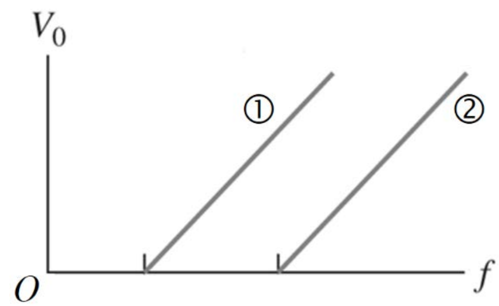
PROBLEM 1 (A1)



Photoelectric experiments are done with the target materials ① and ② with work functions ϕ_1 and ϕ_2, respectively. For each material, the stopping potential V_0 is plotted as a function of the frequency f of the light used. The two straight lines in the adjacent graph show the result.

a. Explain in a few sentences what "stopping potential" means.
b. What is the slope of the straight lines?
c. Which material has the higher work function, ① or ②?

The work function of material ① is $\phi_1 = 4.30\ \text{eV}$.

d. What is the largest wavelength light can have to cause photoemission from material ① ?

ANSWERS

a. Stopping potential is the smallest potential difference between the sample and the anode for which no electrons reach the anode.

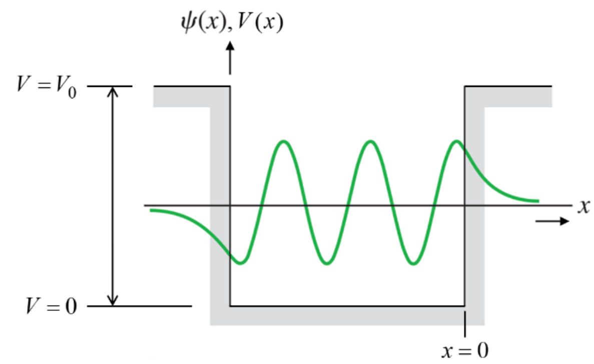
b. We have $hf = \phi + K = \phi + eV_0$, so $V_0 = \frac{h}{e} f - \frac{\phi}{e}$, and the slope is $\frac{h}{e}$.

c. The lines intersect the abscissa when $V_0 = \frac{h}{e} f - \frac{\phi}{e} = 0 \quad \Rightarrow \quad \phi = hf$. This shows that $\phi_2 > \phi_1$

d. For this largest wavelength, $K = 0$, so
$$hf_{\text{min}} = \phi = \frac{hc}{\lambda_{\text{max}}} \quad \Rightarrow \quad \lambda_{\text{max}} = \frac{hc}{\phi} = \frac{1240\ \text{eV} \cdot \text{nm}}{4.30\ \text{eV}} = 288\ \text{nm}.$$

PROBLEM A2

In this one-dimensional problem, we consider a stationary state of an electron with energy E in a finite well of depth V_0. The adjacent diagram shows both the potential $V(x)$ and the electron's wave function $\psi(x)$ as a function of position x. The electron is in a bound state, meaning $E < V_0$. We will focus on the part of the wave function in the classically forbidden region $x > 0$.



a. Why is this region called "classically forbidden"?

b. Show that the wave function $\psi(x) = Ae^{-\kappa x}$ (with $\kappa > 0$ and A some constant) satisfies the time-independent Schrödinger equation in this region, and derive an expression for κ.

c. The wave function $\psi(x) = Be^{+\kappa x}$ also satisfies the Schrödinger equation in this region, but must be rejected. Why?

At $x = 0$, we have $\psi(0)$. At a certain position $x_0 > 0$, the value of the wave function has dropped to $\alpha\psi(0)$, with $\alpha < 1$.

d. Find a relationship between κ, x_0, and α.

e. Use your answer of part *d*. and the expression you found for κ in part *b*. to find an expression for V_0.

f. If $E = 2.27\ \text{eV}$, $x_0 = 1.1\ \text{Å}$, and $\alpha = 0.13$, find the depth of the well in eV.

ANSWERS

a. Classically, the total mechanical energy of a particle is $E = K + U$. The particle can only be in locations where it has kinetic energy greater than zero, i.e. $U < E$. For $x > 0$, we have $U > E$, so, classically, the particle cannot dwell here; the region is "forbidden" to the particle.

b. TISE: $-\dfrac{\hbar^2}{2m}\psi'' + V_0\psi = E\psi \;\Rightarrow\; \psi'' = \dfrac{2m(V_0 - E)}{\hbar^2}\psi \overset{\text{def.}}{=} \kappa^2\psi$, with $\kappa = \sqrt{\dfrac{2m(V_0 - E)}{\hbar^2}}$.

The wave function $Ae^{-\kappa x}$ solves this differential equation.

c. The mathematically valid solution $Be^{+\kappa x}$ must be rejected because the wavefunction would not be normalizable. (The normalization integral would diverge.)

d. $\cancel{A}e^{-\kappa x_0} = \alpha \cancel{A}\,\cancel{e^0} \quad \Rightarrow \quad -\kappa x_0 = \ln(\alpha) \quad \Rightarrow \quad \kappa = \dfrac{-\ln(\alpha)}{x_0}$

e. $\kappa = \sqrt{\dfrac{2m(V_0 - E)}{\hbar^2}} = \dfrac{-\ln(\alpha)}{x_0} \quad \Rightarrow \quad \dfrac{2m(V_0 - E)}{\hbar^2} = \left(\dfrac{\ln(\alpha)}{x_0}\right)^2 \quad \Rightarrow \quad V_0 = E + \dfrac{\hbar^2}{2m}\left(\dfrac{\ln(\alpha)}{x_0}\right)^2$

f. With *m* the electron mass, we calculate $\dfrac{\hbar^2}{2m}\left(\dfrac{\ln(\alpha)}{x_0}\right)^2 = 2.10\times10^{-18}\ \mathrm{J} = 13.11\ \mathrm{eV}$. We conclude that $V_0 = 2.27 + 13.11 = 15.4\ \mathrm{eV}$.

PROBLEM A3

. In this one-dimensional problem, a particle of mass m is inside a potential well given by

$$U(x) = U_0 \cosh(bx)$$

When the particle is not far from the equilibrium position at $x = 0$, the potential $U(x)$ may be approximated by a parabolic potential.

a. Calculate the parabolic potential as a function of x.

Close to the equilibrium distance, we may consider the system as a harmonic oscillator.

b. Find the spring constant of this harmonic oscillator.

We have $U_0 = 10.0\ \text{eV}$, $b = 2.00\times10^{9}\ \text{m}^{-1}$, and $m = 9.11\times10^{-31}\ \text{kg}$.

c. Calculate $\hbar\omega$ in joules and in eV.

d. The system makes a transition from the state with *n* = 3 to the state with *n* = 1, emitting a single photon. Calculate the wavelength of this photon in nanometers.

ANSWERS

a. We have $\cosh(x) = 1 + \frac{x^2}{2} + \frac{x^4}{24} + \frac{x^6}{720} + \ldots \approx 1 + \frac{1}{2}x^2$. Hence, $U_0\cosh(bx) \approx U_0 + \frac{1}{2}U_0 b^2 x^2$.

b. The quadratic term is $\frac{1}{2}U_0 b^2 x^2 \stackrel{\text{def.}}{=} \frac{1}{2}kx^2$, so $k = U_0 b^2$

c. $\hbar\omega = \hbar\sqrt{\frac{k}{m}} = \hbar\sqrt{\frac{U_0 b^2}{m}} = 2.80\times10^{-19}\ \text{J} = 1.75\ \text{eV}$

d. The transition energy is $E_3 - E_1 = (3-1)\hbar\omega = 2\hbar\omega = 3.49\ \text{eV}$. Hence, $\lambda = \frac{1240}{3.49} = 355\ \text{nm}$.

PROBLEM A4

We consider a spin-$\frac{7}{2}$ particle (i.e. it has $s = \frac{7}{2}$).

a. What is the magnitude of its spin vector $\vec{S}$?

b. Can the spin vector of this particle be *perpendicular* to the *z*-axis?

c. Calculate the smallest angle the spin vector can make with the positive *z*-axis.

ANSWERS

a. $\left|\vec{\mathbf{S}}\right| = \sqrt{s(s+1)}\,\hbar = \sqrt{\tfrac{7}{2}\cdot\tfrac{9}{2}}\,\hbar = \tfrac{3}{2}\sqrt{7}\,\hbar = 4.19\times 10^{-34}\ \mathrm{J\cdot s}$

b. No. This would mean $S_z = m_s\hbar = 0$. However, $m_s \in \{-\tfrac{7}{2}, -\tfrac{5}{2}, -\tfrac{3}{2}, -\tfrac{1}{2}, +\tfrac{1}{2}, +\tfrac{3}{2}, +\tfrac{5}{2}, +\tfrac{7}{2}\}$, so m_s (and, hence, S_z) cannot be zero.

c. $\cos\theta_{\min} = \dfrac{(S_z)_{\max}}{\left|\vec{\mathbf{S}}\right|} = \dfrac{(m_s)_{\max}\hbar}{\sqrt{s(s+1)}\,\hbar} = \dfrac{\tfrac{7}{2}\hbar}{\tfrac{3}{2}\sqrt{7}\hbar} = \tfrac{1}{3}\sqrt{7} = 0.882 \quad\Rightarrow\quad \theta_{\min} = 28.1°$

PROBLEM B1

The operators L and M are Hermitian (self-adjoint). Operators A and B are not.

a. Is $\exp(\hat{L})$ Hermitian?

b. Is $[\hat{L},\hat{M}]$ Hermitian?

c. Is $[\hat{A}^\dagger,\hat{B}]+[\hat{A},\hat{B}^\dagger]$ Hermitian?

ANSWERS

a. Yes: $\left(\exp(\hat{L})\right)^\dagger = \left(\Sigma_n \dfrac{\hat{L}^n}{n!}\right)^\dagger = \Sigma_n \left(\dfrac{\hat{L}^n}{n!}\right)^\dagger = \Sigma_n \dfrac{1}{n!}(\hat{L}^n)^\dagger = \Sigma_n \dfrac{1}{n!}\hat{L}^n = \exp(\hat{L})$

b. No: $[\hat{L},\hat{M}]^\dagger = \left(\hat{L}\hat{M}-\hat{M}\hat{L}\right)^\dagger = (\hat{L}\hat{M})^\dagger - (\hat{M}\hat{L})^\dagger = \hat{M}^\dagger\hat{L}^\dagger - \hat{L}^\dagger\hat{M}^\dagger = \hat{M}\hat{L}-\hat{L}\hat{M} \neq [\hat{L},\hat{M}]$

c. No:

$$\left([\hat{A}^\dagger,\hat{B}]+[\hat{A},\hat{B}^\dagger]\right)^\dagger = [\hat{A}^\dagger,\hat{B}]^\dagger + [\hat{A},\hat{B}^\dagger]^\dagger = \left(\hat{A}^\dagger\hat{B}-\hat{B}\hat{A}^\dagger\right)^\dagger + \left(\hat{A}\hat{B}^\dagger-\hat{B}^\dagger\hat{A}\right)^\dagger =$$

$$= (\hat{A}^\dagger\hat{B})^\dagger - (\hat{B}\hat{A}^\dagger)^\dagger + (\hat{A}\hat{B}^\dagger)^\dagger - (\hat{B}^\dagger\hat{A})^\dagger = \hat{B}^\dagger\hat{A}^{\dagger\dagger} - \hat{A}^{\dagger\dagger}\hat{B}^\dagger + \hat{B}^{\dagger\dagger}\hat{A}^\dagger - \hat{A}^\dagger\hat{B}^{\dagger\dagger} =$$

$$= \hat{B}^\dagger\hat{A} - \hat{A}\hat{B}^\dagger + \hat{B}\hat{A}^\dagger - \hat{A}^\dagger\hat{B} = [\hat{B}^\dagger,\hat{A}]+[\hat{B},\hat{A}^\dagger] \neq [\hat{A}^\dagger,\hat{B}]+[\hat{A},\hat{B}^\dagger]$$

Problem B2.

Answers:

(a) at $t > 0$

$$\psi(x,t) = \frac{1}{\sqrt{2}} e^{-i\omega t/2}[\varphi_0(x) + \varphi_1(x) e^{-i\omega t}]$$

therefore

$$\langle x \rangle = \frac{1}{2} \int x |\varphi_0(x) + \varphi_1(x) e^{-i\omega t}|^2 dx = \frac{1}{2} \int 2\varphi_0 \varphi_1 \mathrm{Re} e^{-i\omega t} x dx = \sqrt{\frac{\hbar}{2m\omega}} \cos\omega t$$

where we used

$$\int x\varphi_0^2(x) dx = 0, \quad \int x\varphi_1^2(x) dx = 0$$

due to symmetry consideration, and the fact that φ_0, φ_1 are real.

(b) The classical initial-value problem with $x(0) = x_0$, $v(0) = 0$, gives

$$x(t) = x_0 \cos\omega t$$

where x_0 can be found from the energy equation

$$E = \frac{m\omega^2 x_0^2}{2}, \quad x_0 = \sqrt{\frac{2E}{m\omega^2}}.$$

In order for quantum and classical results to coincide we need $E = \hbar\omega/4$.

B3. (a)

$$\psi(x,t) = \sin(k_1 x)e^{-iE_1 t/\hbar} + 2\cos(k_2 x)e^{-iE_2 t/\hbar}$$

where

$$E_1 = \frac{\hbar^2 k_1^2}{2m}, \quad E_2 = \frac{\hbar^2 k_2^2}{2m}$$

(b) Outcomes: k_1, $-k_1$, k_2, $-k_2$.

The probability to get k_1 is

$$P_1 = \frac{1}{1+1+4+4} = \frac{1}{10}$$

similarly for $-k_1$ we get $1/10$, and for k_2, $-k_2$ we get $4/10$ for each.

(c) After measurements

$$\psi(x,t) = Ce^{ik_1 x}, \quad C = 1/2i.$$

This is the energy eigenstate with the energy eigenvalue $E_1 = \hbar^2 k_1^2/2m$.

B4. (a) Possible outcomes: $\hbar/2$ and $-\hbar/2$;

(b) expectation values: $\langle S_z \rangle = \hbar/2$ (eigenvalue) For S_x $\langle S_x \rangle = P_1\hbar/2 + P_2(-\hbar/2) = 0$ since probabilities for spin up and down are equal 1/2 both.

(c) The Hamiltonian of interaction with magnetic field

$$H' = \frac{e}{m} S_z B$$

(assume **B** along the z axis, since the gyromagnetic ratio for electron is e/m. When the spin flips, the igenvalue of S_z changes from $\hbar/2$ to $-\hbar/2$ resulting in the energy change $\Delta E = e\hbar B/m$, therefore frequency of radiation is

$$\omega = \Delta E/\hbar = eB/m = 1.76 \times 10^{11}\mathrm{C/kg} \cdot 1\mathrm{T} = 1.76 \times 10^{11}\mathrm{rad/s} = 2.80 \times 10^{10}\mathrm{Hz}.$$

A1. A metal hollow sphere of radius R is kept under a constant potential Φ_0. Using the Gauss's law, find the electric field **E** and the electrostatic potential Φ inside and outside the sphere and determine the surface charge density σ.

Solution:

1. Assume that the sphere has surface change σ. According to the Gauss's law, we have for the electric field outside the sphere

$$\oint_S \mathbf{E} \cdot d\mathbf{a} = \frac{4\pi R^2 \sigma}{\varepsilon_0}.$$

Using for the surface S the sphere of radius $r > R$ and using symmetry of the problem, we find

$$\mathbf{E} = \frac{\sigma R^2}{\varepsilon_0 r^2}\hat{\mathbf{r}}.$$

Hence the electrostatic potential outside the sphere is

$$\Phi = -\int_{-\infty}^{r} \mathbf{E} \cdot d\mathbf{r} = -\frac{\sigma R^2}{\varepsilon_0}\int_{-\infty}^{r} \frac{1}{r^2} dr = \frac{\sigma R^2}{\varepsilon_0 r}.$$

The surface change density can be found from the given potential on the sphere which leads to

$$\sigma = \frac{\varepsilon_0 \Phi_0}{R}.$$

Therefore, outside the sphere the electric field is

$$\mathbf{E}(r) = \frac{\Phi_0 R}{r^2}\hat{\mathbf{r}},$$

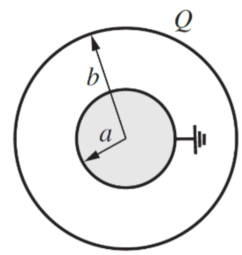
and the potential is

$$\Phi = \frac{\Phi_0 R}{r}.$$

Inside the sphere $r < R$ Gauss's law theorem says that the electric field is zero and consequently the electrostatic potential is constant

$$\Phi(r) = \Phi_0.$$

A2: A spherical conducting shell of radius b is concentric with and encloses a conducting ball of radius a. The ball is grounded, and the shell has charge Q. Argue that the presence of the charge Q on the shell will draw up a charge onto the ball from ground. Find the magnitude of this charge.



Solution

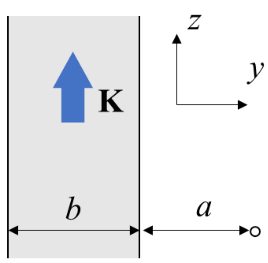
A charge will be induced on the ball to maintain a zero potential. In the absence of this charge the potential would be non-zero, equal to Q/b due to the charge Q on the shell. Drawing up a charge Q' onto the ball from ground establishes zero potential on the ball. The condition of zero potential on the ball determines the magnitude of the charge Q':

$$\frac{Q}{b} + \frac{Q'}{a} = 0\,, \tag{1}$$

from which we obtain

$$Q' = -\frac{a}{b}Q\,. \tag{2}$$

A3. A strip of width b carries a uniform surface current $\mathbf{K} = K\hat{\mathbf{z}}$. Find the magnetic field $\mathbf{B}$ at a point in the plane of the strip that lies at perpendicular distance a from the strip in the $\hat{\mathbf{y}}$-direction.



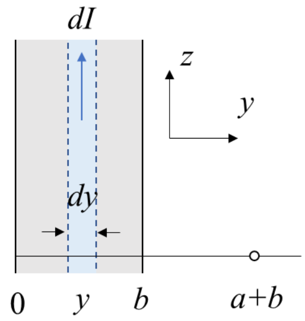
Solution:

The problem amounts to superposing the fields from a collection of long straight wires. An infinitely long filament of width dy at position y carries current $dI = Kdy$. This filament gives a contribution to the magnetic field at the point of consideration

$$d\mathbf{B} = -\frac{\mu_0}{4\pi}\frac{Kdy}{(a+b-y)}\hat{\mathbf{x}}. \tag{1}$$

The total field is obtained by integration of Eq. (1) over the strip width:

$$\mathbf{B} = -\frac{\mu_0}{4\pi}K\int_0^b \frac{dy}{(a+b-y)}\hat{\mathbf{x}} = -\frac{\mu_0}{4\pi}K\ln\left(\frac{a+b}{a}\right)\hat{\mathbf{x}} \tag{2}$$



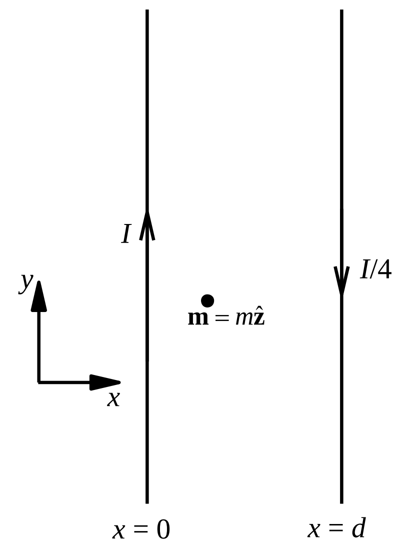
A4. The current is given by $I = e\lambda c$ where λ is the number of protons per unit length, therefore

$$\lambda = \frac{I}{ec} = \frac{5 \times 10^{-3}}{1.6 \times 10^{-19} \cdot 3 \times 10^{8}} = 1.04 \times 10^{8} \mathrm{m}^{-1}.$$

The proton number density in the beam $n = \lambda / A$ where A is the cross section area. Therefore the average distance l is

$$l = n^{-1/3} = \left(\frac{A}{\lambda}\right)^{1/3} = 2.13 \times 10^{-5} \mathrm{m}.$$

B1. Two infinite parallel wires are oriented along the *y*-direction and placed at a distance *d* apart. One wire carries a current *I*, and the other carries current *I*/4 in the opposite direction. Between these two wires there is a particle with position constrained to be in the plane formed by the two wires (the *x*-*y* plane). This particle has magnetic moment, **m**, that is fixed in magnitude and direction along +z, i.e. $\mathbf{m} = m\hat{\mathbf{z}}$. Find the equilibrium position, *x*, between the two wires ($0 < x < d$) of the particle that minimizes the interaction energy of this magnetic moment with the fields generated by the current of the wires.



<u>Solution</u>:

From the Ampere's law, the magnetic field generated in the *x*-*y* plane by the wire at $x = 0$ is

$$\mathbf{B}_1 = -\frac{\mu_0 I}{2\pi x}\hat{\mathbf{z}}. \tag{1}$$

The magnetic field generated by the wire at $x = d$ is

$$\mathbf{B}_2 = \frac{\mu_0 I}{8\pi(x-d)}\hat{\mathbf{z}}. \tag{2}$$

The total field is the sum of these two:

$$\mathbf{B} = -\frac{\mu_0 I}{2\pi x}\hat{\mathbf{z}} + \frac{\mu_0 I}{8\pi(x-d)}\hat{\mathbf{z}}. \tag{3}$$

The interaction energy of the magnetic moment **m** with the field is given by

$$U = -\mathbf{m}\cdot\mathbf{B} = \frac{m\mu_0 I}{2\pi x} - \frac{m\mu_0 I}{8\pi(x-d)}. \tag{4}$$

We must minimize this with respect to *x*. Taking the first derivative, we find:

$$0 = \frac{\partial U}{\partial x} = -\frac{m\mu_0 I}{2\pi x^2} + \frac{m\mu_0 I}{8\pi(x-d)^2} = -\frac{4m\mu_0 I(x-d)^2 - m\mu_0 I x^2}{8\pi x^2 (x-d)^2}. \tag{5}$$

Setting the numerator equal to zero and solving for *x*, we obtain

$$4(x-d)^2 - x^2 = 0 \Rightarrow x = \frac{2}{3}d, 2d. \tag{6}$$

Only one of these, $x = 2d/3$, lies between the wires, i.e. from $0 < x < d$, and this is the minimum. To make sure it is a minimum we can check the second derivative of U:

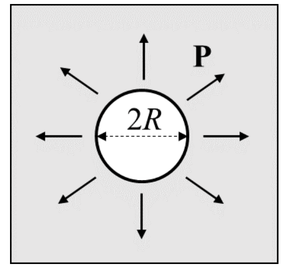
$$\frac{\partial^2 U}{\partial x^2} = \frac{4\mu_0 mI}{4\pi x^3} - \frac{m\mu_0 I}{4\pi(x-d)^3} = \frac{4m\mu_0 I(x-d)^3 - m\mu_0 I x^3}{4\pi x^3 (x-d)^3} = -\frac{m\mu_0 I}{8\pi d^3}, \tag{7}$$

which is negative, ensuring the point is indeed a minimum.

Correction: the second derivative is actually *positive*, coef= +81/8 in front (sorry, could not correct in the equation-IF), which is the requirement for the minimum. It can be also easily seen by sketching function (4) which is positively defined and approaches +\infty when x approaches 0 from the right and d from the left.

2. This is the unstable equilibrium if we allow the dipole to change orientation, since the dipole is originally oriented in such direction that interaction energy $-\mathbf{m}\cdot\mathbf{B}$ is positive (**m** is antiparallel to **B**). The stable equilibrium would correspond to **m** parallel to **B**.

B2. A polarized matter with a radial distribution of polarization, i.e. $\mathbf{P} = P\hat{\mathbf{r}}$, where *P* is constant, has a spherical hole of radius *R* at the origin. Find the polarization charge density and the electric field everywhere.



Solution:

There are two contributions to the polarization charge density: surface and volume. The polarization charge on the surface of the spherical hole is equal to

$$\sigma_P = \mathbf{P}\cdot\mathbf{n} = -\mathbf{P}\cdot\hat{\mathbf{r}} = -P \ . \tag{1}$$

The volume polarization charge at $r \geq R$ is given by

$$\rho_P = -\nabla\cdot\mathbf{P} = -P\nabla\cdot\hat{\mathbf{r}} = -P\nabla\cdot\left(\frac{\mathbf{r}}{r}\right) = -P\left(\frac{\nabla\cdot\mathbf{r}}{r} + \mathbf{r}\cdot\nabla\frac{1}{r}\right) = -P\left(\frac{3}{r} - \mathbf{r}\cdot\frac{\hat{\mathbf{r}}}{r^2}\right) = -\frac{2P}{r} . \tag{2}$$

According to the Gauss's law, σ_P and ρ_P produce purely radial electric fields *outside* the hole. The contribution to the electric field at $r > R$ from the volume charge density is given by

$$\mathbf{E}_\rho(\mathbf{r}) = \frac{Q(r)}{4\pi\varepsilon_0 r^2}\hat{\mathbf{r}} \ , \tag{3}$$

where $Q(r)$ is the volume polarization charge inside the sphere of radius *r*, i.e.

$$Q(r) = \int_R^r \rho_P(r)4\pi r^2 dr = -4\pi\int_R^r \frac{2P}{r}\rho_P(r)r^2 dr = 4\pi P\left(R^2 - r^2\right). \tag{4}$$

The contribution to the electric field at $r > R$ from the surface charge density is

$$\mathbf{E}_\sigma(\mathbf{r}) = \frac{-4\pi PR^2}{4\pi\varepsilon_0 r^2}\hat{\mathbf{r}} = -\frac{PR^2}{\varepsilon_0 r^2}\hat{\mathbf{r}} . \tag{5}$$

Summing up the two contributions, we find for the total electric field:

$$\mathbf{E}(\mathbf{r}) = \mathbf{E}_\rho(\mathbf{r}) + \mathbf{E}_\sigma(\mathbf{r}) = \begin{cases} -\dfrac{P}{\varepsilon_0}\hat{\mathbf{r}} & r > R \\ 0 & r \leq R \end{cases} . \tag{6}$$

B3. Equation for the circuit

$$\mathcal{E} - L\frac{dI}{dt} = IR \quad (1).$$

The current grows from $I = 0$ at $t = 0$ to $\mathcal{E}/R$ at $t \to \infty$. (a) Initially $I = 0$, therefore $dI/dt = \mathcal{E}/L = 3 \times 10^3$ A/s.

(b) For $I = \mathcal{E}/2R$

$$\frac{dI}{dt} = \frac{\mathcal{E} - (\mathcal{E}/2R) \cdot R}{L} = \frac{\mathcal{E}}{2L} = 1.5 \times 10^3 \text{A/s}.$$

(c)

$$I_f = \mathcal{E}/R = 12/150 = 0.08\text{A}$$

(d) The solution of the differential Eq. (1) is

$$I = \frac{\mathcal{E}}{R}(1 - \exp(-Rt/L))$$

therefore

$$t = -\frac{L}{R}\ln\left(1 - \frac{IR}{\mathcal{E}}\right) = -\frac{4 \times 10^{-3}}{150}\ln(0.01) = 0.00123\text{s}.$$

B4. The potential difference is given by the Faraday law

$$V = -\frac{d\Phi}{dt}.$$

The magnetic field at a distance x from the wire is

$$B = \frac{\mu_0 I}{2\pi x}.$$

The corresponding element dx of the wire creates the potential difference

$$dV = -Bdx\frac{dy}{dt}$$

where dy is the element of distance covered by the rod in the direction of the current, therefore $dy/dt = v$ and

$$V = -\frac{\mu_0 I v}{2\pi}\int_d^{d+b}\frac{dx}{x} = -\frac{\mu_0 I v}{2\pi}\ln\frac{d+b}{d}.$$

The sign $-$ represents the Lenz rule: if we had a closed loop whose part is the rod, the current due to the potential difference V would lead to a magnetic force which would slow down the motion of the rod.