UNL - Department of Physics and Astronomy

Preliminary Examination - Day II
Friday, August 8, 2025

This test covers the topics of *Thermodynamics and Statistical Mechanics* (Topic 1) and *Classical Mechanics* (Topic 2). Each topic has 4 "A" questions and 4 "B" questions. Work two problems from each group. Thus, you will work on a total of 8 questions today, 4 from each topic.

Note: If you do more than two problems in a group, only the first two (in the order they appear in this handout) will be graded. For instance, if you do problems A1, A3, and A4, only A1 and A3 will be graded.

WRITE YOUR ANSWERS ON ONE SIDE OF THE PAPER ONLY

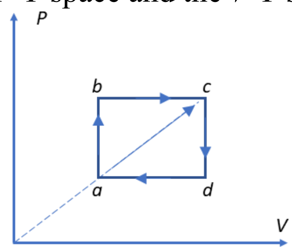
Thermodynamics and Statistical Mechanics Group A

Answer only two Group A questions

A1. A diatomic gas ($C_v = 2.5\ nR$) expands adiabatically to a volume 1.35 times larger than the initial volume. The initial temperature is 18 °C. Find the final temperature.

A2. At 0 °C and 1 atm (1atm = 1.01×10^5 Pa) the thermal expansion coefficient for copper is $\alpha=4.85\times10^{-5}$/K, and the isothermal compressibility is $\kappa=7.8\times10^{-7}$/atm. If the temperature of the cooper is increased to 10 °C, what's the value of pressure if the volume is kept constant?

A3. Consider the five processes in the *P*-*V* space below. Processes *bc* and *da* are isobaric. Processes *ab* and *cd* are isochoric. The extension of the process *ac* passes the origin. Sketch the five processes in the *P*-*T* space and the *V*-*T* space.

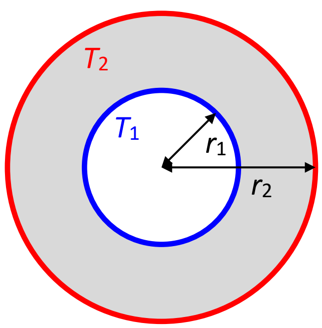


A4. For a diatomic ideal gas ($C_v = 2.5\ nR$) near room temperature, what fraction of the heat supplied is available for external work if the gas is expanded at a constant pressure? At a constant temperature?

Thermodynamics and Statistical Mechanics Group B

Answer only two Group B questions

B1. Consider a cylindrical wall with inner radius r_1, outer radius r_2. The temperature for the inner and outer surfaces are T_1 and T_2, respectively. The heat flow is along the radial direction. Find the thermal conductance $\sigma \equiv I/(T_2-T_1)$ of the cylindrical shell in terms of r_1, r_2, and the length of the cylinder L. Here $I = \lim_{\Delta t\to 0}(\Delta Q/\Delta t)$ is the thermal current, and thermal conductivity is k.



B2. The speed of sound in air is $v_s = [(\partial P/\partial \rho)_S]^{0.5}$, where S is entropy, ρ is density. Assume that air can be approximately treated as ideal gas whose molar mass is M.

(a) Derive the relation between the adiabat exponent γ and C, where $\gamma = C_P/C_V$.

(b) Using the relation C_P-$C_V = nR$, find the expression of C_V from the above relation between γ and v_s.

B3. 10 liters of gas at atmospheric pressure is compressed isothermally to a volume of 1 liter and then allowed to expand adiabatically to 10 liters.

a) Sketch the processes on a pV diagram for a monatomic gas.
b) Make a similar sketch for a diatomic gas.
c) Is a net work done on or by the system?
d) Is it greater or less for the diatomic gas?

For the ratio of the specific heats, recall that $\gamma = 5/3$ for a monatomic gas and $\gamma = 7/5$ for a diatomic gas.

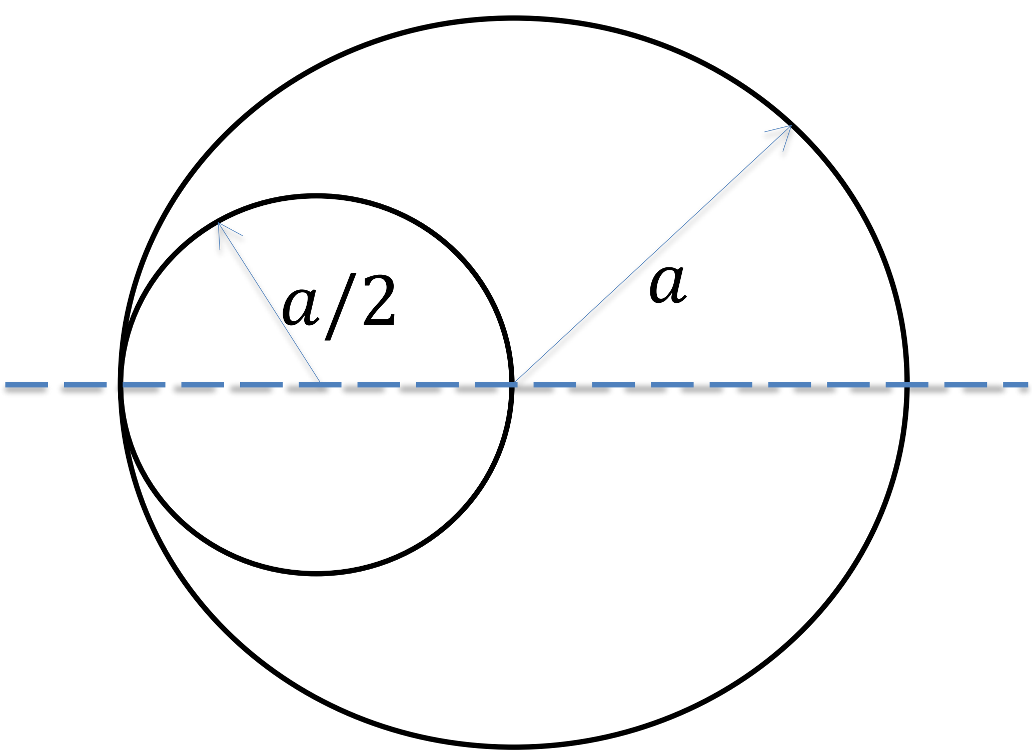
B4. Two bodies with different initial temperatures T_1 and T_2 are brought into contact while being insulated from their surroundings. The heat capacities of the two bodies are equal.

Find the total entropy change accompanying the equilibration of the system and show explicitly that it is always positive.

Classical Mechanics Group A
Answer only two Group A questions

A1. A thin uniform stick of length l is initially at the vertical position at the unstable equilibrium with its bottom end resting on a frictionless table. A small disturbance makes the stick fall. Find the velocity of the center of mass when it reaches the table**.**

A2. A solid uniform sphere of radius a has a spherical cavity of radius $a/2$ centered at a point $a/2$ from the center of the sphere. The mass of the object is m.



(a) Find the position of the center of mass
(b) Find the moment of inertia about the axis passing through the center of the sphere and the center of the cavity

A3. A particle undergoes a simple harmonic motion with the angular frequency ω_0. At the equilibrium position (x=0) its speed is v_0.

(a) Find its speed as a function of displacement x.

(b) Find the displacement x_1 where the kinetic energy equals the potential energy, and the ratio x_1/x_0, where x_0 is the oscillation amplitude.

A4. Write down the Hamiltonian function and find Hamilton's canonical equations for the motion of a projectile in a uniform gravitational field with no air resistance. Solve these equations and show that the results are exactly the same as those obtained from the Newton's law.

Classical Mechanics Group B
Answer only two Group B questions

B1. A particle of mass m is subject to two forces: a central force $\vec{f}_1 = f(r)\frac{\vec{r}}{r}$ and a frictional force $\vec{f}_2 = -\lambda\vec{v}$, where $\lambda > 0$ and $\vec{v}$ is the velocity of the particle.

(a) Write down the equation of motion for the particle's angular momentum;
(b) If the particle initially has angular momentum $\vec{L}_0$ about $\vec{r} = 0$, find its angular momentum for all subsequent times.

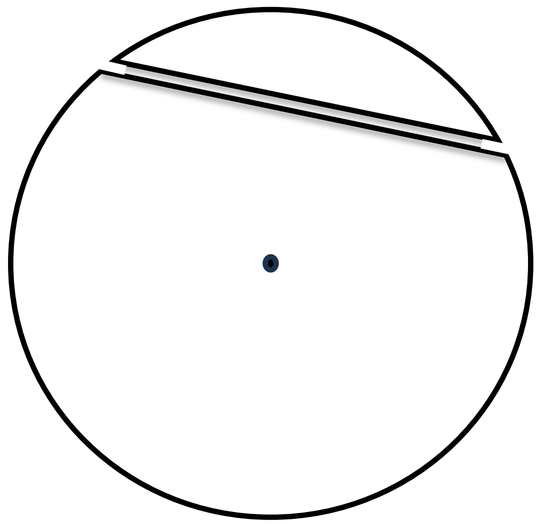
B2. A baseball pitcher can throw a ball more easily horizontally than vertically. Assume that the pitcher's throwing speed varies with elevation angle approximately as $v_0\cos(\theta_0/2)$, where θ_0 is the initial elevation angle and v_0 is the initial velocity when the ball is thrown horizontally. Find the angle θ_0 at which the ball must be thrown to achieve maximum range. Find the value of the maximum range in terms of v_0 and g.

B3. Consider a simple pendulum of length l displaced by a small angle θ_0 and released from rest. The mass m possibly experiences a linear drag force, with the drag coefficient C.

a) Use the Lagrangian formalism to find the equation of motion for the pendulum when there is no drag force. Solve the equation of motion.

b) When the mass m experiences a linear drag force with the drag coefficient C, find the equation of motion using one of the Newton's laws. Solve it for the case of small angle when $g > lC^2/4m^2$.

B4. A particle is sliding in a straight, smooth tube passing obliquely through Earth. Show that the particle is executing simple harmonic motion and find its period in hours. Neglect any resistance



and Earth's rotation. Assume that Earth is a uniform solid sphere with the mass density 5.51 g/cm^3.

Physical Constants

Speed of light .... $c = 2.998\times10^{8}$ m/s

Atmospheric pressure........... 101,325 Pa

Electron mass .... $m_e = 9.109\times10^{-31}$ kg

Avogadro constant .... $N_A = 6.022\times10^{23}$ mol^{-1}

Boltzmann constant.............. $k_B = 1.381\times10^{-23}$ J/K $= 8.617\times10^{-5}$ eV/K

Gas constant .... $R = 8.314$ J/(mol·K)

Atomic mass unit .... 1u $= 1.66\times10^{-27}$ kg

Gravitational constant .... $G = 6.674\times10^{-11}$ m^3/(kg·s^2); $g = 9.8$ m/s^2

Equations That May Be Helpful

TRIGONOMETRY

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$
$$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$
$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$
$$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$
$$\sin(2\theta) = 2\sin\theta\cos\theta$$
$$\cos(2\theta) = \cos^2\theta - \sin^2\theta = 1 - 2\sin^2\theta = 2\cos^2\theta - 1$$
$$\sin\alpha\sin\beta = \tfrac{1}{2}\left[\cos(\alpha-\beta) - \cos(\alpha+\beta)\right]$$
$$\cos\alpha\cos\beta = \tfrac{1}{2}\left[\cos(\alpha-\beta) + \cos(\alpha+\beta)\right]$$
$$\sin\alpha\cos\beta = \tfrac{1}{2}\left[\sin(\alpha+\beta) + \sin(\alpha-\beta)\right]$$
$$\cos\alpha\sin\beta = \tfrac{1}{2}\left[\sin(\alpha+\beta) - \sin(\alpha-\beta)\right]$$

For small x:

$$\sin x \approx x - \tfrac{1}{6}x^3$$
$$\cos x \approx 1 - \tfrac{1}{2}x^2$$
$$\tan x \approx x + \tfrac{1}{3}x^3$$

KINEMATICS IN POLAR COORDINATES

$$\mathbf{r} = r\mathbf{e}_r, \qquad \frac{d\mathbf{r}}{dt} = \frac{dr}{dt}\mathbf{e}_r + r\frac{d\theta}{dt}\mathbf{e}_\theta$$

GAUSS' LAW FOR THE GRAVITY FORCE

$$\oint \mathbf{F}\cdot d\mathbf{a} = 4\pi G M_{\text{enc}} m$$

THERMODYNAMICS

Heat capacity $C_V = N\dfrac{d\langle E\rangle}{dT}$.

Molar heat capacity of diatomic gas: $C_V = \tfrac{5}{2}R$.

For adiabatic processes in an ideal gas with constant heat capacity, $pV^{\gamma} = \text{const}$.

$$dU = TdS - pdV$$
$$dF = -SdT - pdV$$

$$H = U + pV$$
$$F = U - TS$$
$$G = F + pV$$
$$\Omega = F - \mu N$$

$$C_V = \left(\frac{\delta Q}{dT}\right)_V = T\left(\frac{\partial S}{\partial T}\right)_V$$
$$C_p = \left(\frac{\delta Q}{dT}\right)_p = T\left(\frac{\partial S}{\partial T}\right)_p$$
$$TdS = C_V dT + T\left(\frac{\partial S}{\partial V}\right)_T dV$$

$$\kappa = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_T$$
$$\alpha = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_p$$

Efficiency of a heat engine: $\eta = \frac{W}{|Q_{in}|} = 1 - \frac{|Q_{out}|}{|Q_{in}|}$

Carnot engine: ΔS=0

Carnot efficiency = $1 - T_c/T_h$.

The cyclic rule:

$$\left(\frac{\partial P}{\partial T}\right)_V \left(\frac{\partial T}{\partial V}\right)_P \left(\frac{\partial V}{\partial P}\right)_T = -1$$

Stefan-Boltzmann's law:

$$P = \sigma T^4; \quad \sigma = 5.67\times 10^{-8} W\cdot m^{-2}\cdot K^{-4}$$

TABLE 9.2 Moments of Inertia of Various Bodies

(a) Slender rod, axis through center

$$I = \frac{1}{12}ML^2$$

(b) Slender rod, axis through one end

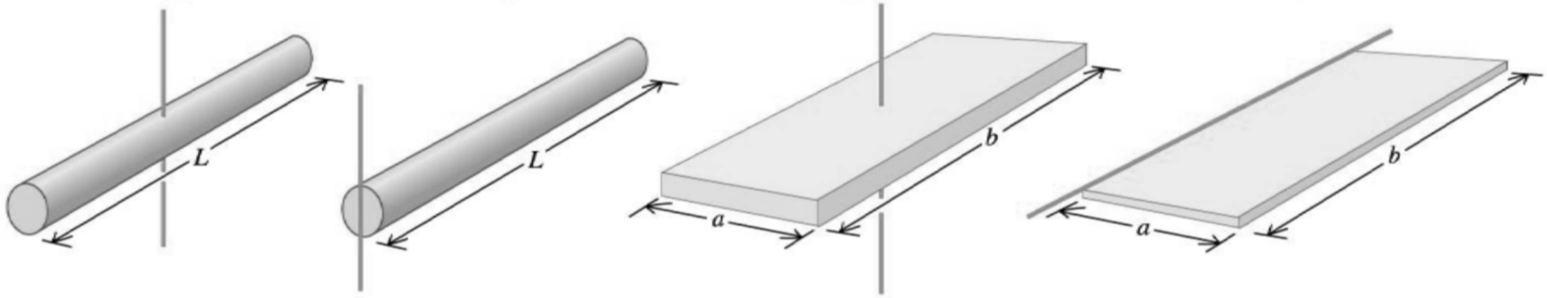
$$I = \frac{1}{3}ML^2$$

(c) Rectangular plate, axis through center

$$I = \frac{1}{12}M\left(a^2 + b^2\right)$$

(d) Thin rectangular plate, axis along edge

$$I = \frac{1}{3}Ma^2$$



(e) Hollow cylinder

$$I = \frac{1}{2}M\left(R_1^2 + R_2^2\right)$$

(f) Solid cylinder

$$I = \frac{1}{2}MR^2$$

(g) Thin-walled hollow cylinder

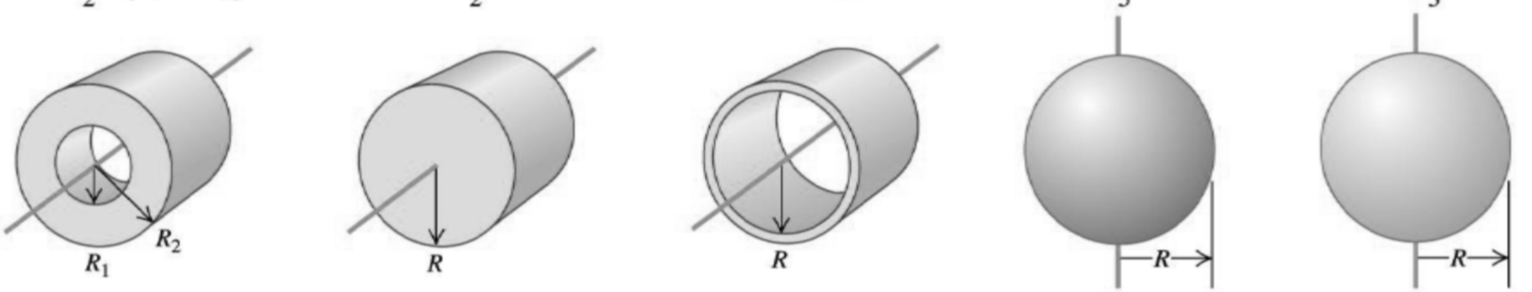
$$I = MR^2$$

(h) Solid sphere

$$I = \frac{2}{5}MR^2$$

(i) Thin-walled hollow sphere

$$I = \frac{2}{3}MR^2$$



VECTOR DERIVATIVES

Cartesian. $d\mathbf{l} = dx\,\hat{\mathbf{x}} + dy\,\hat{\mathbf{y}} + dz\,\hat{\mathbf{z}}; \quad d\tau = dx\,dy\,dz$

Gradient: $$\nabla t = \frac{\partial t}{\partial x}\hat{\mathbf{x}} + \frac{\partial t}{\partial y}\hat{\mathbf{y}} + \frac{\partial t}{\partial z}\hat{\mathbf{z}}$$

Divergence: $$\nabla \cdot \mathbf{v} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$$

Curl: $$\nabla \times \mathbf{v} = \left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z}\right)\hat{\mathbf{x}} + \left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x}\right)\hat{\mathbf{y}} + \left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right)\hat{\mathbf{z}}$$

Laplacian: $$\nabla^2 t = \frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2}$$

Spherical. $d\mathbf{l} = dr\,\hat{\mathbf{r}} + r\,d\theta\,\hat{\boldsymbol{\theta}} + r\sin\theta\,d\phi\,\hat{\boldsymbol{\phi}}; \quad d\tau = r^2\sin\theta\,dr\,d\theta\,d\phi$

Gradient: $$\nabla t = \frac{\partial t}{\partial r}\hat{\mathbf{r}} + \frac{1}{r}\frac{\partial t}{\partial \theta}\hat{\boldsymbol{\theta}} + \frac{1}{r\sin\theta}\frac{\partial t}{\partial \phi}\hat{\boldsymbol{\phi}}$$

Divergence: $$\nabla \cdot \mathbf{v} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta\, v_\theta) + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi}$$

Curl: $$\nabla \times \mathbf{v} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial \theta}(\sin\theta\, v_\phi) - \frac{\partial v_\theta}{\partial \phi}\right]\hat{\mathbf{r}}$$
$$+ \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial v_r}{\partial \phi} - \frac{\partial}{\partial r}(r v_\phi)\right]\hat{\boldsymbol{\theta}} + \frac{1}{r}\left[\frac{\partial}{\partial r}(r v_\theta) - \frac{\partial v_r}{\partial \theta}\right]\hat{\boldsymbol{\phi}}$$

Laplacian: $$\nabla^2 t = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial t}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial t}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 t}{\partial \phi^2}$$

Cylindrical. $d\mathbf{l} = ds\,\hat{\mathbf{s}} + s\,d\phi\,\hat{\boldsymbol{\phi}} + dz\,\hat{\mathbf{z}}; \quad d\tau = s\,ds\,d\phi\,dz$

Gradient: $$\nabla t = \frac{\partial t}{\partial s}\hat{\mathbf{s}} + \frac{1}{s}\frac{\partial t}{\partial \phi}\hat{\boldsymbol{\phi}} + \frac{\partial t}{\partial z}\hat{\mathbf{z}}$$

Divergence: $$\nabla \cdot \mathbf{v} = \frac{1}{s}\frac{\partial}{\partial s}(s v_s) + \frac{1}{s}\frac{\partial v_\phi}{\partial \phi} + \frac{\partial v_z}{\partial z}$$

Curl: $$\nabla \times \mathbf{v} = \left[\frac{1}{s}\frac{\partial v_z}{\partial \phi} - \frac{\partial v_\phi}{\partial z}\right]\hat{\mathbf{s}} + \left[\frac{\partial v_s}{\partial z} - \frac{\partial v_z}{\partial s}\right]\hat{\boldsymbol{\phi}} + \frac{1}{s}\left[\frac{\partial}{\partial s}(s v_\phi) - \frac{\partial v_s}{\partial \phi}\right]\hat{\mathbf{z}}$$

Laplacian: $$\nabla^2 t = \frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\partial t}{\partial s}\right) + \frac{1}{s^2}\frac{\partial^2 t}{\partial \phi^2} + \frac{\partial^2 t}{\partial z^2}$$

VECTOR IDENTITIES

Triple Products

(1) $\mathbf{A}\cdot(\mathbf{B}\times\mathbf{C}) = \mathbf{B}\cdot(\mathbf{C}\times\mathbf{A}) = \mathbf{C}\cdot(\mathbf{A}\times\mathbf{B})$

(2) $\mathbf{A}\times(\mathbf{B}\times\mathbf{C}) = \mathbf{B}(\mathbf{A}\cdot\mathbf{C}) - \mathbf{C}(\mathbf{A}\cdot\mathbf{B})$

Product Rules

(3) $\nabla(fg) = f(\nabla g) + g(\nabla f)$

(4) $\nabla(\mathbf{A}\cdot\mathbf{B}) = \mathbf{A}\times(\nabla\times\mathbf{B}) + \mathbf{B}\times(\nabla\times\mathbf{A}) + (\mathbf{A}\cdot\nabla)\mathbf{B} + (\mathbf{B}\cdot\nabla)\mathbf{A}$

(5) $\nabla\cdot(f\mathbf{A}) = f(\nabla\cdot\mathbf{A}) + \mathbf{A}\cdot(\nabla f)$

(6) $\nabla\cdot(\mathbf{A}\times\mathbf{B}) = \mathbf{B}\cdot(\nabla\times\mathbf{A}) - \mathbf{A}\cdot(\nabla\times\mathbf{B})$

(7) $\nabla\times(f\mathbf{A}) = f(\nabla\times\mathbf{A}) - \mathbf{A}\times(\nabla f)$

(8) $\nabla\times(\mathbf{A}\times\mathbf{B}) = (\mathbf{B}\cdot\nabla)\mathbf{A} - (\mathbf{A}\cdot\nabla)\mathbf{B} + \mathbf{A}(\nabla\cdot\mathbf{B}) - \mathbf{B}(\nabla\cdot\mathbf{A})$

Second Derivatives

(9) $\nabla\cdot(\nabla\times\mathbf{A}) = 0$

(10) $\nabla\times(\nabla f) = 0$

(11) $\nabla\times(\nabla\times\mathbf{A}) = \nabla(\nabla\cdot\mathbf{A}) - \nabla^2\mathbf{A}$

FUNDAMENTAL THEOREMS

Gradient Theorem: $\int_{\mathbf{a}}^{\mathbf{b}}(\nabla f)\cdot d\mathbf{l} = f(\mathbf{b}) - f(\mathbf{a})$

Divergence Theorem: $\int(\nabla\cdot\mathbf{A})\,d\tau = \oint\mathbf{A}\cdot d\mathbf{a}$

Curl Theorem: $\int(\nabla\times\mathbf{A})\cdot d\mathbf{a} = \oint\mathbf{A}\cdot d\mathbf{l}$

CARTESIAN AND SPHERICAL UNIT VECTORS

$$\hat{\mathbf{x}} = (\sin\theta\cos\phi)\hat{\mathbf{r}} + (\cos\theta\cos\phi)\hat{\boldsymbol{\theta}} - \sin\phi\,\hat{\boldsymbol{\varphi}}$$

$$\hat{\mathbf{y}} = (\sin\theta\sin\phi)\hat{\mathbf{r}} + (\cos\theta\sin\phi)\hat{\boldsymbol{\theta}} + \cos\phi\,\hat{\boldsymbol{\varphi}}$$

$$\hat{\mathbf{z}} = \cos\theta\,\hat{\mathbf{r}} - \sin\theta\,\hat{\boldsymbol{\theta}}$$

INTEGRALS

$\int_0^\infty \frac{1}{1+bx^2}dx = \frac{\pi}{2b^{1/2}}$	$\int_0^\infty e^{-x^2}dx = \frac{\sqrt{\pi}}{2\sqrt{a}}$
$\int_0^\infty x^n e^{-bx}dx = \frac{n!}{b^{n+1}}$	$\int_0^\infty xe^{-x^2}dx = \frac{1}{2a}$
$\int (x^2+b^2)^{-1/2}dx = \ln\left(x+\sqrt{x^2+b^2}\right)$	$\int_0^\infty x^2e^{-x^2}dx = \frac{\sqrt{\pi}}{2a^{3/2}}$
$\int (x^2+b^2)^{-1}dx = \frac{1}{b}\arctan\left(\frac{x}{b}\right)$	$\int_0^\infty x^3e^{-x^2}dx = \frac{1}{2a^2}$
$\int (x^2+b^2)^{-3/2}dx = \frac{x}{b^2\sqrt{x^2+b^2}}$	$\int_0^\infty x^4e^{-x^2}dx = \frac{3\sqrt{\pi}}{8a^{5/2}}$
$\int (x^2+b^2)^{-2}dx = \frac{\frac{bx}{x^2+b^2}+\arctan\left(\frac{x}{b}\right)}{2b^3}$	$\int_0^\infty x^5e^{-x^2}dx = \frac{1}{a^3}$
$\int \frac{x\,dx}{x^2+b^2} = \frac{1}{2}\ln\left(x^2+b^2\right)$	$\int_0^\infty x^6e^{-x^2}dx = \frac{15\sqrt{\pi}}{16a^{7/2}}$
$\int \frac{dx}{x(x^2+b^2)} = \frac{1}{2b^2}\ln\left(\frac{x^2}{x^2+b^2}\right)$	
$\int \frac{dx}{a^2x^2-b^2} = \frac{1}{2ab}\ln\left(\frac{ax-b}{ax+b}\right) = -\frac{1}{ab}\operatorname{artanh}\left(\frac{ax}{b}\right)$	