A1

For circular orbit

CM 6 $$\frac{mv^2}{r} = \frac{GmM}{r^2}$$

3 $$v = \left(\frac{GM}{r}\right)^{1/2}$$

4. $$\omega = \frac{v}{r} = \frac{(GM)^{1/2}}{r^{3/2}}$$

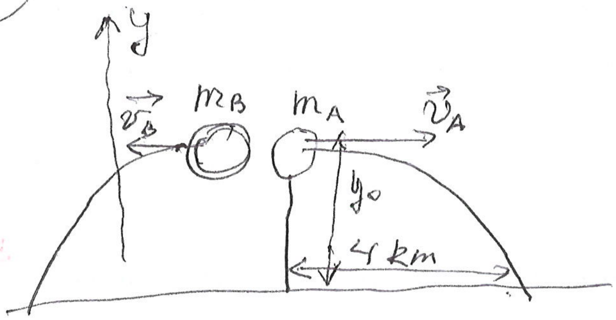
on the earth surface

6 $$\frac{GM}{R^2} = g \rightarrow GM = gR^2$$

3 $$\omega = \frac{Rg^{1/2}}{r^{3/2}}$$

3 $$T = \frac{2\pi}{\omega} = \frac{2\pi r^{3/2}}{Rg^{1/2}}$$

A2 CM



conservation of momentum

5 $$\vec{p}_A + \vec{p}_B = 0$$

2 $$\vec{v}_B = -\frac{m_A}{m_B}\vec{v}_A$$

Eq. of motion in the vertical direction

5 $$y = y_0 - \frac{gt^2}{2} \quad (\text{since } v_y = 0)$$

5 for $y=0$ $t = \sqrt{\frac{2y_0}{g}}$, therefore both fragments hit the ground at the same time

since $v_A t = 4\text{ km}$,

8 $$v_B t = -\frac{m_A}{m_B} v_A t = \begin{cases} -2\text{ km} & \text{if } m_B = 2m_A \\ -0.4\text{ km} & \text{if } m_B = 10\, m_A \end{cases}$$

CM

A3

Eq. for a damped oscillator

setup 3 $\ddot{x} + 2\gamma\dot{x} + \omega_0^2 x = 0$

Complex solution

$$x = e^{\lambda t}$$

solve

$$\lambda^2 + 2\gamma\lambda + \omega_0^2 = 0$$

8

$$\lambda = -\gamma + \sqrt{\gamma^2 - \omega_0^2} = -\gamma + i\,\omega_d\,, \text{ (underdamping)}$$

$$\omega_d^2 = \omega_0^2 - \gamma^2 \qquad (1)$$

General solution in the real form

4 $x = A e^{-\gamma t}\cos(\omega_d t + \phi)$

4 for $t = \overset{n}{\cancel{10}}\,T \qquad A e^{-\overset{n}{\cancel{10}}\gamma T} = A e^{-1}$

3 $\gamma = \dfrac{1}{n\,\cancel{10}\,T} = \dfrac{\omega_d}{\underset{n}{\cancel{10}} \cdot 2\pi} = \dfrac{\omega_d}{2\cancel{0}\pi n}$

3 from (1) $\omega_0^2 = \omega_d^2 + \left(\dfrac{\omega_d}{2\pi n}\right)^2$

A4.   CM

2 $$\vec{v}_{cm} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2} \qquad \vec{v} = \vec{v}_2 - \vec{v}_1$$

solve for $\vec{v}_1$ and $\vec{v}_2$

3 $$\begin{cases} m_1\vec{v}_1 + m_2\vec{v}_2 = m\vec{v}_{cm} \\ -\vec{v}_1 + \vec{v}_2 = \vec{v} \end{cases} \qquad m = m_1 + m_2$$

solve

8

$$\frac{m_2}{m_1}\vec{v}_2 + \vec{v}_2 = \frac{m}{m_1}\vec{v}_{cm} + \vec{v}$$

$$\vec{v}_2 = \frac{\frac{m}{m_1}\vec{v}_{cm} + \vec{v}}{1 + \frac{m_2}{m_1}} = \vec{v}_{cm} + \frac{m_1}{m}\vec{v}$$

$$\vec{v}_1 = \vec{v}_2 - \vec{v} = \vec{v}_{cm} - \frac{m_2}{m}\vec{v}$$

setup 2 $$\frac{m_1 v_1^2}{2} + \frac{m_2 v_2^2}{2} = \frac{m_1}{2}\left(v_{cm}^2 + \left(\frac{m_2}{m}\right)^2 v^2 - \frac{2m_2}{m}\vec{v}_{cm}\cdot\vec{v}\right)$$

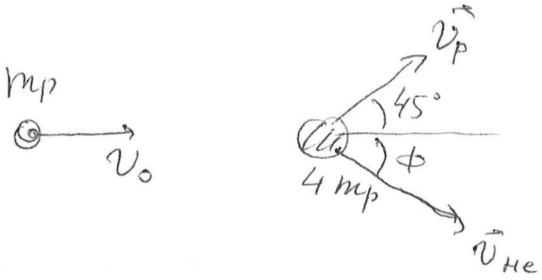
$$+ \frac{m_2}{2}\left(v_{cm}^2 + \left(\frac{m_1}{m}\right)^2 v^2 + \frac{2m_1}{m}\vec{v}_{cm}\cdot\vec{v}\right)$$

algebra

10

$$= \frac{m v_{cm}^2}{2} + \frac{1}{2}\,\frac{m_1 m_2^2 + m_2 m_1^2}{(m_1 + m_2)^2}\,v^2 = \frac{m v_{cm}^2}{2} + \frac{1}{2}\mu v^2$$

~~7.14~~ BI CM



cons. of momentum

4 $$m_p v_0 = m_p v_p \cos 45^\circ + 4 m_p v_{He} \cos\phi \qquad (1)$$

$$v_p \sin 45^\circ = v_{He} \sin\phi \qquad (2)$$

$$\cos 45^\circ = \sin 45^\circ = \frac{1}{\sqrt{2}}$$

cons. of energy

2 $$\frac{1}{2} m_p v_0^2 = \frac{1}{2} m_p v_p^2 + \overset{\cancel{4}}{2} m_p v_{He}^2 \qquad (3)$$

From first two equations

$$\frac{v_p}{\sqrt{2}} - v_{He} \sin\phi = 0$$

$$\frac{v_p}{\sqrt{2}} + 4 v_{He} \cos\phi = v_0$$

Solve for V_{He}, V_p 5

subtract first from second

$$v_{He}(4\cos\phi + \sin\phi) = v_0 \longrightarrow v_{He} = \frac{v_0}{4\cos\phi + \sin\phi}, \quad v_p = \frac{v_0\sqrt{2}\sin\phi}{4\cos\phi + \sin\phi}$$

eq. for φ

substitute into (3), $v_0^2 = v_p^2 + 4 v_{He}^2$

8 $$v_0^2 (4\cos\phi + \sin\phi)^2 = 2 v_0^2 \sin^2\phi + 4 v_0^2$$

$$16\cos^2\phi + \sin^2\phi + 8\cos\phi\sin\phi = 2\sin^2\phi + 4(\cos^2\phi + \sin^2\phi)$$

$$12\cos^2\phi - 5\sin^2\phi + 8\cos\phi\sin\phi = 0$$

solve 4 $$5\tan^2\phi - 8\tan\phi - 12 = 0$$

$$\tan\phi = \frac{4 \pm \sqrt{16+60}}{5} = 2.5436$$ (negative root is unphysical)

$$\phi = 1.1962 \text{ rad} \qquad 4\cos\phi + \sin\phi = 2.3942$$

2 $$v_{He} = \frac{v_0}{2.3942} = 0.4177 v_0 \qquad v_p = \frac{v_0\sqrt{2}\sin\phi}{2.3942} = 0.5496 v_0$$

(CM) (B2.)

(CM)

motions in x and y directions are independent, therefore

$x = A_1 \cos\omega t + B_1 \sin\omega t, \quad \omega = \sqrt{\frac{k}{m}}$

5

$y = A_2 \cos 2\omega t + B_2 \sin 2\omega t$

3

$\dot{x} = -A_1\omega \sin\omega t + B_1\omega\cos\omega t$

$\dot{y} = -2A_2\omega \sin 2\omega t + 2B_2\omega\cos 2\omega t$

2. $t = 0$ $\quad A_1 = a \quad A_2 = 0$

$B_1 = 0 \quad 2B_2\omega = v_0$

5

$x = a\cos\omega t$

$\dot{x} = -a\omega\sin\omega t$

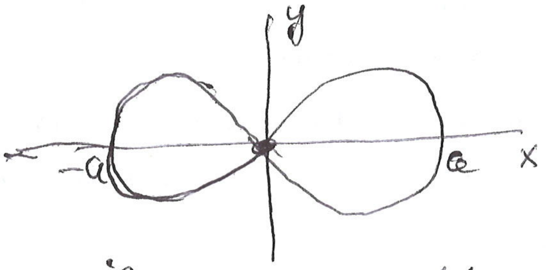
$y = \frac{v_0}{2\omega}\sin 2\omega t \quad \dot{y} = v_0\cos 2\omega t$

3. xy plane

5

$y = \frac{v_0}{\omega}\sin\omega t\cos\omega t = \pm\frac{v_0}{\omega}\frac{x}{a}\sqrt{1-\frac{x^2}{a^2}}$

3



Symmetoric in x and y axis

4. 2 $\frac{x^2}{a^2} + \frac{\dot{x}^2}{(a\omega)^2} = 1$ - ellipse 2

(CM) (B3)

3 $V = mg\frac{l}{2}(1-\cos\theta)$

4 1. $L = \frac{I\dot{\theta}^2}{2} - \frac{mgl}{2}(1-\cos\theta)$

Lagr equation of motion

4 $I\ddot{\theta} + mg\frac{l}{2}\sin\theta = 0$

$I = \frac{1}{3}ml^2$, $\ddot{\theta} + \frac{3g}{2l}\sin\theta = 0$

2. For small θ the general solution is

$\theta = A\cos\omega t + B\sin\omega t$ $\omega = \sqrt{\frac{3g}{2l}}$

2 $\dot{\theta} = -\omega A\sin\omega t + \omega B\cos\omega t$

initial conditions: $t=0$: $\theta = 0$, $\dot{\theta} = \frac{v_0}{l}$

$\rightarrow$ $A = 0$ $\omega B = \frac{v_0}{l}$, $B = \frac{v_0}{l\omega}$

4 $\theta = \frac{v_0}{l\omega}\sin\omega t = \frac{v_0}{\sqrt{\frac{3}{2}gl}}\sin\omega t$

3. 3 Small amplitude: $v_0 \ll \sqrt{\frac{3}{2}gl}$

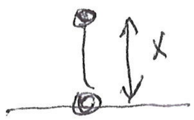
4. From the conservation of energy

$$\frac{I[\dot{\theta}(0)]^2}{2} > mgl$$

5 $\frac{ml^2}{6}\frac{v_0^2}{l^2} > mgl$

$v_0^2 > 6gl$ or $v_0 > \sqrt{6gl}$

B4. CM



1. 3 $m\dot{v} = -cv - mg$

$$-\frac{m\,dv}{cv+mg} = dt$$

$$-\frac{m}{c}\ln\left(v+\frac{mg}{c}\right) = t + \text{const}$$

$$v + \frac{mg}{c} = Ce^{-\frac{ct}{m}}$$

3 $$v = Ce^{-\frac{ct}{m}} - \frac{mg}{c}$$

~~$C = mg$~~ $v_0 = C - \frac{mg}{c}$

c: $\frac{M}{T}$ 2 $C = v_0 + \frac{mg}{c}$ $$\boxed{v = \left(v_0 + \frac{mg}{c}\right)e^{-\frac{ct}{m}} - \frac{mg}{c}}$$

2 $$x = -\frac{m}{c}\left(v_0 + \frac{mg}{c}\right)e^{-\frac{ct}{m}} - \frac{mg}{c}t + \text{const}$$

$x(0) = 0 \quad -\frac{m}{c}\left(v_0 + \frac{mg}{c}\right) + \text{const} = 0$

2 $$\text{const} = \frac{m}{c}\left(v_0 + \frac{mg}{c}\right)$$

$$\boxed{x = \frac{m}{c}\left(v_0 + \frac{mg}{c}\right)\cancel{e}^{c}\left(1 - e^{-ct/m}\right) - \frac{mg}{c}t}$$

2. max height: 2 $v = 0 \to \left(v_0 + \frac{mg}{c}\right)e^{-\frac{ct}{m}} = \frac{mg}{c}$

2 $$e^{-\frac{ct}{m}} = \frac{\frac{mg}{c}}{v_0 + \frac{mg}{c}} = \frac{1}{\frac{cv_0}{mg} + 1} \qquad t = \frac{m}{c}\ln\left(1 + \frac{cv_0}{mg}\right)$$

3. 4 $$x_{max} = \frac{m}{c}\left(v_0 + \frac{mg}{c}\right)\left(1 - \frac{1}{\frac{cv_0}{mg} + 1}\right) - \frac{mg}{c}\frac{m}{c}\ln\left(1 + \frac{cv_0}{mg}\right)$$

$$= \frac{m}{c}\left(v_0 + \frac{mg}{c}\right)\frac{\frac{cv_0}{mg}}{\frac{cv_0}{mg} + 1} - \frac{m^2 g}{c^2}\ln\left(1 + \frac{cv_0}{mg}\right)$$

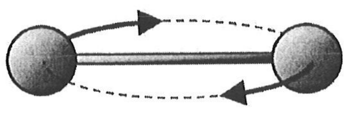
$$\boxed{x_{max} = \frac{mv_0}{c} - \frac{m^2 g}{c^2}\ln\left(1 + \frac{cv_0}{mg}\right)}$$

5 4. en. dissipated $\frac{mv_0^2}{2} - mg\left[\frac{mv_0}{c} - \frac{m^2 g}{c^2}\ln\left(1 + \frac{cv_0}{mg}\right)\right]$

$$= \frac{mv_0^2}{2} - \frac{m^2 g}{c}\left[v_0 - \frac{mg}{c}\ln\left(1 + \frac{cv_0}{mg}\right)\right]$$

(B4:) (Th)

~~B3~~ The quantum mechanical energy levels of a rotating dumbbell are given by



$$E_\ell = B\ell(\ell+1) \qquad \text{for } \ell = 0,1,2,3,4,\ldots$$

where B is the rotational constant. For each value of ℓ there are $2\ell+1$ possible quantum states of the same energy.

a. Give the partition function Z.

In parts *b.* and *c.* we assume $T = 2B/k_B$.

b. Make a table of the probabilities P_ℓ that the energy of the dumbbell is E_ℓ, for the values $\ell = 0$, 1, 2, and 3 only. Explain why higher values of ℓ are barely relevant at this temperature.

c. Give the average energy of the dumbbell in terms of T.

d. What is the average energy for $T \ll B/k_B$?

$1.\ 3\ B = \frac{\hbar^2}{2I}$ since $H = \frac{\vec{L}^2}{2I}$

ANSWERS

$4\ 2.\ m = -l, \ldots, l$ degeneracy $= 2l+1$

$3.\ Z = \sum_{\ell=0}^{\infty}(2\ell+1)e^{-\beta B\ell(\ell+1)}$

4 ~~b.~~

$$T = 2B/k_B \Rightarrow \frac{B}{k_B T} = \tfrac{1}{2} = \beta B \Rightarrow$$

$5\ 4.\ Z = \sum_{\ell=0}^{\infty}(2\ell+1)e^{-\frac{1}{2}\ell(\ell+1)} = 1 + 3e^{-1} + 5e^{-3} + 7e^{-6} + 9e^{-10} + \ldots \approx 1 + 3e^{-1} + 5e^{-3} + 7e^{-6} = 2.36992$

At this temperature, the terms $(2\ell+1)e^{-\frac{1}{2}\ell(\ell+1)}$ in the partition function rapidly approach zero for increasing values of ℓ. The probability for particles to be in higher energy states $\ell \geq 4$ becomes negligibly small. The term for $\ell+1$ is smaller than the term for ℓ by a factor of approximately $\frac{1}{e^{\ell+1}}$, and for $\ell = 3$ this is already a factor ~ 0.01 .

5 – ~~4~~ We decide to take the value Z = 2.36992 and calculate the probabilities:

$P_0 = 42.20\%$ $P_1 = 46.57\%$ $P_2 = 10.50\%$ $P_3 = 0.73\%$

$4\ 5.\ E_{av} = \sum_{l=0}^{3} \underbrace{\left(\tfrac{1}{2} k_B T\right)}_{B} P_l \cdot l(l+1) = B(P_0 \cdot 0 + P_1 \cdot 2 + P_2 \cdot 6 + P_3 \cdot 12)$

$= 1.65\,B$

A1. 300 g of aluminum block at 100 °C is placed in a calorimeter cup with 400 g of water. The mass of the copper calorimeter cup is 80 g. The initial temperature of the water and the cup is 22 °C. What's the final temperature? ($c_{\text{water}} = 4.184\ \text{J/g}\cdot\text{K}$; $c_{\text{Al}} = 0.9\ \text{J/g}\cdot\text{K}$ $c_{\text{Cu}} = 0.385\ \text{J/g}\cdot\text{K}$)

Solution: (25 pt)

The heat input of the water and the cup is the same as the heat output of the aluminum block. Assuming the final temperature is *T*:

$(100 - T)C_{\text{Al}} = (T - 22)\,(C_{\text{water}} + C_{\text{Cu}})$ (5 pt)
$C_{\text{Al}} = 300 \times 0.9\ \text{J/K} = 270\ \text{J/K}$ (5 pt)
$C_{\text{water}} = 400 \times 4.184\ \text{J/K} = 1674\ \text{J/K}$ (5 pt)
$C_{\text{Cu}} = 80 \times 0.385\ \text{J/K} = 30.8\ \text{J/K}$ (5 pt)

Hence: $T = 32.7$ °C (5 pt)

A2. Consider an ideal gas system that changes temperature from T_1 to T_2. Show that the entropy change of the isobaric process is γ times of that of the isochoric process, where $\gamma = C_P/C_V$.

Solution: (25 pt)

To calculate entropy change, we use:

$dS = \bar{d}Q/T$, and $\bar{d}Q = dU + \bar{d}W = dU + PdV$. (5 pt)

In isochoric process: $dV = 0$, $\bar{d}Q = dU = C_v dT$,

$dS = \bar{d}Q/T = C_v dT/T$. (5 pt)

In isobaric process: $\bar{d}Q = C_v dT + PdV$

Because *P* is constant: $PdV = d(PV)$, $\bar{d}Q = C_v dT + d(PV)$ (5 pt)

Because $PV = nRT$, $\bar{d}Q = C_v dT + d(nRT) = C_v dT + nRdT = (C_v + nR)dT$ (5 pt)

$dS = \bar{d}Q/T = (C_v + nR)dT/T$.

Therefore, the ratio of the entropy change is

$dS_{isobaric}/dS_{isochoric} = (C_v + nR)/C_v = C_p/C_v = \gamma$. (5 pt)

A3. Show that for any *PVT* system with two degrees of freedom, the equation of state can be found using $\ln(V) = \int \alpha\, dT - k_T dP$, where $\alpha = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P$ is the thermal expansion coefficient, $k_T = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T$ is the isothermal compressibility.

Solution: (25 pt)

$$dV = \left(\frac{\partial V}{\partial T}\right)_P dT + \left(\frac{\partial V}{\partial P}\right)_T dP. \qquad \text{(5 pt)}$$

Hence:

$$\frac{dV}{V} = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P dT + \frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T dP. \qquad \text{(5 pt)}$$

Therefore,

$$\ln(V) = \int \left(\frac{\partial \ln V}{\partial T}\right)_P dT - \left(\frac{\partial \ln V}{\partial P}\right)_T dP. \qquad \text{(5 pt)}$$

We can rewrite the definition:

$$\alpha = \left(\frac{\partial \ln V}{\partial T}\right)_P, \qquad \text{(5 pt)}$$

$$k_T = -\left(\frac{\partial \ln V}{\partial P}\right)_T, \qquad \text{(5 pt)}$$

which means:

$$\ln(V) = \int \alpha dT - k_T dP.$$

B1. Using the relation $\left(\frac{\partial U}{\partial V}\right)_T = T\left(\frac{\partial P}{\partial T}\right)_V - P$, show that the Joule Thompson coefficient $\left(\frac{\partial T}{\partial P}\right)_H = 1/C_P[T\left(\frac{\partial V}{\partial T}\right)_P - V]$ in the throttling process. (Hint: use the cyclic rule).

Solution: (25 pt)

Use the cyclic rule, we have:

$$\left(\frac{\partial T}{\partial P}\right)_H = -\left(\frac{\partial T}{\partial H}\right)_P\left(\frac{\partial H}{\partial P}\right)_T = -\left(\frac{\partial H}{\partial P}\right)_T/\left(\frac{\partial H}{\partial T}\right)_P = -\frac{1}{C_P}\left(\frac{\partial H}{\partial P}\right)_T. \qquad \text{(3 pt)}$$

Here,

$$\left(\frac{\partial H}{\partial P}\right)_T = \left(\frac{\partial U}{\partial P}\right)_T + \left[\frac{\partial (PV)}{\partial P}\right]_T. \quad (1) \qquad \text{(3 pt)}$$

$$\left(\frac{\partial U}{\partial P}\right)_T = \left(\frac{\partial U}{\partial V}\right)_T\left(\frac{\partial V}{\partial P}\right)_T \qquad \text{(3 pt)}$$

Given $\left(\frac{\partial U}{\partial V}\right)_T = T\left(\frac{\partial P}{\partial T}\right)_V - P$, this becomes:

$$\left(\frac{\partial U}{\partial P}\right)_T = \left[T\left(\frac{\partial P}{\partial T}\right)_V - P\right]\left(\frac{\partial V}{\partial P}\right)_T = T\left(\frac{\partial P}{\partial T}\right)_V\left(\frac{\partial V}{\partial P}\right)_T - P\left(\frac{\partial V}{\partial P}\right)_T. \qquad \text{(3 pt)}$$

Using the cyclic rule, it follows that:

$$\left(\frac{\partial P}{\partial T}\right)_V\left(\frac{\partial V}{\partial P}\right)_T = -\left(\frac{\partial V}{\partial T}\right)_P. \qquad \text{(3 pt)}$$

Hence,

$$\left(\frac{\partial U}{\partial P}\right)_T = -T\left(\frac{\partial V}{\partial T}\right)_P - P\left(\frac{\partial V}{\partial P}\right)_T. \quad (2) \qquad \text{(3 pt)}$$

We also have:

$$\left[\frac{\partial (PV)}{\partial P}\right]_T = V + P\left(\frac{\partial V}{\partial P}\right)_T. \quad (3) \qquad \text{(3 pt)}$$

Substitute (2) and (3) into Eq. (1), one gets

$$\left(\frac{\partial H}{\partial P}\right)_T = -T\left(\frac{\partial V}{\partial T}\right)_P - P\left(\frac{\partial V}{\partial P}\right)_T + V + P\left(\frac{\partial V}{\partial P}\right)_T = -T\left(\frac{\partial V}{\partial T}\right)_P + V. \quad \text{(3 pt)}$$

Hence, the Joule Thompson coefficient is

$$\left(\frac{\partial T}{\partial P}\right)_H = -\frac{1}{C_P}\left(\frac{\partial H}{\partial P}\right)_T = -\frac{1}{C_P}\left[-T\left(\frac{\partial V}{\partial T}\right)_P + V\right] = \frac{1}{C_P}\left[T\left(\frac{\partial V}{\partial T}\right)_P - V\right] \qquad \text{(1 pt)}$$

B2. Consider a gas system that satisfies $\left(\frac{\partial v}{\partial T}\right)_P = \frac{R}{P} + \frac{a}{T^2}$ and $\left(\frac{\partial v}{\partial P}\right)_T = -Tf(P)$, where v is the molar volume, a and R are constants, and $f(p)$ is a function of P. Show that:

(a) $f(P) = \frac{R}{P^2}$.

(b) The equation of state of this system is $Pv = RT - aP/T$.

Solution: (25 pt)

(a) Because the 2nd order partial derivative:

$$\frac{\partial^2 v}{\partial T \partial P} = \left(\partial \left(\frac{\partial v}{\partial T}\right)_P / \partial P\right)_T = \left(\partial \left(\frac{\partial v}{\partial P}\right)_T / \partial T\right)_P, \qquad \text{(3 pt)}$$

and $\left(\partial \left(\frac{\partial v}{\partial T}\right)_P / \partial P\right)_T = -R/P^2$ (3 pt)

$$\left(\partial \left(\frac{\partial v}{\partial P}\right)_T / \partial T\right)_P = -f(P), \qquad \text{(3 pt)}$$

one has

$f(P) = R/P^2.$ (3 pt)

(b) We can integrate to find the *v*,

$$v = \int \left(\frac{\partial v}{\partial T}\right)_P dT = \int [R/P + a/T^2]\, dT = RT/P - a/T + f(P) + c, \qquad \text{(3 pt)}$$

$$v = \int \left(\frac{\partial v}{\partial P}\right)_T dP = \int [-Tf(P)] dP = \int [-TR/P^2] dP = RT/P + g(T) + c. \qquad \text{(3 pt)}$$

After comparison, one gets

$f(P) = 0, g(T) = -a/T.$ (2 pt)

Hence

$$v = \frac{RT}{P} - \frac{a}{T} + c. \qquad \text{(2 pt)}$$

For gas system at high-*T* limit, $v = RT/P$. So, $c = 0$. (2 pt)

Finally, the equation of state is

$v = RT/P - a/T.$ (1 pt)

B3 2nd page.

(8)

$$(5) \Rightarrow \left(P+\frac{a}{V^2}\right)=\frac{RT}{V-b} \Rightarrow P=\frac{RT}{V-b}-\frac{a}{V^2}$$

$$\Rightarrow \left(\frac{\partial P}{\partial T}\right)_V=\frac{R}{V-b} \qquad (6)$$

and $\left(\frac{\partial V}{\partial T}\right)_P=\frac{1}{\left(\frac{\partial T}{\partial V}\right)_P}$

$$(5) \Rightarrow T=\left(P+\frac{a}{V^2}\right)\frac{V-b}{R} \quad \text{or} \quad P+\frac{a}{V^2}=\frac{RT}{V-b}$$

then: $\left(\frac{\partial T}{\partial V}\right)_P=-\frac{2a}{V^3}\,\frac{V-b}{R}+\frac{P+\frac{a}{V^2}}{R}=\frac{RT}{(V-b)R}-\frac{2a(V-b)}{RV^3}$

$$\Rightarrow \left(\frac{\partial V}{\partial T}\right)_P=R\Big/\left[\frac{RT}{V-b}-\frac{2a(V-b)}{V^3}\right] \qquad (7)$$

(4) in (3): $C_p-C_v=\left[T\left(\frac{\partial P}{\partial T}\right)_V\right]\left(\frac{\partial V}{\partial T}\right)_P$

and using (6) and (7), one gets:

$$C_p-C_v=\frac{R^2T}{(V-b)\left[\frac{RT}{V-b}-\frac{2a(V-b)^2}{V^3}\right]}$$

$$\boxed{C_p-C_v=\frac{R}{1-\frac{2a(1-b/V)^2}{VRT}}} \xrightarrow[V\to\infty]{} R$$

, which is the result for an ideal gas.

B3 1^{st} page

⑦

$$C_v = \left(\frac{\partial U}{\partial T}\right)_v \quad , \quad C_p = \left(\frac{\partial H}{\partial T}\right)_p$$

a) show that $C_p - C_v = \left[p + \left(\frac{\partial U}{\partial V}\right)_T\right]\left(\frac{\partial V}{\partial T}\right)_p$

Starting with the first law of thermodynamics, one has: $H = U + pV$

$$\Longrightarrow \left(\frac{\partial H}{\partial T}\right)_p = \left(\frac{\partial U}{\partial T}\right)_p + p\left(\frac{\partial V}{\partial T}\right)_p . \qquad (1)$$

Let $U = U[T, V(T,p)]$, then

$$\left(\frac{\partial U}{\partial T}\right)_p = \left(\frac{\partial U}{\partial T}\right)_v + \left(\frac{\partial U}{\partial V}\right)_T \left(\frac{\partial V}{\partial T}\right)_p \qquad (2)$$

Inserting (2) in (1) leads to:

$$\left(\frac{\partial H}{\partial T}\right)_p = \left(\frac{\partial U}{\partial T}\right)_v + \left(\frac{\partial U}{\partial V}\right)_T \left(\frac{\partial V}{\partial T}\right)_p + p\left(\frac{\partial V}{\partial T}\right)_p$$

$$\Longrightarrow \boxed{C_p - C_v = \left[p + \left(\frac{\partial U}{\partial V}\right)_T\right]\left(\frac{\partial V}{\partial T}\right)_p} \qquad (3)$$

b) Use the relation $p + \left(\frac{\partial U}{\partial V}\right)_T = T\left(\frac{\partial p}{\partial T}\right)_V$ (4) to find $C_p - C_v$ for a Van der Waals gas:

$$\left(p + \frac{a}{V^2}\right)(V - b) = RT \qquad (5)$$

A4

⑥ pb # 5 : Thermo (easy)

(a) $$W = \int_{V_0}^{10V_0} p\,dV = \int_{V_0}^{10V_0} \frac{RT}{V}\,dV = RT\ln(10) = 5.2\times10^{3}\,J$$

since $pV \underset{\substack{\uparrow \\ n=1 \text{ mole.}}}{=} RT$ and $T = 0°C = 273\,K$

$R = 8.314\ J.mol^{-1}.K^{-1}$

(b) adiabatic process: $pV^{\gamma} = \text{constant}$, $\gamma = 5/3$ for an ideal gas molecule, which is monoatomic.

Ideal gas: $pV = RT$.

$$pV^{\gamma} = \text{const} \iff pV\,V^{\gamma-1} = \text{constant}$$
$$\iff RT\,V^{\gamma-1} = \text{constant}$$
$$\iff T\,V^{\gamma-1} = \text{constant}$$
$$\Rightarrow T_i V_i^{\gamma-1} = T_f V_f^{\gamma-1}$$
$$\Rightarrow \boxed{T_f = T_i\left(\frac{V_i}{V_f}\right)^{\gamma-1}}$$

$T_i = 0° = 273K$, $V_i = V_0$, $V_f = 10V_0$, $\gamma = 5/3$

$$\Rightarrow \underline{\underline{T_f = 59\,K \text{ or } -214\,°C}}$$