged harmonic oscillator (mass m, frequency ω) in an external static electric field is

$$H=H_0-Fx$$

where H_0 is the Hamiltonian of the unperturbed oscillator, and F is the electric force.

(a) Find the energy spectrum of this Hamiltonian.
(b) Find the ground-state wavefunction for this Hamiltonian.
(c) Suppose at $t<0$ the unperturbed oscillator is in its ground state. At $t=0$ the external field is suddenly switched on. What is the probability that the oscillator remains in its ground state?
(d) Suppose for some value of F and ω the probability is 0.1. How will this value change if the field has increased by a factor of two.

Hint: start with completing square in the Hamiltonian H.

B3. Consider a harmonic oscillator (mass m, frequency ω) at $t=0$ in a superposition state

$$\psi(x) = \frac{1}{\sqrt{2}}[\psi_0(x) + \psi_1(x)]$$

Where ψ_0 and ψ_1 are energy eigenstates.

(a) Calculate the expectation values of position x, momentum p_x, Hamiltonian H and parity P at $t=0$.
(b) Which of the observables listed above are conserved?
(c) Find the wavefunction at $t>0$.
(d) If you decided that some of the observables listed in part (a) above are not conserved, calculate the time dependence of their expectation values.

B4. A particle of mass m is moving in the 1-dimensional potential

$V(x)=\infty$, $x<0$; $V(x)=0$, $0<x<a$; $V(x)=V_0$, $x>0$.

(a) Find the particle's wavefunction if its energy $E<V_0$
(b) Set up an equation allowing determination of the particle's eigenenergies.
(c) Show that in the limit $V_0\to\infty$ the particle's wavefunctions and its energy eigenvalues turn into those for the infinite potential well problem.

Electrodynamics Group A

Answer only two Group A questions

A1. An infinite, linear dielectric slab of thickness d and permittivity ε occupies the region $-d/2 < z < d/2$. The slab is placed in an otherwise uniform electric field

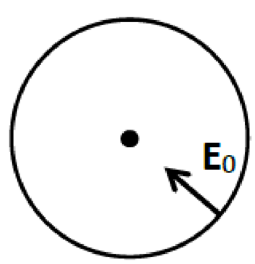
$$\mathbf{E}_0 = E_0\hat{\mathbf{z}}.$$

Assume vacuum outside the slab.

1. Determine the electric field $\mathbf{E}(z)$everywhere.
2. Find the bound surface charge densities at $z = \pm d/2$.
3. Compute the polarization **P** inside the slab and verify consistency with boundary conditions.

A2. A conducting rod of negligible resistance and length $L = 0.50$ slides on two parallel rails with a constant speed of $v = 8.0$ m/s. The rails are connected through a resistor of resistance $R = 4.0\ \Omega$. A uniform magnetic field of magnitude $B = 0.30$ T points perpendicular to the plane of the rails. What external power is required to keep the rod moving at constant speed?

A3. A capacitor comprises a long straight thin wire and a long thin cylindrical shell of radius R. The electric field outside the shell is zero everywhere. The field immediately inside the cylinder has magnitude E_0 and points towards the wire. Calculate the linear charge density on the wire and the surface charge density on the cylinder.



A4. In a certain region of space the free current density is given by $\mathbf{j}=j_0(y^2+z^2)\mathbf{e}_x$ where $j_0=200$ A/m^4. Find the B-field vector at $(x,y,z)=(1,2,1)$ m.

<u>Electrodynamics Group B</u>

Answer only two Group B questions

B1. A grounded conducting sphere of radius R is centered at the origin. A point charge q is placed on the z-axis at position $z = a$, where $a > R$.

1. Use the method of images to find the electrostatic potential outside the sphere.
2. Determine the magnitude and position of the image charge.
3. Find the total induced charge on the conducting sphere.
4. Find the force acting on the point charge q.

B2. A resistor R and capacitor C are connected in series with a battery of emf V_0. At time $t = 0$, the switch is closed and the capacitor is initially uncharged.

1. Find the current $I(t)$ in the circuit.
2. Find the charge $Q(t)$ on the capacitor.
3. Find the voltage across the capacitor $V_C(t)$.
4. Determine the characteristic time scale of the charging process.
5. Find the energy stored in the capacitor after a long time.

B3. A thin spherical shell of radius R carries a uniform surface charge density σ. The shell rotates with a constant angular velocity

$$\boldsymbol{\omega} = \omega\hat{\mathbf{z}}.$$

Assume the rotation is nonrelativistic, so electrostatic charge redistribution can be neglected.

1. Find the surface current density **K**.
2. Using the Biot–Savart law, find the magnetic field at the center of the sphere.

B4. A long solenoid of radius a, length L (L>>a), and number of loops N, is driven by an alternating current $I(t)$.

1. Find the magnetic field inside and outside the solenoid.
2. Find the induced electric field inside and outside the solenoid.

3. Does your calculation of the magnetic field contain an approximation? If yes, find the condition under which this approximation is valid. Check this condition quantitatively if N=1000, L=0.1 m, a=0.01m, $I=I_0\cos(\omega t)$, ω=100 s^{-1}. *Hint*: think about the Maxwell's generalization of the Ampere's law.

Physical constants

Speed of light .... $c = 2.998 \times 10^{8}$ m/s
Planck's constant .... $h = 6.626 \times 10^{-34}$ J·s
Planck's constant / 2π.... $\hbar = 1.055 \times 10^{-34}$ J·s
Electron mass .... $m_e = 9.109 \times 10^{-31}$ kg
Electron's rest energy .... 511.0 keV
Boltzmann constant .... $k_B = 1.381 \times 10^{-23}$ J/K
Compton wavelength ..... $\lambda_C = \dfrac{h}{m_e c} = 2.426$ pm
Elementary charge .... $e = 1.602 \times 10^{-19}$ C
Proton mass .... $m_p = 1.673 \times 10^{-27}\,\text{kg} = 1836\, m_e$
Atomic mass unit .... 1 u=1.66 × 10^{-27} kg
Electric permittivity ...... $\varepsilon_0 = 8.854 \times 10^{-12}$ F/m
Bohr radius..................... $a_0 = \dfrac{4\pi\varepsilon_0 \hbar^2}{e^2 m_e} = 0.5292$ Å
Magnetic permeability.... $\mu_0 = 1.257 \times 10^{-6}$ H/m
Rydberg unit of energy ... $Ry = 13.6$ eV
Rydberg constant……..... R=1.097x10^{7} m^{-1}
1 hartree (= 2 Ry)............. $E_h = \dfrac{\hbar^2}{m_e a_0^2} = 27.21$ eV
Molar gas constant.......... $R = 8.314$ J / mol·K
Gravitational constant...... $G = 6.674 \times 10^{-11}$ m^3 / kg s^2
Avogadro constant .... $N_A = 6.022 \times 10^{23}$ mol^{-1}
hc .... $hc = 1240$ eV·nm
Fine structure constant .. $\alpha = \dfrac{1}{4\pi\varepsilon_0}\dfrac{e^2}{\hbar c}$

Relativistic dynamics
$E=mc^2\gamma$ $\quad$ $p=mv\gamma$ $\quad$ $\gamma=(1-v^2/c^2)^{-1/2}$

TRIGONOMETRY

$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$\sin(2\theta) = 2\sin\theta\cos\theta$

$\cos(2\theta) = \cos^2\theta - \sin^2\theta = 1 - 2\sin^2\theta = 2\cos^2\theta - 1$

$\sin\alpha\sin\beta = \frac{1}{2}\left[\cos(\alpha-\beta) - \cos(\alpha+\beta)\right]$

$\cos\alpha\cos\beta = \frac{1}{2}\left[\cos(\alpha-\beta) + \cos(\alpha+\beta)\right]$

$\sin\alpha\cos\beta = \frac{1}{2}\left[\sin(\alpha+\beta) + \sin(\alpha-\beta)\right]$

$\cos\alpha\sin\beta = \frac{1}{2}\left[\sin(\alpha+\beta) - \sin(\alpha-\beta)\right]$

For small x:

$\sin x \approx x - \frac{1}{6}x^3$

$\cos x \approx 1 - \frac{1}{2}x^2$

$\tan x \approx x + \frac{1}{3}x^3$

QUANTUM MECHANICS

Current density

$$j = \frac{\hbar}{2mi}(\psi^*\nabla\psi - \psi\nabla\psi^*)$$

Ehrenfest theorem

$$\frac{d\langle x\rangle}{dt} = \frac{\langle p\rangle}{m} \qquad \frac{d\langle p\rangle}{dt} = \langle F\rangle$$

Angular momentum: $[L_x, L_y] = i\hbar L_z \quad et\ cycl.$

Wave function of a particle in an infinite square well with the walls at $x=0$ and $x=L$

$$\psi_n(x) = \left(\frac{2}{L}\right)^{1/2} \sin\left(\frac{\pi n}{L}x\right), \quad n = 1, 2, \ldots$$

Table Spherical harmonics and their expressions in Cartesian coordinates.

$Y_{lm}(\theta,\varphi)$	$Y_{lm}(x,y,z)$
$Y_{00}(\theta,\varphi)=\frac{1}{\sqrt{4\pi}}$	$Y_{00}(x,y,z)=\frac{1}{\sqrt{4\pi}}$
$Y_{10}(\theta,\varphi)=\sqrt{\frac{3}{4\pi}}\cos\theta$	$Y_{10}(x,y,z)=\sqrt{\frac{3}{4\pi}}\frac{z}{r}$
$Y_{1,\pm1}(\theta,\varphi)=\mp\sqrt{\frac{3}{8\pi}}e^{\pm i\varphi}\sin\theta$	$Y_{1,\pm1}(x,y,z)=\mp\sqrt{\frac{3}{8\pi}}\frac{x\pm iy}{r}$
$Y_{20}(\theta,\varphi)=\sqrt{\frac{5}{16\pi}}(3\cos^2\theta-1)$	$Y_{20}(x,y,z)=\sqrt{\frac{5}{16\pi}}\frac{3z^2-r^2}{r^2}$
$Y_{2,\pm1}(\theta,\varphi)=\mp\sqrt{\frac{15}{8\pi}}e^{\pm i\varphi}\sin\theta\cos\theta$	$Y_{2,\pm1}(x,y,z)=\mp\sqrt{\frac{15}{8\pi}}\frac{(x\pm iy)z}{r^2}$
$Y_{2,\pm2}(\theta,\varphi)=\sqrt{\frac{15}{32\pi}}e^{\pm 2i\varphi}\sin^2\theta$	$Y_{2,\pm2}(x,y,z)=\mp\sqrt{\frac{15}{32\pi}}\frac{x^2-y^2\pm 2ixy}{r^2}$

Stationary states of harmonic oscillator for $n = 0$ and $n = 1$:

$$\varphi_0(x)=\left(\frac{\alpha}{\pi^{1/2}}\right)^{1/2}\mathrm{e}^{-\frac{\alpha^2x^2}{2}},$$

$$\varphi_1(x)=\left(\frac{\alpha}{\pi^{1/2}}\right)^{1/2}2ax\,\mathrm{e}^{-\frac{\alpha^2x^2}{2}},$$

where $\alpha=\left(\frac{m\omega}{\hbar}\right)^{1/2}$.

Hydrogen atom: $E_n=-\frac{Ry}{n^2},\ Ry=\frac{me^4}{2(4\pi\varepsilon_0)^2\hbar^2}$

Radial functions for the hydrogen atom $R_{nl}(r)$:

$$R_{10}(r)=\frac{2}{a_0^{3/2}}\exp\left(-\frac{r}{a_0}\right),$$

$$R_{20}(r)=\frac{2}{(2a_0)^{3/2}}\left[1-\frac{r}{2a_0}\right]\exp\left(-\frac{r}{2a_0}\right),$$

$$R_{21}(r)=\frac{r}{24^{1/2}a_0^{5/2}}\exp\left(-\frac{r}{2a_0}\right).$$

ELECTROSTATICS

$\oiint_S \mathbf{E}\cdot\hat{\mathbf{n}}\,da = \frac{q_{\text{encl}}}{\varepsilon_0}$; $\mathbf{E} = -\nabla\Phi$; $\int_{\mathbf{r}_1}^{\mathbf{r}_2} \mathbf{E}\cdot d\boldsymbol{\ell} = \Phi(\mathbf{r}_1) - \Phi(\mathbf{r}_2)$; $\Phi(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\frac{q(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}$.

Work done: $W = -\int_{\mathbf{a}}^{\mathbf{b}} q\mathbf{E}\cdot d\boldsymbol{\ell} = q\left[\Phi(\mathbf{b}) - \Phi(\mathbf{a})\right]$.

Energy stored in electric field: $W = \frac{1}{2}\varepsilon_0\int_V E^2 d\tau = Q^2/2C$.

Multipole expansion: $\Phi(\mathbf{r}) = \frac{q}{4\pi\varepsilon_0 r} + \frac{1}{4\pi\varepsilon_0}\frac{\mathbf{r}\cdot\mathbf{p}}{r^3} + \frac{1}{4\pi\varepsilon_0}\frac{1}{2}\sum_{ij} Q_{ij}\frac{x_i x_j}{r^5} + \ldots$

Field of electric dipole:

$$\mathbf{E}(\mathbf{r}) = \frac{3\hat{\mathbf{r}}(\mathbf{p}\cdot\hat{\mathbf{r}}) - \mathbf{p}}{4\pi\epsilon_0 r^3}$$

Monopole moment: $q = \int \rho(\mathbf{r})d^3\mathbf{r}$.

Dipole moment: $\mathbf{p} = \int \rho(\mathbf{r})\mathbf{r}d^3\mathbf{r}$.

Quadrupole moment : $Q_{ij} = \int \rho(\mathbf{r})\left[3r_i r_j - r^2\delta_{ij}\right]d^3\mathbf{r}$ (notation: $r_1 = x,\ r_2 = y,\ r_3 = z$).

Parallel-plate capacitor: $C = \varepsilon_0\frac{A}{d}$.

Spherical capacitor: $C = 4\pi\varepsilon_0\frac{ab}{b-a}$.

Cylindrical capacitor: $C = 2\pi\varepsilon_0\frac{L}{\ln(b/a)}$ (for a length L).

Relative permittivity: $\varepsilon_{\text{r}} = 1 + \chi_{\text{e}}$.

Bound charges: $\rho_{\text{b}} = -\nabla\cdot\mathbf{P}$; $\sigma_{\text{b}} = \mathbf{P}\cdot\hat{\mathbf{n}}$.

MAGNETOSTATICS

Relative permeability: $\mu_{\text{r}} = 1 + \chi_{\text{m}}$.

Lorentz force: $\mathbf{F} = q\mathbf{E} + q(\mathbf{v}\times\mathbf{B})$.

Current densities: $I = \int \mathbf{J}\cdot d\mathbf{A}$, $I = \int \mathbf{K}\cdot d\boldsymbol{\ell}$.

Biot-Savart Law: $\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi}\int\frac{Id\boldsymbol{\ell}\times\hat{\mathbf{R}}}{R^2}$ ($\mathbf{R}$ is vector from source point to field point $\mathbf{r}$).

B-field inside of an infinitely long solenoid: $\mathbf{B} = \mu_0 nI\hat{\boldsymbol{\varphi}}$ (*n* is the number of turns per unit length).

Ampere's law: $\oint \mathbf{B}\cdot d\boldsymbol{\ell} = \mu_0 I_{\text{encl}}$.

Magnetic dipole moment of a planar current distribution: $\mathbf{m} = I\int d\mathbf{a}$.

Force on a magnetic dipole: $\mathbf{F} = \nabla(\mathbf{m}\cdot\mathbf{B})$.

Torque on a magnetic dipole: $\boldsymbol{\tau} = \mathbf{m}\times\mathbf{B}$.

B-field of magnetic dipole: $\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi}\frac{3\hat{\mathbf{r}}(\mathbf{m}\cdot\hat{\mathbf{r}})-\mathbf{m}}{r^3}$.

Bound currents: $J_{\text{b}} = \nabla\times\mathbf{M}$; $K_{\text{b}} = \mathbf{M}\times\hat{\mathbf{n}}$.

<u>Maxwell's equations in vacuum</u>

$\nabla\cdot\mathbf{E} = \frac{\rho}{\varepsilon_0}$	Gauss' law
$\nabla\cdot\mathbf{B} = 0$	no magnetic charge
$\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t}$	Faraday's law
$\nabla\times\mathbf{B} = \mu_0\mathbf{J} + \varepsilon_0\mu_0\frac{\partial\mathbf{E}}{\partial t}$	Ampere's law with Maxwell's correction

<u>Maxwell's equations in linear, isotropic, and homogeneous media</u>

$\nabla\cdot\mathbf{D} = \rho_{\text{f}}$	Gauss' law
$\nabla\cdot\mathbf{B} = 0$	no magnetic charge
$\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t}$	Faraday's law
$\nabla\times\mathbf{H} = \mathbf{J}_{\text{f}} + \frac{\partial\mathbf{D}}{\partial t}$	Ampere's law with Maxwell's correction

Alternative way of writing Faraday's law: $\oint\mathbf{E}\cdot d\boldsymbol{\ell} = -\frac{d\mathsf{F}_B}{dt}$.

Mutual and self inductance: $\mathsf{F}_2 = M_{21}I_1$; $\mathsf{F} = LI$.

Energy stored in magnetic field: $W = \frac{1}{2}\mu_0^{-1}\int_V B^2 d\tau = \frac{1}{2}LI^2 = \frac{1}{2}\oint\mathbf{A}\cdot\mathbf{I}\,d\ell$.

Wave equations in a conducting medium:

$$\nabla^2\mathbf{E} = \mu\sigma\frac{\partial\mathbf{E}}{\partial t} + \mu\varepsilon\frac{\partial^2\mathbf{E}}{\partial t^2};\quad \nabla^2\mathbf{B} = \mu\sigma\frac{\partial\mathbf{B}}{\partial t} + \mu\varepsilon\frac{\partial^2\mathbf{B}}{\partial t^2}.$$

VECTOR IDENTITIES

Triple Products

(1) $\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = \mathbf{B} \cdot (\mathbf{C} \times \mathbf{A}) = \mathbf{C} \cdot (\mathbf{A} \times \mathbf{B})$

(2) $\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \mathbf{B}(\mathbf{A} \cdot \mathbf{C}) - \mathbf{C}(\mathbf{A} \cdot \mathbf{B})$

Product Rules

(3) $\nabla(fg) = f(\nabla g) + g(\nabla f)$

(4) $\nabla(\mathbf{A} \cdot \mathbf{B}) = \mathbf{A} \times (\nabla \times \mathbf{B}) + \mathbf{B} \times (\nabla \times \mathbf{A}) + (\mathbf{A} \cdot \nabla)\mathbf{B} + (\mathbf{B} \cdot \nabla)\mathbf{A}$

(5) $\nabla \cdot (f\mathbf{A}) = f(\nabla \cdot \mathbf{A}) + \mathbf{A} \cdot (\nabla f)$

(6) $\nabla \cdot (\mathbf{A} \times \mathbf{B}) = \mathbf{B} \cdot (\nabla \times \mathbf{A}) - \mathbf{A} \cdot (\nabla \times \mathbf{B})$

(7) $\nabla \times (f\mathbf{A}) = f(\nabla \times \mathbf{A}) - \mathbf{A} \times (\nabla f)$

(8) $\nabla \times (\mathbf{A} \times \mathbf{B}) = (\mathbf{B} \cdot \nabla)\mathbf{A} - (\mathbf{A} \cdot \nabla)\mathbf{B} + \mathbf{A}(\nabla \cdot \mathbf{B}) - \mathbf{B}(\nabla \cdot \mathbf{A})$

Second Derivatives

(9) $\nabla \cdot (\nabla \times \mathbf{A}) = 0$

(10) $\nabla \times (\nabla f) = 0$

(11) $\nabla \times (\nabla \times \mathbf{A}) = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A}$

FUNDAMENTAL THEOREMS

Gradient Theorem: $\int_{\mathbf{a}}^{\mathbf{b}} (\nabla f) \cdot d\mathbf{l} = f(\mathbf{b}) - f(\mathbf{a})$

Divergence Theorem: $\int (\nabla \cdot \mathbf{A})\, d\tau = \oint \mathbf{A} \cdot d\mathbf{a}$

Curl Theorem: $\int (\nabla \times \mathbf{A}) \cdot d\mathbf{a} = \oint \mathbf{A} \cdot d\mathbf{l}$

CARTESIAN AND SPHERICAL UNIT VECTORS

$$\hat{\mathbf{x}} = (\sin\theta\cos\phi)\hat{\mathbf{r}} + (\cos\theta\cos\phi)\hat{\boldsymbol{\theta}} - \sin\phi\,\hat{\boldsymbol{\varphi}}$$
$$\hat{\mathbf{y}} = (\sin\theta\sin\phi)\hat{\mathbf{r}} + (\cos\theta\sin\phi)\hat{\boldsymbol{\theta}} + \cos\phi\,\hat{\boldsymbol{\varphi}}$$
$$\hat{\mathbf{z}} = \cos\theta\,\hat{\mathbf{r}} - \sin\theta\,\hat{\boldsymbol{\theta}}$$

INTEGRALS

$$\int_0^\infty \frac{1}{1+bx^2}dx = \frac{\pi}{2b^{1/2}}$$

$$\int_0^\infty x^n e^{-bx}dx = \frac{n!}{b^{n+1}}$$

$$\int (x^2+b^2)^{-1/2}dx = \ln\left(x+\sqrt{x^2+b^2}\right)$$

$$\int (x^2+b^2)^{-1}dx = \frac{1}{b}\arctan\left(\frac{x}{b}\right)$$

$$\int (x^2+b^2)^{-3/2}dx = \frac{x}{b^2\sqrt{x^2+b^2}}$$

$$\int (x^2+b^2)^{-2}dx = \frac{\frac{bx}{x^2+b^2}+\arctan\left(\frac{x}{b}\right)}{2b^3}$$

$$\int \frac{x\,dx}{x^2+b^2} = \frac{1}{2}\ln\left(x^2+b^2\right)$$

$$\int \frac{dx}{x(x^2+b^2)} = \frac{1}{2b^2}\ln\left(\frac{x^2}{x^2+b^2}\right)$$

$$\int \frac{dx}{a^2x^2-b^2} = \frac{1}{2ab}\ln\left(\frac{ax-b}{ax+b}\right) =$$
$$= -\frac{1}{ab}\operatorname{artanh}\left(\frac{ax}{b}\right)$$

$$\int x^4 e^{-x}dx = -e^{-x}\left(x^4+4x^3+12x^2+24x+24\right.$$

$$\int_0^\infty x^n e^{-x}dx = n!$$

$$\int_0^\infty e^{-x^2}dx = \pi^{1/2}$$

$$\int_0^\infty x^{2n}e^{-x^2}dx = \frac{(2n-1)!!}{2^n}\pi^{1/2}, n = 1, 2, .$$

$$\int_0^\infty x^{2n+1}e^{-x^2}dx = \frac{n!}{2}, n = 0, 1, \ldots$$