

(A1)

For circular orbit

(CM) 6 $\frac{mv^2}{r} = \frac{GmM}{r^2}$

3 $v = \left(\frac{GM}{r}\right)^{1/2}$

4 $\omega = \frac{v}{r} = \frac{(GM)^{1/2}}{r^{3/2}}$

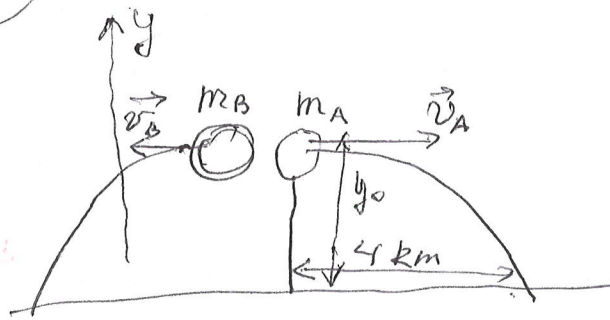
on the earth surface

6 $\frac{GM}{R^2} = g \rightarrow GM = gR^2$

3 $\omega = \frac{Rg^{1/2}}{r^{3/2}}$

3 $T = \frac{2\pi}{\omega} = \frac{2\pi r^{3/2}}{Rg^{1/2}}$

A2 CM



conservation of momentum

5

$$\vec{p}_A + \vec{p}_B = 0$$

2

$$\vec{v}_B = -\frac{m_A}{m_B} \vec{v}_A$$

Eq. of motion in the vertical direction

5

$$y = y_0 - \frac{gt^2}{2} \quad (\text{since } v_y = 0)$$

5

for $y=0$ $t = \sqrt{\frac{2y_0}{g}}$, therefore both

fragments hit the ground at the same time

8

since $v_A t = 4 \text{ km}$,

$$v_B t = -\frac{m_A}{m_B} v_A t = \begin{cases} -2 \text{ km} & \text{if } m_B = 2 m_A \\ -0.4 \text{ km} & \text{if } m_B = 10 m_A \end{cases}$$

(CM)

(A3)

Eg. for a damped oscillator

step 3 $\ddot{x} + 2\gamma\dot{x} + \omega_0^2 x = 0$

Complex Solution
 $x = e^{\lambda t}$

solve

$$\lambda^2 + 2\gamma\lambda + \omega_0^2 = 0$$

2

$$\lambda = -\gamma + \sqrt{\gamma^2 - \omega_0^2} = -\gamma + i\omega_d, \text{ (underdamping)}$$

$$\omega_d^2 = \omega_0^2 - \gamma^2 \quad (1)$$

General solution in the real form

$$4 \quad x = A e^{-\gamma t} \cos(\omega_d t + \phi)$$

$$4 \text{ for } t = \frac{n}{10} T \quad A e^{-\frac{n}{10}\gamma T} = A e^{-1}$$

$$3 \quad \gamma = \frac{1}{n \frac{10}{10} T} = \frac{\omega_d}{\frac{10}{n} \cdot 2\pi} = \frac{\omega_d}{2\pi T n}$$

$$3 \text{ from (1)} \quad \omega_0^2 = \omega_d^2 + \left(\frac{\omega_d}{2\pi n}\right)^2$$

A4

CM

2 $\vec{v}_{cm} = \frac{m_1 \vec{v}_1 + m_2 \vec{v}_2}{m_1 + m_2}$ $\vec{v} = \vec{v}_2 - \vec{v}_1$

Solve for \vec{v}_1 and \vec{v}_2

3
$$\begin{cases} m_1 \vec{v}_1 + m_2 \vec{v}_2 = m \vec{v}_{cm} \\ -\vec{v}_1 + \vec{v}_2 = \vec{v} \end{cases} \quad m = m_1 + m_2$$

Solve

8

$$\frac{m_2}{m_1} \vec{v}_1 + \vec{v}_2 = \frac{m}{m_1} \vec{v}_{cm} + \vec{v}$$

$$\vec{v}_2 = \frac{\frac{m}{m_1} \vec{v}_{cm} + \vec{v}}{1 + \frac{m_2}{m_1}} = \vec{v}_{cm} + \frac{m_1}{m} \vec{v}$$

$$\vec{v}_1 = \vec{v}_2 - \vec{v} = \vec{v}_{cm} - \frac{m_2}{m} \vec{v}$$

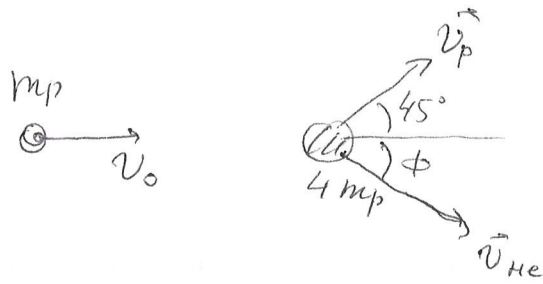
Setup 2
$$\frac{m_1 v_1^2}{2} + \frac{m_2 v_2^2}{2} = \frac{m_1}{2} \left(v_{cm}^2 + \left(\frac{m_2}{m}\right)^2 v^2 - \frac{2m_2}{m} \vec{v}_{cm} \cdot \vec{v} \right)$$

algebra
$$+ \frac{m_2}{2} \left(v_{cm}^2 + \left(\frac{m_1}{m}\right)^2 v^2 + \frac{2m_1}{m} \vec{v}_{cm} \cdot \vec{v} \right)$$

10

$$= \frac{m v_{cm}^2}{2} + \frac{1}{2} \frac{m_1 m_2^2 + m_2 m_1^2}{(m_1 + m_2)^2} v^2 = \frac{m v_{cm}^2}{2} + \frac{1}{2} \mu v^2$$

7.14 (BI) (CM)



Cons. of momentum

4 $m_p v_0 = m_p v_p \cos 45^\circ + 4m_p v_{He} \cos \phi$ (1)

$v_p \sin 45^\circ = v_{He} \sin \phi$ (2)

$\cos 45^\circ = \sin 45^\circ = \frac{1}{\sqrt{2}}$

Cons. of energy

2 $\frac{1}{2} m_p v_0^2 = \frac{1}{2} m_p v_p^2 + 2 m_p v_{He}^2$ (3)

From first two equations

$\frac{v_p}{\sqrt{2}} - v_{He} \sin \phi = 0$

$\frac{v_p}{\sqrt{2}} + 4 v_{He} \cos \phi = v_0$

Solve for v_{He}, v_p 5

subtract first from second

$v_{He} (4 \cos \phi + \sin \phi) = v_0 \rightarrow v_{He} = \frac{v_0}{4 \cos \phi + \sin \phi}, v_p = \frac{v_0 \sqrt{2} \sin \phi}{4 \cos \phi + \sin \phi}$

eq. for ϕ

Substitute into (3), $v_0^2 = v_p^2 + 4 v_{He}^2$

8 $v_0^2 (4 \cos \phi + \sin \phi)^2 = 2 v_0^2 \sin^2 \phi + 4 v_0^2$

$16 \cos^2 \phi + \sin^2 \phi + 8 \cos \phi \sin \phi = 2 \sin^2 \phi + 4 (\cos^2 \phi + \sin^2 \phi)$

$12 \cos^2 \phi - 5 \sin^2 \phi + 8 \cos \phi \sin \phi = 0$

Solve 4

$5 \tan^2 \phi - 8 \tan \phi - 12 = 0$

$\tan \phi = \frac{4 \pm \sqrt{16 + 60}}{5} = 2.5436$ (negative root is unphysical)

$\phi = 1.1962 \text{ rad}$ $4 \cos \phi + \sin \phi = 2.3942$

2 $v_{He} = \frac{v_0}{2.3942} = 0.4177 v_0$ $v_p = \frac{v_0 \sqrt{2} \sin \phi}{2.3942} = 0.5496 v_0$

CM B2.

CM

Motions in x and y directions are independent, therefore

$$x = A_1 \cos \omega t + B_1 \sin \omega t, \quad \omega = \sqrt{\frac{k}{m}}$$

5 $y = A_2 \cos 2\omega t + B_2 \sin 2\omega t$

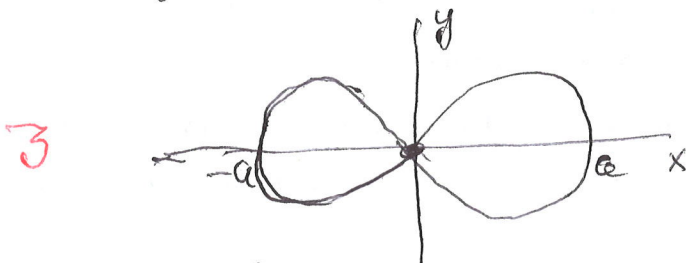
3 $\dot{x} = -A_1 \omega \sin \omega t + B_1 \omega \cos \omega t$
 $\dot{y} = -2A_2 \omega \sin 2\omega t + 2B_2 \omega \cos 2\omega t$

2. $t=0$ $A_1 = a$ $A_2 = 0$
 $B_1 = 0$ $2B_2 \omega = v_0$

5 $x = a \cos \omega t$
 $\dot{x} = -a\omega \sin \omega t$
 $y = \frac{v_0}{2\omega} \sin 2\omega t$ $\dot{y} = v_0 \cos 2\omega t$

3. xy plane

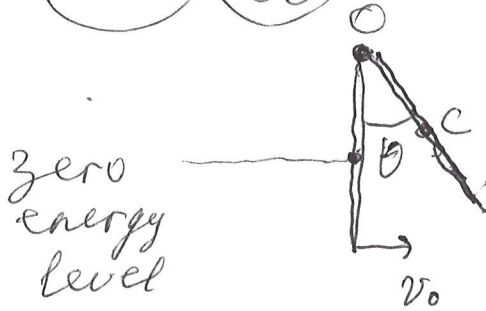
5 $y = \frac{v_0}{\omega} \sin \omega t \cos \omega t = \pm \frac{v_0}{\omega} \frac{x}{a} \sqrt{1 - \frac{x^2}{a^2}}$



Symmetric in x and y axis

4. $\frac{x^2}{a^2} + \frac{\dot{x}^2}{(a\omega)^2} = 1$ - ellipse 2

CM B3



3 $V = mg \frac{l}{2} (1 - \cos \theta)$

4 1. $L = \frac{I \dot{\theta}^2}{2} - \frac{mgl}{2} (1 - \cos \theta)$

Lagr equation of motion

4 $I \ddot{\theta} + mg \frac{l}{2} \sin \theta = 0$

$I = \frac{1}{3} ml^2, \quad \ddot{\theta} + \frac{3g}{2l} \sin \theta = 0$

2. For small θ the general solution is

2 $\theta = A \cos \omega t + B \sin \omega t \quad \omega = \sqrt{\frac{3g}{2l}}$

$\dot{\theta} = -\omega A \sin \omega t + \omega B \cos \omega t$

initial conditions: $t=0; \theta=0, \dot{\theta} = \frac{v_0}{l}$

$\rightarrow A=0 \quad \omega B = \frac{v_0}{l}, \quad B = \frac{v_0}{l\omega}$

4 $\theta = \frac{v_0}{l\omega} \sin \omega t = \frac{v_0}{\sqrt{\frac{3}{2}gl}} \sin \omega t$

3. 3 Small amplitude: $v_0 \ll \sqrt{\frac{3}{2}gl}$

4. From the conservation of energy

$\frac{I [\dot{\theta}(0)]^2}{2} > mgl$

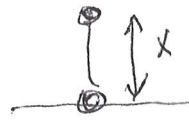
5 $\frac{ml^2}{6} \frac{v_0^2}{l^2} > mgl$

$v_0^2 > 6gl \quad \text{or} \quad v_0 > \sqrt{6gl}$

B4. CM

1. $3 m \dot{v} = -cv - mg$

$$-\frac{m dv}{cv + mg} = dt$$



$$-\frac{m}{c} \ln(v + \frac{mg}{c}) = t + \text{const}$$

$$v + \frac{mg}{c} = C e^{-\frac{ct}{m}}$$

3.
$$v = C e^{-\frac{ct}{m}} - \frac{mg}{c}$$

c: $\frac{M}{T}$ 2

$$C = v_0 + \frac{mg}{c}$$

$$v = (v_0 + \frac{mg}{c}) e^{-\frac{ct}{m}} - \frac{mg}{c}$$

2

$$x = -\frac{m}{c} (v_0 + \frac{mg}{c}) e^{-\frac{ct}{m}} - \frac{mg}{c} t + \text{const}$$

$$x(0) = 0 \Rightarrow -\frac{m}{c} (v_0 + \frac{mg}{c}) + \text{const} = 0$$

2

$$\text{const} = \frac{m}{c} (v_0 + \frac{mg}{c})$$

$$x = \frac{m}{c} (v_0 + \frac{mg}{c}) (1 - e^{-\frac{ct}{m}}) - \frac{mg}{c} t$$

2. max height: 2 $v = 0 \rightarrow (v_0 + \frac{mg}{c}) e^{-\frac{ct}{m}} = \frac{mg}{c}$

2
$$e^{-\frac{ct}{m}} = \frac{\frac{mg}{c}}{v_0 + \frac{mg}{c}} = \frac{1}{\frac{cv_0}{mg} + 1} \quad t = \frac{m}{c} \ln(1 + \frac{cv_0}{mg})$$

3. 4
$$x_{\text{max}} = \frac{m}{c} (v_0 + \frac{mg}{c}) (1 - \frac{1}{\frac{cv_0}{mg} + 1}) - \frac{mg}{c} \frac{m}{c} \ln(1 + \frac{cv_0}{mg})$$

$$= \frac{m}{c} (v_0 + \frac{mg}{c}) \frac{\frac{cv_0}{mg}}{\frac{cv_0}{mg} + 1} - \frac{m^2 g}{c^2} \ln(1 + \frac{cv_0}{mg})$$

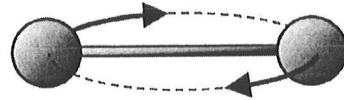
$$x_{\text{max}} = \frac{m v_0}{c} - \frac{m^2 g}{c^2} \ln(1 + \frac{c v_0}{m g})$$

5 4. en. dissipated
$$\frac{m v_0^2}{2} - m g \left[\frac{m v_0}{c} - \frac{m^2 g}{c^2} \ln(1 + \frac{c v_0}{m g}) \right]$$

$$= \frac{m v_0^2}{2} - \frac{m^2 g}{c} \left[v_0 - \frac{m g}{c} \ln(1 + \frac{c v_0}{m g}) \right]$$

B3 The quantum mechanical energy levels of a rotating dumbbell are given by

$$E_\ell = B\ell(\ell+1) \quad \text{for } \ell = 0, 1, 2, 3, 4, \dots$$



where B is the rotational constant. For each value of ℓ there are $2\ell+1$ possible quantum states of the same energy.

a. Give the partition function Z .

In parts b. and c. we assume $T = 2B/k_B$.

b. Make a table of the probabilities P_ℓ that the energy of the dumbbell is E_ℓ , for the values $\ell = 0, 1, 2$, and 3 only. Explain why higher values of ℓ are barely relevant at this temperature.

c. Give the average energy of the dumbbell in terms of T .

d. What is the average energy for $T \ll B/k_B$?

ANSWERS

3
4

$$Z = \sum_{\ell=0}^{\infty} (2\ell+1)e^{-\beta B\ell(\ell+1)}$$

$1.3 B = \frac{\hbar^2}{2I}$ since $H = \frac{L^2}{2I}$
 $42. m = -l, \dots, l$ degeneracy $= 2l+1$

$$T = 2B/k_B \Rightarrow \frac{B}{k_B T} = \frac{1}{2} = \beta B \Rightarrow$$

$$4. Z = \sum_{\ell=0}^{\infty} (2\ell+1)e^{-\frac{1}{2}\ell(\ell+1)} = 1 + 3e^{-1} + 5e^{-3} + 7e^{-6} + 9e^{-10} + \dots \approx 1 + 3e^{-1} + 5e^{-3} + 7e^{-6} = 2.36992$$

At this temperature, the terms $(2\ell+1)e^{-\frac{1}{2}\ell(\ell+1)}$ in the partition function rapidly approach zero for increasing values of ℓ . The probability for particles to be in higher energy states $\ell \geq 4$ becomes negligibly small. The term for $\ell+1$ is smaller than the term for ℓ by a factor of approximately $\frac{1}{e^{\ell+1}}$, and for $\ell=3$ this is already a factor ~ 0.01 .

5

We decide to take the value $Z = 2.36992$ and calculate the probabilities:

$$P_0 = 42.20\% \quad P_1 = 46.57\% \quad P_2 = 10.50\% \quad P_3 = 0.73\%$$

4

$$5. E_{av} = \sum_{\ell=0}^{\infty} \left(\frac{1}{Z} k_B T \right) P_\ell \cdot \ell(\ell+1) = B (P_0 \cdot 0 + P_1 \cdot 2 + P_2 \cdot 6 + P_3 \cdot 12) = 1.65 B$$

A1. 300 g of aluminum block at 100 °C is placed in a calorimeter cup with 400 g of water. The mass of the copper calorimeter cup is 80 g. The initial temperature of the water and the cup is 22 °C. What's the final temperature? ($c_{\text{water}} = 4.184 \text{ J/g} \cdot \text{K}$; $c_{\text{Al}} = 0.9 \text{ J/g} \cdot \text{K}$ $c_{\text{Cu}} = 0.385 \text{ J/g} \cdot \text{K}$)

Solution: (25 pt)

The heat input of the water and cup is the same as the heat output of the aluminum block. Assuming the final temperature is T :

$$(100 - T)C_{\text{Al}} = (T - 22) (C_{\text{water}} + C_{\text{Cu}}) \quad (5 \text{ pt})$$

$$C_{\text{Al}} = 300 \times 0.9 \text{ J/K} = 270 \text{ J/K} \quad (5 \text{ pt})$$

$$C_{\text{water}} = 400 \times 4.184 \text{ J/K} = 1674 \text{ J/K} \quad (5 \text{ pt})$$

$$C_{\text{Cu}} = 80 \times 0.385 \text{ J/K} = 30.8 \text{ J/K} \quad (5 \text{ pt})$$

$$\text{Hence: } T = 32.7 \text{ }^\circ\text{C} \quad (5 \text{ pt})$$

A2. Consider an ideal gas system that changes temperature from T_1 to T_2 . Show that the entropy change of the isobaric process is γ times of that of the isochoric process, where $\gamma = C_p/C_v$.

Solution: (25 pt)

To calculate entropy change, we use:

$$dS = \delta Q/T, \text{ and } \delta Q = dU + \delta W = dU + PdV. \quad (5 \text{ pt})$$

In isochoric process: $dV = 0$, $\delta Q = dU = C_v dT$,

$$dS = \delta Q/T = C_v dT/T. \quad (5 \text{ pt})$$

In isobaric process: $\delta Q = C_v dT + PdV$

$$\text{Because } P \text{ is constant: } PdV = d(PV), \delta Q = C_v dT + d(PV) \quad (5 \text{ pt})$$

$$\text{Because } PV = nRT, \delta Q = C_v dT + d(nRT) = C_v dT + nR dT = (C_v + nR) dT \quad (5 \text{ pt})$$

$$dS = \delta Q/T = (C_v + nR) dT/T.$$

Therefore, the ratio of the entropy change is

$$dS_{\text{isobaric}}/dS_{\text{isochoric}} = (C_v + nR)/C_v = C_p/C_v = \gamma. \quad (5 \text{ pt})$$

A3. Show that for any PVT system with two degrees of freedom, the equation of state can be found using $\ln(V) = \int \alpha dT - k_T dP$, where $\alpha = \frac{1}{V} \left(\frac{\partial V}{\partial T} \right)_P$ is the thermal expansion coefficient, $k_T = -\frac{1}{V} \left(\frac{\partial V}{\partial P} \right)_T$ is the isothermal compressibility.

Solution: (25 pt)

$$dV = \left(\frac{\partial V}{\partial T}\right)_P dT + \left(\frac{\partial V}{\partial P}\right)_T dP. \quad (5 \text{ pt})$$

Hence:

$$\frac{dV}{V} = \frac{1}{V} \left(\frac{\partial V}{\partial T}\right)_P dT + \frac{1}{V} \left(\frac{\partial V}{\partial P}\right)_T dP. \quad (5 \text{ pt})$$

Therefore,

$$\ln(V) = \int \left(\frac{\partial \ln V}{\partial T}\right)_P dT - \left(\frac{\partial \ln V}{\partial P}\right)_T dP. \quad (5 \text{ pt})$$

We can rewrite the definition:

$$\alpha = \left(\frac{\partial \ln V}{\partial T}\right)_P, \quad (5 \text{ pt})$$

$$k_T = -\left(\frac{\partial \ln V}{\partial P}\right)_T, \quad (5 \text{ pt})$$

which means:

$$\ln(V) = \int \alpha dT - k_T dP.$$

B1. Using the relation $\left(\frac{\partial U}{\partial V}\right)_T = T\left(\frac{\partial P}{\partial T}\right)_V - P$, show that the Joule Thompson coefficient $\left(\frac{\partial T}{\partial P}\right)_H = 1/C_P[T\left(\frac{\partial V}{\partial T}\right)_P - V]$ in the throttling process. (Hint: use the cyclic rule).

Solution: (25 pt)

Use the cyclic rule, we have:

$$\left(\frac{\partial T}{\partial P}\right)_H = -\left(\frac{\partial T}{\partial H}\right)_P \left(\frac{\partial H}{\partial P}\right)_T = -\left(\frac{\partial H}{\partial P}\right)_T / \left(\frac{\partial H}{\partial T}\right)_P = -\frac{1}{C_P} \left(\frac{\partial H}{\partial P}\right)_T. \quad (3 \text{ pt})$$

Here,

$$\left(\frac{\partial H}{\partial P}\right)_T = \left(\frac{\partial U}{\partial P}\right)_T + \left[\frac{\partial(PV)}{\partial P}\right]_T. \quad (1) \quad (3 \text{ pt})$$

$$\left(\frac{\partial U}{\partial P}\right)_T = \left(\frac{\partial U}{\partial V}\right)_T \left(\frac{\partial V}{\partial P}\right)_T \quad (3 \text{ pt})$$

Given $\left(\frac{\partial U}{\partial V}\right)_T = T\left(\frac{\partial P}{\partial T}\right)_V - P$, this becomes:

$$\left(\frac{\partial U}{\partial P}\right)_T = \left[T\left(\frac{\partial P}{\partial T}\right)_V - P \right] \left(\frac{\partial V}{\partial P}\right)_T = T\left(\frac{\partial P}{\partial T}\right)_V \left(\frac{\partial V}{\partial P}\right)_T - P\left(\frac{\partial V}{\partial P}\right)_T. \quad (3 \text{ pt})$$

Using the cyclic rule, it follows that:

$$\left(\frac{\partial P}{\partial T}\right)_V \left(\frac{\partial V}{\partial P}\right)_T = -\left(\frac{\partial V}{\partial T}\right)_P. \quad (3 \text{ pt})$$

Hence,

$$\left(\frac{\partial U}{\partial P}\right)_T = -T\left(\frac{\partial V}{\partial T}\right)_P - P\left(\frac{\partial V}{\partial P}\right)_T. \quad (2) \quad (3 \text{ pt})$$

We also have:

$$\left[\frac{\partial(PV)}{\partial P}\right]_T = V + P\left(\frac{\partial V}{\partial P}\right)_T. \quad (3) \quad (3 \text{ pt})$$

Substitute (2) and (3) into Eq. (1), one gets

$$\left(\frac{\partial H}{\partial P}\right)_T = -T\left(\frac{\partial V}{\partial T}\right)_P - P\left(\frac{\partial V}{\partial P}\right)_T + V + P\left(\frac{\partial V}{\partial P}\right)_T = -T\left(\frac{\partial V}{\partial T}\right)_P + V. \quad (3 \text{ pt})$$

Hence, the Joule Thompson coefficient is

$$\left(\frac{\partial T}{\partial P}\right)_H = -\frac{1}{C_P} \left(\frac{\partial H}{\partial P}\right)_T = -\frac{1}{C_P} \left[-T\left(\frac{\partial V}{\partial T}\right)_P + V \right] = \frac{1}{C_P} \left[T\left(\frac{\partial V}{\partial T}\right)_P - V \right] \quad (1 \text{ pt})$$

B2. Consider a gas system that satisfies $\left(\frac{\partial v}{\partial T}\right)_P = \frac{R}{P} + \frac{a}{T^2}$ and $\left(\frac{\partial v}{\partial P}\right)_T = -Tf(P)$, where v is the molar volume, a and R are constants, and $f(p)$ is a function of P . Show that:

(a) $f(P) = \frac{R}{P^2}$.

(b) The equation of state of this system is $Pv = RT - aP/T$.

Solution: (25 pt)

(a) Because the 2nd order partial derivative:

$$\frac{\partial^2 v}{\partial T \partial P} = \left(\partial \left(\frac{\partial v}{\partial T} \right)_P / \partial P \right)_T = \left(\partial \left(\frac{\partial v}{\partial P} \right)_T / \partial T \right)_P, \quad (3 \text{ pt})$$

$$\text{and } \left(\partial \left(\frac{\partial v}{\partial T} \right)_P / \partial P \right)_T = -R/P^2 \quad (3 \text{ pt})$$

$$\left(\partial \left(\frac{\partial v}{\partial P} \right)_T / \partial T \right)_P = -f(P), \quad (3 \text{ pt})$$

one has

$$f(P) = R/P^2. \quad (3 \text{ pt})$$

(b) We can integrate to find the v ,

$$v = \int \left(\frac{\partial v}{\partial T} \right)_P dT = \int [R/P + a/T^2] dT = RT/P - a/T + f(P) + c, \quad (3 \text{ pt})$$

$$v = \int \left(\frac{\partial v}{\partial P} \right)_T dP = \int [-Tf(P)] dP = \int [-TR/P^2] dP = RT/P + g(T) + c. \quad (3 \text{ pt})$$

After comparison, one gets

$$f(P) = 0, g(T) = -a/T. \quad (2 \text{ pt})$$

Hence

$$v = \frac{RT}{P} - \frac{a}{T} + c. \quad (2 \text{ pt})$$

For gas system at high- T limit, $v = RT/P$. So, $c = 0$. (2 pt)

Finally, the equation of state is

$$v = RT/P - a/T. \quad (1 \text{ pt})$$

B3 2nd page.

$$(8) \quad (5) \Rightarrow \left(P + \frac{a}{V^2} \right) = \frac{RT}{V-b} \Rightarrow P = \frac{RT}{V-b} - \frac{a}{V^2}$$

$$\Rightarrow \left(\frac{\partial P}{\partial T} \right)_V = \frac{R}{V-b} \quad (6)$$

$$\text{and } \left(\frac{\partial V}{\partial T} \right)_P = \frac{1}{\left(\frac{\partial P}{\partial V} \right)_P}$$

$$(5) \Rightarrow T = \left(P + \frac{a}{V^2} \right) \frac{V-b}{R} \quad \text{or } P + \frac{a}{V^2} = \frac{RT}{V-b}$$

$$\text{then : } \left(\frac{\partial T}{\partial V} \right)_P = -\frac{2a}{V^3} \frac{V-b}{R} + \frac{P + \frac{a}{V^2}}{R} = \frac{RT}{(V-b)R} - \frac{2a(V-b)}{R V^3}$$

$$\Rightarrow \left(\frac{\partial V}{\partial T} \right)_P = R / \left[\frac{RT}{V-b} - \frac{2a(V-b)}{V^3} \right] \quad (7)$$

$$(4) \text{ in } (3) : C_p - C_v = \left[T \left(\frac{\partial P}{\partial T} \right)_V \right] \left(\frac{\partial V}{\partial T} \right)_P$$

and using (6) and (7), one gets:

$$C_p - C_v = \frac{R^2}{(V-b) \left[\frac{RT}{V-b} - \frac{2a(V-b)^2}{V^3} \right]}$$

$$C_p - C_v = \frac{R}{1 - \frac{2a(1-b/V)^2}{VRT}}$$

$\xrightarrow{V \rightarrow \infty}$ R , which is the result for an ideal gas.

$$\textcircled{7} \quad C_V = \left(\frac{\partial U}{\partial T} \right)_V, \quad C_P = \left(\frac{\partial H}{\partial T} \right)_P$$

a) Show that $C_P - C_V = \left[P + \left(\frac{\partial U}{\partial V} \right)_T \right] \left(\frac{\partial V}{\partial T} \right)_P$

Starting with the first law of thermodynamics, one has: $H = U + PV$

$$\Rightarrow \left(\frac{\partial H}{\partial T} \right)_P = \left(\frac{\partial U}{\partial T} \right)_P + P \left(\frac{\partial V}{\partial T} \right)_P \quad (1)$$

Let $U = U[T, V(T, P)]$, then

$$\left(\frac{\partial U}{\partial T} \right)_P = \left(\frac{\partial U}{\partial T} \right)_V + \left(\frac{\partial U}{\partial V} \right)_T \left(\frac{\partial V}{\partial T} \right)_P \quad (2)$$

Inserting (2) in (1) leads to:

$$\left(\frac{\partial H}{\partial T} \right)_P = \left(\frac{\partial U}{\partial T} \right)_V + \left(\frac{\partial U}{\partial V} \right)_T \left(\frac{\partial V}{\partial T} \right)_P + P \left(\frac{\partial V}{\partial T} \right)_P$$

$$\Rightarrow C_P - C_V = \left[P + \left(\frac{\partial U}{\partial V} \right)_T \right] \left(\frac{\partial V}{\partial T} \right)_P \quad (3)$$

b) Use the relation $P + \left(\frac{\partial U}{\partial V} \right)_T = T \left(\frac{\partial P}{\partial T} \right)_V$ to find $C_P - C_V$ for a Van der Waals gas:

$$\left(P + \frac{a}{V^2} \right) (V - b) = RT \quad (5)$$

A4

(6) pb # 5 : Thermo (easy)

$$(a) \quad W = \int_{V_0}^{10V_0} p \, dV = \int_{V_0}^{10V_0} \frac{RT}{V} \, dV = RT \ln(10) = 5.2 \times 10^3 \text{ J}$$

since $pV = \underset{n=1 \text{ mole.}}{\uparrow} RT$ and $T = 0^\circ\text{C} = 273 \text{ K}$
 $R = 8.314 \text{ J} \cdot \text{mol}^{-1} \cdot \text{K}^{-1}$

(b) adiabatic process: $pV^\gamma = \text{constant}$, $\gamma = 5/3$ for an ideal gas molecule, which is monoatomic.

Ideal gas: $pV = RT$.

$$pV^\gamma = \text{const} \Rightarrow pV V^{\gamma-1} = \text{constant}$$

$$\Rightarrow RT V^{\gamma-1} = \text{constant}$$

$$\Rightarrow T V^{\gamma-1} = \text{constant}$$

$$\Rightarrow T_i V_i^{\gamma-1} = T_f V_f^{\gamma-1}$$

$$\Rightarrow \boxed{T_f = T_i \left(\frac{V_i}{V_f} \right)^{\gamma-1}}$$

$$T_i = 0^\circ = 273 \text{ K}, \quad V_i = V_0, \quad V_f = 10V_0, \quad \gamma = 5/3$$

$$\Rightarrow \underline{\underline{T_f = 59 \text{ K or } -214^\circ\text{C}}}$$