Preliminary Thermal – August 2024

Easy Problems:

A1. For a wire with equation of state $F = bT\left(\frac{L}{L_0} - \frac{L_0^2}{L^2}\right)$, where L is the length, $L_0(T)$ is the length when the tension F is zero, b is a constant. Calculate the work done by the environment when the length changes from L_0 to $\frac{L_0}{2}$ at constant temperature.

Solution:

$$W = \int F dL = \int_{L_0}^{\frac{L_0}{2}} bT\left(\frac{L}{L_0} - \frac{L_0^2}{L^2}\right) dL$$

$$= bT\left(\frac{L^2}{2L_0} + \frac{L_0^2}{L}\right)\Bigg|_{L_0}^{\frac{L_0}{2}} = \frac{5}{8} bTL_0$$

A2. At low temperature, the constant volume molar specific heat of a solid is $C_v = a\left(\frac{T}{\theta_D}\right)^3$, where θ_D is called Debye temperature. Consider a solid with a = 1.94 kJ/mol/K and θ_D = 27 °C, calculate the heat per mole the solid needs to absorb to increase temperature from 5 K to 10 K.

Solution:

$$Q = \int n\, C_v\, dT = \int_5^{10} 1.94 \times 10^3 \left(\frac{T}{300}\right)^3 dT = 0.168\, J.$$

A3. Calculate $C_P = \left(\frac{\partial H}{\partial T}\right)_P$ using the chain rule for a simple solid for which $H(T, V) = Mc_0 T + (a_0 T - b_0)V$ and $V = V_0 \exp[(a_0 T - P)/b_0]$. Show that the result is $C_P = Mc_0 + \frac{a_0^2}{b_0} TV$.

Solution:

Using the chain rule, we have

$$C_P = \left(\frac{\partial H}{\partial T}\right)_P = \left(\frac{\partial H}{\partial T}\right)_V + \left(\frac{\partial H}{\partial V}\right)_T \left(\frac{\partial V}{\partial T}\right)_P$$

$$\left(\frac{\partial H}{\partial T}\right)_V = Mc_0 + a_0 V$$

$$\left(\frac{\partial H}{\partial V}\right)_T = a_0 T - b_0$$

$$\left(\frac{\partial V}{\partial T}\right)_P = \frac{a_0}{b_0} V$$

Therefore,

$$C_P = Mc_0 + a_0 V + a_0 T \frac{a_0}{b_0} V - b_0 \frac{a_0}{b_0} V = Mc_0 + \frac{a_0^2}{b_0} TV$$

Problem 5: (Easy) key

the state equation of ideal gas is

$$pV = nRT$$

The equation of adiabatic process is

$$p\left(\frac{V}{V_0}\right)^{\gamma} = P_0 \qquad \underline{P_0^{He} = P_0^{air}}$$

where $\gamma = C_P/C_V$, P_0 and p are starting and final pressures, respectively, and V_0 ~~ad~~ and V are volumes. Because $V_0 > V$ and $\gamma_{He} > \gamma_{air}$

one gets:

$$\frac{T_{He}}{T_{air}} = \frac{P_{He}}{P_{air}} = \frac{\left(\frac{V_0}{V}\right)^{\gamma_{He}}}{\left(\frac{V_0}{V}\right)^{\gamma_{air}}} = \left(\frac{V_0}{V}\right)^{\gamma_{He}-\gamma_{air}}$$

$$\frac{T_{He}}{T_{air}} = \left(\frac{V_0}{V}\right)^{\frac{7}{5}-\frac{5}{3}} = \left(\frac{V_0}{V}\right)^{\frac{6}{15}} > 1$$

since $V_0 > V$. $\Rightarrow$ $\underline{\underline{T_{He} > T_{air}}}$

Hard Problems:

B1. For ideal gas, if the heat capacity of a process is constant, then the process is polytropic $PV^l = constant$. Assuming that C_p and C_v are constants, find *l*.

Solution:

$$dU = C_n dT - PdV$$

$$(C_v - C_n)dT = -PdV = -nRT\frac{dV}{V}$$

$$(C_v - C_n)\frac{dT}{T} = -nR\frac{dV}{V}$$

$$(C_v - C_n)\ln T = -nR \ln V + constant$$

$$\ln(T^{C_v - C_n}) + \ln(V^{nR}) = constant$$

$$\exp[\ln(T^{C_v - C_n}) + \ln(V^{nR})] = constant$$

$$T^{C_v - C_n}V^{nR} = constant$$

$$(PV/nR)^{C_v - C_n}V^{nR} = constant$$

$$P^{C_v - C_n}V^{C_v - C_n + nR} = constant$$

$$P^{C_v - C_n}V^{C_p - C_n} = constant$$

$$P\,V^l = constant$$

$$l = (C_p - C_n)/(C_v - C_n)$$

B2. In the Einstein model of specific heat for diatomic molecules, atomic vibrations can be treated as independent quantum oscillators with quantized energy: $\mathcal{E}_n = n\hbar\omega$ and the population of excited oscillators follow the Boltzmann distribution.

a) At thermal equilibrium, show that the average energy for the quantum oscillators is given by: $\bar{\mathcal{E}} = \frac{\hbar\omega}{e^{\hbar\omega/k_BT} - 1}$.

b) Find expression for the specific heat C_v of 1 mole of the diatomic molecules. Show that it approaches the $3R$ limit at high temperatures, where $R = N_A k_B$ is the universal gas constant.

Solution:

a) For the Boltzmann distribution $e^{-\mathcal{E}_n/k_BT}$

$$\bar{\mathcal{E}} = \frac{\sum_{n=0}^{\infty}\mathcal{E}_n e^{-\mathcal{E}_n/k_BT}}{\sum_{n=0}^{\infty} e^{-\mathcal{E}_n/k_BT}} = -\frac{\partial}{\partial\beta} ln\left[\sum_{n=0}^{\infty} e^{-\mathcal{E}_n\beta}\right] \text{ with } \beta = \frac{1}{k_BT}$$

$$\sum_{n=0}^{\infty} e^{-n\hbar\omega\beta} = \frac{1}{1 - e^{-\hbar\omega\beta}}$$

$$\bar{\mathcal{E}} = \frac{\partial}{\partial\beta} ln\left[1 - e^{-\hbar\omega\beta}\right] = \frac{\hbar\omega}{e^{\hbar\omega/k_{BT}} - 1}$$

b) For 1 mole of the molecules,

$$U = 3N_A \frac{\hbar\omega}{e^{\hbar\omega/k_B T} - 1}$$

$$C_v = \left(\frac{\partial U}{\partial T}\right)_v = 3N_A k_B \left(\frac{\hbar\omega}{k_B T}\right)^2 \frac{e^{\hbar\omega/k_B T}}{(e^{\hbar\omega/k_B T} - 1)^2}$$

At high temperatures, $k_B T \gg \hbar\omega$

$$e^{\hbar\omega/kBT} \approx 1 + \hbar\omega/k_B T$$

$$U = 3N_A k_B T$$

$$C_v = 3N_A k_B = 3R$$

B3. A solid with heat capacity C_A at temperature T_A is placed in contact with another solid with heat capacity C_B at a lower temperature T_B. What is the change in entropy of the system after the two bodies have reached thermal equilibrium?

Solution:

Assuming that the final temperature is *T*, according to energy conservation,

$$Q_A + Q_B = 0$$

Or $\quad C_A(T - T_A) + C_B(T - T_B) = 0$

So $\quad T = (C_A T_A + C_B T_B)/(C_A + C_B).$

In general, $dS = đQ/T = C/T dT$

$$\Delta S = \int dS = \int C dlnT = ln(T_f/T_i).$$

For the two systems:

$$\Delta S_A = C_A \ln(T/T_A).$$

$$\Delta S_B = C_B \ln(T/T_B).$$

$$\Delta S = \Delta S_A + \Delta S_B = C_A \ln(T/T_A) + C_B ln(T/T_B)$$

$$= C_A \ln[(1 + C_B T_B/C_A T_A)/(1 + C_B/C_A)] + C_B \ln[(1 + C_A T_A/C_B T_B)/(1 + C_A/C_B)]$$

Problem 6 : (Hard) key

a) We consider nitrogen to be an ideal gas
The heat required is:

$$Q = n(C_v + R)\Delta T$$

$$= \frac{1000}{28}(5+2) \times 120 = \underline{30 \times 10^3 \text{ cal}}.$$

b) The increase of internal energy is

$$\Delta U = n C_v \Delta T = \frac{1000}{28} \times 5 \times 120$$

$$= 21.4 \times 10^3 \text{ cal}$$

c) The external work done is:

$$W = Q - \Delta U = 8.6 \times 10^3 \text{ cal}$$

d) If it is a process of constant volume, the required heat is

$$Q = n C_v \Delta T = 21.4 \times 10^3 \text{ cal}$$

CM Peter 1

A1 5 $\frac{\Delta L}{\Delta t} = \tau$

5 (a) $\Delta L = \tau \Delta t = F R \Delta t$

5 $I\omega = F R \Delta t$

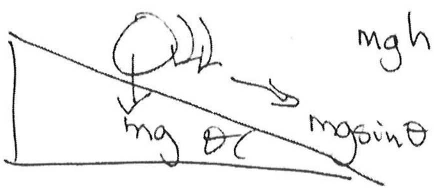
5 $\omega = \frac{F R \Delta t}{I} = \frac{18 \cdot 24 \cdot 15}{2100} = 3.09 \frac{rad}{s}$

5 (b) $W = \frac{I\omega^2}{2} = \frac{2100 \cdot 3.09^2}{2} = 10^4 J = 10 kJ$

CMA2

2) A hoop of radius 0.10 m and mass of 0.20 kg rolls down an inclined plane. If it starts from rest at a height of 2.0 m,

a) what is the total kinetic energy at the bottom of the incline (Use g=9.8m/s²)?



$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 \qquad I_{hoop} = mR^2$$

7

$$= \frac{1}{2}mv^2 + \frac{1}{2}mR^2\omega^2$$

$$= \underbrace{0.2\,kg}_{m} \times \underbrace{9.8\,\frac{m}{s^2}}_{g} \times \underbrace{2m}_{h} = \underline{3.92\,J}$$

b) what is the rotational kinetic energy at the bottom of the incline (the moment of inertia for a hoop is mR²) ?

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

6

$$= \frac{1}{2}mv^2 + \frac{1}{2}mR^2\omega^2$$

$$= \frac{1}{2}mv^2 + \frac{1}{2}mv^2$$

$$\frac{1}{2}I\omega^2 = \frac{1}{2}mgh = \frac{1}{2}\,3.92\,J = \underline{1.96\,J}$$

c) what is the translational kinetic energy at the bottom of the incline ?

6

$$\frac{1}{2}mv^2 = mgh - \frac{1}{2}I\omega^2$$

$$\frac{1}{2}mv^2 = \frac{1}{2}mgh = \frac{1}{2}\,3.92\,J = \underline{\underline{1.96\,J}}$$

6 (d) $\omega = \sqrt{\frac{2E_{rot}}{I}} = \sqrt{\frac{2 \cdot 1.96}{0.2 \cdot 0.01}} = 44.3\ \frac{rad}{s}$

(9) pb #5 (CMA3)

perfectly inelastic collision

a) ~~momentum~~ momentum conservation for the perfectly inelastic collision

$$m_1\vec{v}_{1i} + \vec{0} = (m_1+m_2)\vec{v}_f \ , \quad \vec{v}_f = \vec{v}_{1f} = \vec{v}_{2f}$$

$$\Rightarrow \vec{v}_f = \frac{m_1}{m_1+m_2}\vec{v}_{1i}$$

$$Q = T_i - T_f = \frac{1}{2}m_1 v_{1i}^2 - \frac{1}{2}(m_1+m_2)v_f^2$$

2

$$= \frac{1}{2}m_1 v_{1i}^2 - \frac{1}{2}(m_1+m_2)\frac{m_1^2}{(m_1+m_2)^2}v_{1i}^2$$

$$= \frac{1}{2}m_1 v_{1i}^2\left(1 - \frac{m_1}{m_1+m_2}\right)$$

5

$$\boxed{Q = \frac{m_2}{m_1+m_2}T_i = \frac{m_1 m_2}{2(m_1+m_2)}v_{1i}^2 = \frac{1}{2}\mu v_{1i}^2}$$ ✓

Alternative method: $Q = \frac{1}{2}\mu v_{1i}^2(1-e^2)$, $e=0$ for perfectly inelastic

3

$Q>0 \Rightarrow$ an ~~the~~ energy loss does occur, thus this process is called an exoergic collision.

b)

(10) CM A3, p.2

$$Q = \bar{T}_i - \bar{T}_f \qquad \underline{\underline{\bar{T}_f = 0}} \quad \text{key result}$$

2

$$= \frac{1}{2} m_1 \bar{v}_1^2 + \frac{1}{2} m_2 \bar{v}_2^2 - 0$$

$$= \frac{1}{2} m_1 \left(\frac{m_2}{m_1+m_2}\right)^2 v_{1i}^2 + \frac{1}{2} m_2 \left(\frac{m_1}{m_1+m_2}\right)^2 v_{1i}^2$$

$$\begin{cases} m_1 \vec{\bar{v}}_{1i} + m_2 \vec{\bar{v}}_{2i} = \vec{0} \\ m_1 \vec{\bar{v}}_{1f} + m_2 \vec{\bar{v}}_{2f} = \vec{0} \end{cases}$$

$$(m_1+m_2) \vec{\bar{v}}_{1f} = \vec{0}$$

$$\vec{\bar{v}}_{1f} = \vec{\bar{v}}_{2f} \equiv \vec{\bar{v}}_f$$

5

$$= \frac{1}{2} \frac{m_1 m_2}{(m_1+m_2)^2} (m_2 + m_1) v_{1i}^2$$

$$\boxed{Q = \frac{1}{2} \frac{m_1 m_2}{m_1+m_2} v_{1i}^2}$$

the same as in the lab frame.

⑥ problem #3: CMA4

a) $F_d = -b\dot{x}$, find $v(x)$.

$$-b\dot{x} = m\frac{d\dot{x}}{dt} = m\frac{d\dot{x}}{dx}\frac{dx}{dt} = m\dot{x}\frac{d\dot{x}}{dx}$$

$$\Rightarrow \int_{x_0}^{x} -\frac{b}{m}\,dx = \int_{v_0}^{v_0} d\dot{x}$$

$$\Rightarrow v - v_0 = -\frac{b}{m}(x - x_0) \quad , \quad \text{At } t=0,\ x = x_0 = 0$$

$$\Rightarrow \boxed{v = v_0 - \frac{b}{m}x} \qquad \text{when } v = 0,\ \boxed{x = \frac{mv_0}{b}}$$

which is finite.

$$m\dot{v} = -c\dot{x}^2 \to m\frac{dv}{dx}\dot{x} = -c\dot{x}^2$$

8 A4

b) $F_d = -c\dot{x}^2$, find $v(x)$ and $x(t)$

$$-c\dot{x}^2 = m\dot{x}\frac{d\dot{x}}{dx} \Leftrightarrow \int_{v_0}^{v}\frac{d\dot{x}}{\dot{x}} = -\int_{x_0}^{x}\frac{c}{m}\,dx$$

7

$$\Rightarrow \ln\left(\frac{v}{v_0}\right) = -\frac{c}{m}x \quad , \quad x_0 = 0$$

7

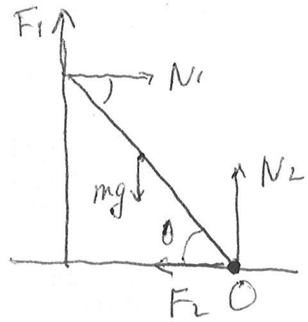
$$\Rightarrow \boxed{v(x) = v_0 e^{-\frac{c}{m}x}}$$

3

$v \to 0$ only when $x \to \infty$ (which is not finite), which means that the mass never comes to rest.

(CM)

(B1)



5

4 $N_1 = F_2$ $\quad F_1 = \mu N_1$

$F_1 + N_2 = mg$ $\quad F_2 = \mu N_2$

$\downarrow$

3 $\left.\begin{array}{l} N_1 = \mu N_2 \\ \mu N_1 + N_2 = mg \end{array}\right\}$ $N_1 = \mu(mg - \mu N_1)$

$N_1 = \frac{\mu mg}{1+\mu^2}$

equation for torques about point O:

3 $N_1 l \sin\theta + \mu N_1 l \cos\theta - mg\frac{l}{2}\cos\theta = 0$

$\tan\theta = \frac{\frac{1}{2}mg - \mu N_1}{N_1} = \frac{mg}{2N_1} - \mu$

4 $= \frac{mg(1+\mu^2)}{2\mu mg} - \mu = \frac{1}{2}\left(\frac{1}{\mu} + \mu\right) - \mu$

$= \frac{1}{2}\left(\frac{1}{\mu} - \mu\right)$

2 Solve for μ $\quad \mu^2 + 2\mu\tan\theta - 1 = 0$

$\theta = 60^\circ$

$\cos\theta = \frac{1}{2}$ $\tan\theta = \sqrt{3}$

$\mu_{1,2} = -\tan\theta \pm \sqrt{\tan^2\theta + 1} =$

$= -\tan\theta \pm \frac{1}{\cos\theta}$

2 take positive root $\mu = -\sqrt{3} + 2 = 0.268$

(6)

$F_2 = N_1 = \cancel{\mu(mg - \mu} \; \frac{\mu mg}{1+\mu^2} = \frac{0.268 \cdot 70 \cdot 9.8}{1 + 0.268^2} = 172 \, N$

2

CM (B2)

4) A string is wrapped around a hoop of radius 3 cm and mass 20 gms. The moment of inertia is mR^2

a) what is the angular acceleration in terms of g(=9.8m/s²) ?

8 .7

$\tau = r \times F = I\alpha = TR \rightarrow T = \frac{I\alpha}{R}$

$mg - T = ma = m\alpha R$

$T = mg - ma = mg - m\alpha R$

7 algebra .3

$mg = m\alpha R + \frac{I\alpha}{R}$

$\alpha = \frac{mg}{mR + \frac{I}{R}}$

$mg = \alpha(mR + mR)$

$\alpha = \frac{mgR}{mR + I}$

$\boxed{\alpha = \frac{g}{2R}}$

[this is sufficient]

$\alpha = 163.3 \frac{rad}{s^2} = 16.79$

.5 b) calculate the time it takes for the hoop to descend 0.5 meters ?

$\frac{h}{2\pi R} = N = 2.65$ rev

$\theta = 2\pi N = 16.7$ rad

$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

5

$\alpha = \frac{g}{2R}$ $a = R\alpha$ so $a = \frac{g}{2} = 4.9 \frac{M}{s^2}$

$d = \frac{1}{2}at^2$

$\sqrt{\frac{2d}{a}} = t$ $t = 0.45 sec$

2.2 $\frac{m}{s}$

.5 c) Calculate the angular velocity of the rotating hoop after it has descended 0.5 m.

$x = \frac{v_f^2 - 0}{2a} \Rightarrow v_f$

$v_f = R\omega$

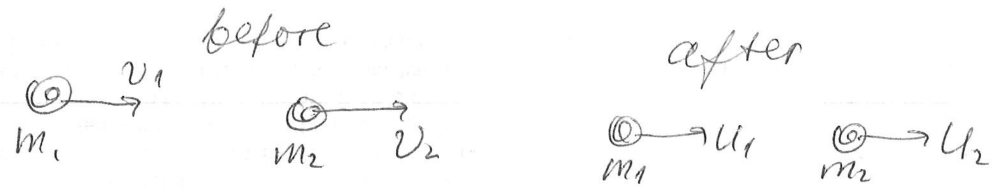
5

$\omega = \alpha t = 163.3 \frac{rad}{sec^2}\, 0.45\, sec = 73.5 \frac{rad}{sec}$

$= 11.7 \frac{rev.}{sec.}$

CM

B 3)



6
$$\begin{cases} \dfrac{m_1 v_1^2}{2} + \dfrac{m_2 v_2^2}{2} = \dfrac{m_1 u_1^2}{2} + \dfrac{m_2 u_2^2}{2} \\ m_1 v_1 + m_2 v_2 = m_1 u_1 + m_2 u_2 \end{cases}$$

3
$$\begin{cases} m_1 (v_1^2 - u_1^2) = m_2 (u_2^2 - v_2^2) & (1) \\ m_1 (v_1 - u_1) = m_2 (u_2 - v_2) & (2) \end{cases}$$

3 1st solution : $v_1 = u_1$, $v_2 = u_2$

corresponds to no collisions

so assume $u_1 \neq v_1$, $u_2 \neq v_2$

divide (1) by (2)

then $v_1 + u_1 = u_2 + v_2$

and we have a linear system

2 mult. by m_1

$$u_1 - u_2 = v_2 - v_1$$

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$m_2 u_2 + m_1 u_2 = m_1 v_1 + m_2 v_2 - m_1 (v_2 - v_1)$$

$$u_2 = \frac{(m_2 - m_1) v_2 + 2 m_1 v_1}{m_1 + m_2}$$

5

$1 \rightleftarrows 2$: $$u_1 = \frac{(m_1 - m_2) v_1 + 2 m_2 v_2}{m_1 + m_2}$$

3 (a) $m_1 = m_2$ $\quad u_2 = v_1 \quad u_1 = v_2$

3 (b) $m_1 \gg m_2$ $\quad u_2 = \dfrac{2 v_1 - v_2}{1} = 2 v_1 - v_2 \quad u_1 = 2 v_2 - v_1$

<u>alternatively</u>, one can use Cm frame

with $m_1 u_{1cm} = - m_2 u_{2cm}$ $\quad |v_1 - v_2| = |u_{1cm} - u_{2cm}|$ relative velocities

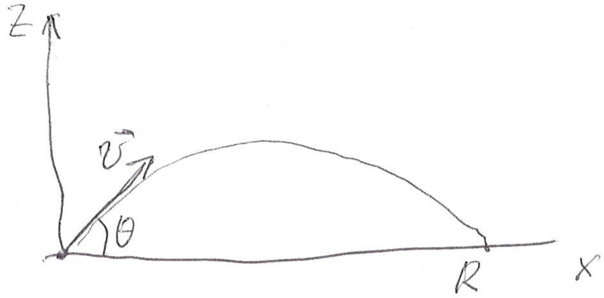
and then add $u_1 = u_{1cm} + \dfrac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$

$$u_{1cm} = \frac{m_2 (v_2 - v_1)}{m_1 + m_2}$$

$$u_{2cm} = \frac{m_1 (v_1 - v_2)}{m_1 + m_2}$$

CM (B4)

(a)



$$m\dot{v}_x = 0 \qquad m\dot{v}_z = -g$$

2 $$x = v_{x0} t \qquad z = v_{z0} t - \frac{g t^2}{2}$$

for $x = R$ $z = 0$ $\rightarrow$ $t = \frac{R}{v_{x0}}$

$$0 = \frac{v_{z0}}{v_{x0}} R - \frac{g}{2}\left(\frac{R}{v_{x0}}\right)^2 = 0 \qquad R = \frac{2 v_{x0} v_{z0}}{g} = \frac{2 v_0^2 \sin\theta \cos\theta}{g}$$

3 $$= \frac{v_0^2 \sin 2\theta}{g} = \frac{5^2 \times 10^4 \cdot \frac{\sqrt{3}}{2}}{9.8} = 2.21 \times 10^4 m = 22.1 km$$

(b) $$m\dot{v}_x = -c_1 v_x, \quad m\dot{v}_z = -c_1 v_z - g$$

2 $$v_x = v_{x0} e^{-\gamma t}, \ \gamma = \frac{c_1}{m} \qquad v_z = \left(v_{z0} + \frac{g}{\gamma}\right) e^{-\gamma t} - \frac{g}{\gamma}$$

$$x = -\frac{v_{x0}}{\gamma} e^{-\gamma t} + const \qquad x(0) = 0 \rightarrow const = \frac{v_{x0}}{\gamma}$$

2 $$\underline{x = \frac{v_{x0}}{\gamma}\left(1 - e^{-\gamma t}\right)} \quad (1)$$

$$z = -\frac{1}{\gamma}\left(v_{z0} + \frac{g}{\gamma}\right) e^{-\gamma t} - \frac{g}{\gamma} t + const$$

$$z(0) = 0 \rightarrow \quad const = \frac{1}{\gamma}\left(v_{z0} + \frac{g}{\gamma}\right)$$

2 $$\underline{z = \frac{1}{\gamma}\left(v_{z0} + \frac{g}{\gamma}\right)\left(1 - e^{-\gamma t}\right) - \frac{g}{\gamma} t} \quad (2)$$

for $x = R$ $z = 0$

2 From (1) $1 - e^{-\gamma t} = \frac{R\gamma}{v_{x0}}$, $t = -\frac{1}{\gamma} \ln\left(1 - \frac{R\gamma}{v_{x0}}\right)$

substitute into (2) with $z = 0$

CM B4, p. 2

$$0=\frac{1}{\gamma'}\left(v_{z0}+\frac{g}{\gamma'}\right)\frac{R\gamma'}{v_{x0}}+\frac{g}{\gamma'^2}\ln\left(1-\frac{R\gamma'}{v_{x0}}\right)$$

denote $u=\frac{R\gamma'}{v_{x0}}$, then

2 $$\left(v_{z0}+\frac{g}{\gamma'}\right)u+\frac{g}{\gamma'}\left(-u-\frac{u^2}{2}-\frac{u^3}{3}\right)=0$$

where we used $\ln(1-u)\approx-u-\frac{u^2}{2}-\frac{u^3}{3}$

$$u\,v_{z0}-u^2\frac{g}{2\gamma'}-u^3\frac{g}{3\gamma'}=0$$

$$\frac{1}{3}u^3+\frac{1}{2}u-v_{z0}\frac{\gamma'}{g}=0$$

5 $$u=\frac{-\frac{1}{2}+\sqrt{\frac{1}{4}+\frac{4}{3}v_{z0}\frac{\gamma'}{g}}}{2/3}=\frac{3}{4}\left[-1+\sqrt{1+\frac{16}{3}v_{z0}\frac{\gamma'}{g}}\right]$$

$\gamma'=\frac{c_1}{m}$ $$\gamma'=\frac{6.65\times10^{-5}\,\frac{kg}{s}}{0.05\,kg}=1.33\times10^{-3}\,s^{-1}$$

$v_{x0}=500\,\frac{m}{s}\cdot\cos 30^\circ=433\,\frac{m}{s}$ $v_{z0}=500\,\frac{m}{s}\cdot\frac{1}{2}=250\,\frac{m}{s}$

5 $$u=\frac{3}{4}\left(-1+\sqrt{1+\frac{16}{3}\cdot250\cdot\frac{1.33\times10^{-3}}{9.8}}\right)=0.0650$$

$$R=u\frac{v_{x0}}{\gamma'}=0.065\frac{433}{1.33\times10^{-3}}=21161\,m=21.16\,km$$