EM A1

2) A spin 1/2 neutral Cu atom beam (copper $m=1.05\times10^{-25}$ kg) enters a Stern-Gerlach experiment 1 m long with a filed gradient $\partial B_z/\partial z$ of 12 T/m. The moment on each spin 1/2 atom is $e\hbar/2m_e = 9.274\times10^{-24}$ J/T and the atoms enter with velocity 500 m/s.

a) If the direction motion along the Stern-Gerlach is 'x', what is the velocity of the atoms along the field gradient z, at the end of the Stern-Gerlach apparatus ?

$$V_z = a_z t = \frac{F}{m} t = \frac{\mu}{m}\frac{dB}{dz}\cdot t$$

5 pts

$$= \frac{\mu}{m}\frac{dB}{dz}\frac{x}{V_x}$$

$$\left(F = ma,\ F = -\frac{dU}{d\ell} = \mu\frac{dB}{dz}\right)$$

$$F\cdot d\ell = dU$$

$$V_z = a_z t$$

$$t = \frac{x}{V_x}$$

$$\left(F = \frac{e\hbar}{2m_e}\frac{dB}{dz}\right)$$

5 pts

$$= \frac{9.3\times10^{-24}\ J/T}{1.05\times10^{-25}\ kg}\left(12\ \frac{T}{m}\right)\left(\frac{1\ m}{500\ \frac{m}{s}}\right)$$

$$= 2.13\ \frac{m}{s}$$

5 pts

b) How big is the deflection at the end of the Stern-Gerlach experiment

$$z = \frac{1}{2}at^2 = \frac{1}{2}\frac{F}{m}t^2 \quad (F = ma)$$

$$= \frac{1}{2}\frac{\mu}{m}\frac{dB}{dz}t^2 \quad F = \mu\frac{dB}{dz}$$

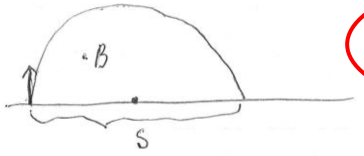
5 pts

$$z = \frac{1}{2}\frac{\mu}{m}\frac{dB}{dz}\left(\frac{x}{V_x}\right)^2 = \frac{1}{2}\frac{9.3\times10^{-24}\ J/T}{1.05\times10^{-25}\ kg}\left(12\ \frac{T}{m}\right)\left(\frac{1}{500\ \frac{m}{s}}\right)^2$$

$$= 2.13\times10^{-3}\ m$$

5 pts

EM A2



$$qvB = \frac{mv^2}{r}$$ 10 pts

$$r = \frac{mv}{qB} \qquad s = 2r = \frac{2mv}{qB}$$

5 pts

$$S = \frac{2\sqrt{2mE}}{qB} = \frac{2}{B}\sqrt{\frac{2mU}{q}}$$

where we used
$E = qU$
$U = 2\times10^{4}\,V$

$$= \frac{2}{2}\sqrt{\frac{2\times40\times1.66\times10^{-27}\cdot 2\times10^{4}}{1.6\times10^{-19}}} = 0.129\,m$$

10 pts

EM A3

$B = B_0 e^{-t/\tau}$

$\vec{B}$

Faraday law

5 pts

$$2\pi r \cdot E = -\pi r^2 \frac{\partial B}{\partial t}$$

$$E = -\frac{r}{2} \frac{\partial B}{\partial t}$$

5 pts

$$\frac{\partial B}{\partial t} = -\frac{1}{\tau} B_0 e^{-t/\tau}$$

Could derive from the Ohms law

$$j = \sigma E = \frac{\sigma r}{2\tau} B_0 e^{-t/\tau}$$

10 pts

The direction (counterclockwise) follows from the Lenz law – Since $\vec{B}$ is decreasing, the induced current create B_{ind} in the same direction

5 pts

EM A4

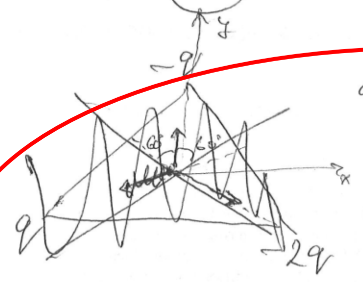
(a) $B_0 = \frac{E_0}{C} = \frac{5\times10^{-3}\ V/m}{3\times10^{8}\ m/s} = 1.67\times10^{-11}\ T$ 12 pts

(b) $I = \frac{\varepsilon_0 E_0^2 C}{2} = \frac{8.854\times10^{-12}\cdot(5\times10^{-3})^2\cdot 3\times10^{8}}{2}$

13 pts

$= 3.32\times10^{-8}\ \frac{W}{m^2}$

EM (B1)

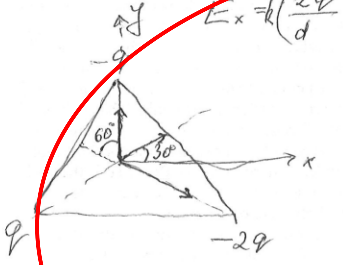


distance to any corner

$$d = \frac{a}{2\sin 60^\circ} = \frac{a}{\sqrt{3}}$$

5 pts

$$V = k\frac{(q-q-2q)}{d} = -k\frac{2q}{d} = -k\frac{2\sqrt{3}\,q}{a} \qquad k = \frac{1}{4\pi\epsilon_0}$$



$$E_x = k\left(\frac{2q}{d}\cos 30^\circ + \frac{q}{d}\cos 30^\circ\right) = k\frac{3q}{d}\frac{\sqrt{3}}{2} = k\frac{9q}{2a}$$

$$E_y = k\left(\frac{q}{r} + \frac{q}{r}\cos 60^\circ - \frac{2q}{r}\cos 60^\circ\right)$$

$$= k\frac{q}{r}(1-\cos 60^\circ) = k\frac{q}{2r} = k\frac{\sqrt{3}}{2}\frac{q}{a}$$

$$E = \sqrt{E_x^2+E_y^2} = k\frac{q}{a}\sqrt{\frac{81}{4}+\frac{3}{4}} = k\frac{q}{a}\sqrt{21}$$

10 pts

angle which makes $\vec{E}$ with the x axis

$$\tan\alpha = \frac{E_y}{E_x} = \frac{\sqrt{3}}{2} \Big/ \frac{9}{2} = \frac{\sqrt{3}}{9}$$

$$\alpha = 10.9^\circ$$

(b) pot energy

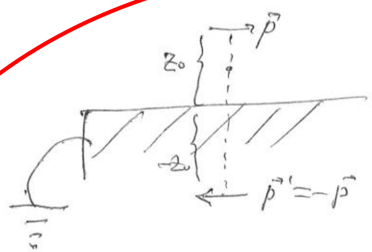
$$U_0 = k\left(-\frac{q^2}{a} - \frac{2q^2}{a} + \frac{2q^2}{a}\right) = -k\frac{q^2}{a}$$

increase length from a to 2a $\quad U_1 = -k\frac{q^2}{2a}$

Work $= +(U_1 - U_0) = +k\frac{q^2}{2a}$ ~~negative~~ positive since U is increasing

10 pts

EM (B2)



15 pts

Use method of images. The actual dipole create the field

$$\vec{E}(\vec{r}) = -k\frac{\vec{p}}{r^3} \quad k = \frac{1}{4\pi\varepsilon_0}$$ since $\vec{p} \perp \hat{r}$ on the z axis

$$\vec{E}_0 = -k\,\vec{p}\,\frac{1}{|z-z_0|^3}$$

Similar the image charge create the field

$$\vec{E}_1 = k\,\vec{p}\,\frac{1}{|z+z_0|^3}$$

$$\vec{E} = k\vec{p}\left[-\frac{1}{|z-z_0|^3} + \frac{1}{|z+z_0|^3}\right]$$

(b) for $z >> z_0$

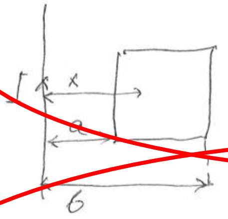
$$\frac{1}{(z+z_0)^3} - \frac{1}{(z-z_0)^3} \approx \frac{-3z^2z_0 - z_0^3 - 3z^2z_0 - z_0^3}{z^6} \approx \frac{-6z_0}{z^4}$$

$$\vec{E} = k\vec{p}\,\frac{-6z_0}{z^4}$$

this is a quadrupole field

10 pts

EM B3



$\mathcal{E} = -\frac{d}{dt}\int B\,dA$ 5 pts

$B = \frac{\mu_0 I}{2\pi x}$ from the Ampere law

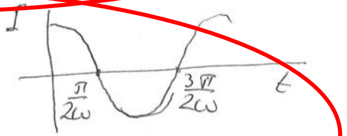
$\int B\,dA = \int_a^b \frac{\mu_0 I}{2\pi x}(b-a)\,dx$

$= \frac{\mu_0 I}{2\pi}(b-a)\ln\frac{b}{a}$ 10 pts

$\mathcal{E} = -\frac{\mu_0}{2\pi}(b-a)\ln\frac{b}{a}\frac{dI}{dt} = \frac{\mu_0}{2\pi}(b-a)\ln\frac{b}{a}\,\omega I_0\sin\omega t$

$I_{ind} = \mathcal{E}/R$

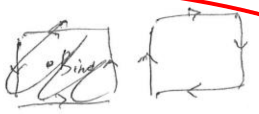
(b) at $t = \frac{\pi}{2\omega}$ current is decreasing therefore the direction of $\vec{B}_{ind}$ should be ~~the same as $\vec{B}$~~ positive



current

5 pts

Similar at $t = \frac{3\pi}{2\omega}$



5 pts

EM (B4)

(i)

volume charge density

$\rho_b = \vec{\nabla} \cdot \vec{P} = 0$

5 pts

Surface charge density

$\sigma_b(\theta) = \hat{r} \cdot \vec{P} = P\cos\theta \quad (1)$

5 pts

(ii) we have to solve electrostatic problem with the charge distribution given by (1). The Laplace equation for $r<a$ has the general solution

$V = A r\cos\theta$ ($\frac{1}{r^2}$ solution does not satisfy boundary condition at $r=0$)

5 pts

for $r>a$ $V = \frac{B\cos\theta}{r^2}$ since $V \to 0$ @ $r \to \infty$

match potential

$V(a-0) = V(a+0)$

$Aa = \frac{B}{a^2}$

5 pts

The field has a discontinuity for the normal component

$E(a+0,\theta) - E(a-0,\theta) = \frac{\sigma_b(\theta)}{\varepsilon_0} = \frac{P\cos\theta}{\varepsilon_0} \quad (E_r = -\frac{\partial V}{\partial r})$

or $\quad \frac{2B}{a^3} + A = \frac{P}{\varepsilon_0}$

$A\left(\frac{2a^3}{a^3} + 1\right) = \frac{P}{\varepsilon_0} \rightarrow A = \frac{P}{3\varepsilon_0} \quad B = \frac{P}{3\varepsilon_0} a^3$

Therefore $V = \frac{P}{3\varepsilon_0} r\cos\theta \quad r<a$

$\frac{Pa^3}{3\varepsilon_0} \frac{\cos\theta}{r^2} \quad r>a$

5 pts

~~Independence Difference~~

A 1 QUANTUM

a) Non relativistic

7 pts.

$$K = \frac{p^2}{2m_e} = \frac{h^2}{2m_e\lambda^2} \Rightarrow \lambda = \sqrt{\frac{h^2}{2m_e K}}$$

For photon

$$\left[E = pc = \frac{h}{\lambda}c = \frac{\cancel{h}\sqrt{2m_e K}\,c}{\cancel{h}}\right].$$

b) Relativistic
For electron

10 pts.

$$E_{electron} = K + m_e c^2 = \sqrt{p^2c^2 + m_e^2c^4}.$$

$$K^2 + 2Km_ec^2 + \cancel{m_e^2c^4} = p^2c^2 + \cancel{m_e^2c^4}$$

$$K^2 + 2Km_ec^2 = \frac{h^2}{\lambda^2}c^2$$

$$\left[E_{photon} = pc = \frac{h}{\lambda}c = \sqrt{K^2 + 2Km_ec^2}\right]$$

c) For $K = 1$ MeV we use the relativistic case because $K \sim$ Rest mass energy

8 pts.

$$E_{photon} = \sqrt{(10^6 \times 1.6 \times 10^{-19})^2 + 2(10^6 \times 1.6 \times 10^{-19}) \times 9.11 \times 10^{-31} \times (3\times10^8)^2}.$$

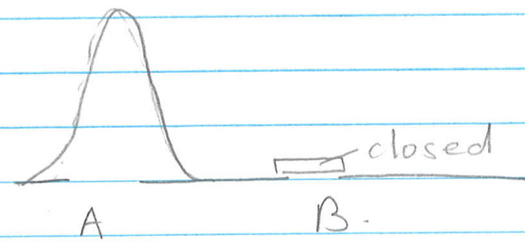
OR $= \sqrt{10^{12} + 2(10^6)\,0.511 \times 10^6} = 10^6\sqrt{1 + 2(0.51)}$
$= 10^6\sqrt{}$

QUANTUM

2

A2

8 points a)



8 pts.

8 points b) No $I_{AB} \neq I_A + I_b$ because of interference

8 pts

8pts c) Here $I_A + I_b$ consist of particles in different eigen states ∴ ~~No interference~~

8pts

∴ $I_{AB} = I_A + I_b$

For part (b) + (c) the reason is worth 4 points in each part.

A 3

a) Show $\langle x \rangle \neq 0$ for $\Psi = A\Psi_0 + B\Psi_1$

13 points

$$\left[\langle x \rangle = \int (A\Psi_0 + B\Psi_1)^* x \, (A\Psi_0 + B\Psi_1)\, dx \right]$$ 4 pts.

Since A + B are real.

$$\langle x \rangle = A^2 \underbrace{\int \Psi_0^* x \Psi_0}_{\langle x \rangle_{n=0} = 0} + B^2 \underbrace{\int \Psi_1^* x \Psi_1}_{\langle x \rangle_{n=1} = 0}$$

$$+ AB \underbrace{\int \Psi_0^* x \Psi_1}_{\neq 0} + BA \underbrace{\int \Psi_1^* x \Psi_0}_{\neq 0}$$ 6 points

because

$$= 2AB \int \Psi_0 x \Psi_1$$ because $\Psi_0 + \Psi_1$ are REAL for HO. 3 pts.

$$= 2AB \langle \Psi_0 | x | \Psi_1 \rangle$$

12 points

b) We rewrite $A\Psi_0 + B\Psi_1$ to normalize it

5 points

i.e.

$$\left[\int (A\Psi_0 + B\Psi_1)^2 dx = 1 \Rightarrow A^2 \underbrace{\int \Psi_0 \Psi_0}_{=1} + B^2 \underbrace{\int \Psi_1 \Psi_1}_{=1} + AB \underbrace{\int \Psi_0 \Psi_1}_{=0} + BA \underbrace{\int \Psi_1 \Psi_0}_{=0} = 1 \right.$$

(orthonormality)

$$\boxed{A^2 + B^2 = 1}$$ Normalization. 1 point.

$\langle x \rangle$ is proportional to $2AB$.

$$2AB = 2A(1-A^2)^{1/2}$$

We want to maximize this wrt A

$$\frac{\partial}{\partial A} 2A(1-A^2)^{1/2} = 2A \frac{1 \cdot -\cancel{2}A}{\cancel{2}(1-A^2)^{1/2}} + 2(1-A^2)^{1/2} = 0$$

6 points.

$$\frac{\cancel{2}A^2}{(1-A^2)^{1/2}} = \cancel{2}(1-A^2)^{1/2}.$$

$$A^2 = 1 - A^2 \Longrightarrow A = \pm 1/\sqrt{2} \;;\; B = \pm 1/\sqrt{2}$$

A 4

a) Because the muon is so much heavier than an electron, it is a significant fraction of the mass of the proton.

$$m_p = 1836\, m_e$$

$$\therefore m_{\mu on} = \frac{m_p}{1836} \times 207 \sim 0.11\, m_p$$

7 points

If do not use m_{eff} give 0 for this section

$\therefore$ We cannot just substitute $m_{\mu on}$ for m_e in the Rydberg formula; rather we need to use the effective mass of the proton + muon system.

$$\frac{1}{m_{eff}} = \frac{1}{m_p} + \frac{1}{m_{muon}} \Rightarrow m_{eff} = \frac{m_p m_{\mu on}}{m_p + m_{\mu on}} = 186\, m_e$$

b) Energy of ground state + first excited state

$$E_n = -\frac{me^4}{2(4\pi\epsilon_0)^2\hbar^2}\frac{1}{n^2}$$

10 points

Here m is the effective mass, as compared to the $m_{electron}$ for the Hydrogen atom

We know $E_{n=1} = -13.6\,eV$ for Hydrogen

Here we have $m = 186\, m_e$

$$\left[\therefore E_{n=1} = -13.6 \times 186 = -2529.6\, eV\right]$$

$$\left[E_{1st\,excited} = -\frac{13.6}{4} \times 186 = -632.4\, eV\right]$$

A 4 (cont)

c) Wavelength of photon

8 pts.

$$E_{n=2} - E_{n=1} = \frac{hc}{\lambda} = 1897.2\,eV$$

$$\frac{hc}{1897.2\,eV} = \therefore \lambda = \frac{6.626 \times 10^{-34} \times 3 \times 10^{8}}{1897.2 \times \underbrace{1.6 \times 10^{-19}}_{\text{Convert to Joules}}}$$

$$= 6.54 \times 10^{-3-34+8+19}$$

$$= 6.54 \times 10^{-10}\,m$$

$$= 0.654\,nm \text{ or } 6.54\,\mathring{A}$$

B1

7 points

a) Time dependent Sch Eqn.

$$-\frac{\hbar^2}{2m}\nabla^2\Psi = i\hbar\frac{\partial\Psi}{\partial t} \qquad \Psi(\bar{r},t)$$

OR

$$-\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} - \frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial y^2} - \frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial z^2} = i\hbar\frac{\partial\Psi}{\partial t}$$

Inside the well.

8 points

b) The 3 dim time ind wave function is

$$\Psi(x,y,z) = \left(\sqrt{\frac{2}{L}}\right)^3 \sin\frac{n_x\pi x}{L}\sin\frac{n_y\pi y}{L}\sin\frac{n_z\pi z}{L}$$

where n_x, n_y + n_z are integers.

The lowest energy state is where $n_{x,y,z}$ is lowest + this occurs for $n_x = n_y = n_z = 1$.

$$\therefore \Psi = \left(\sqrt{\frac{2}{L}}\right)^3 \sin\frac{\pi x}{L}\sin\frac{\pi y}{L}\sin\frac{\pi z}{L}$$

9 points

c) Which is the lowest energy state with degeneracy > 3

Ground 1 1 1 — 3

{1 2 1, 1 1 2, 2 1 1} 1st exc. — 6

{3 1 1} × 3 3rd exc.

{2 2 1} × 3 2nd exc.

{1 2 3, 2 1 3, 3 1 2, 3 2 1, 1 3 2, 2 3 1} 4th excited

E (~~3~~ 4th excited state 6 fold degenerate)

$$E = \underbrace{1^2+2^2+3^2}_{=14}\frac{\pi^2\hbar^2}{2mL^2}$$

Part (a) 17 points
Part (b) 18 points

B2

a) Operator $= A s_y + B s_z$

$$= A\frac{\hbar}{2}\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} + B\frac{\hbar}{2}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$= \frac{\hbar}{2}\begin{bmatrix} B & -iA \\ iA & -B \end{bmatrix}$$

3 points

17 points

Eigen values $\frac{\hbar}{2}\begin{vmatrix} B-\lambda & -iA \\ iA & -B-\lambda \end{vmatrix} = 0$

$$\frac{\hbar}{2}\left[(B-\lambda)(-B-\lambda) - A^2\right] = 0$$

$$\frac{\hbar}{2}\left(-B^2 - \cancel{B\lambda} + \cancel{B\lambda} + \lambda^2 - A^2\right) = 0$$

The 2 eigen values

$$\lambda = +\frac{\hbar}{2}(A^2+B^2)^{1/2}$$

$$\lambda = -\frac{\hbar}{2}(A^2+B^2)^{1/2}$$

4 points

10 points

Eigen vectors

$$\frac{\hbar}{2}\begin{bmatrix} B & -iA \\ iA & -B \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \pm\frac{\hbar}{2}(A^2+B^2)^{1/2}\begin{bmatrix} a \\ b \end{bmatrix}$$

For positive eigen value

Then $aB - biA = +(A^2+B^2)^{1/2}\, a$

and $a\, iA - bB = +(A^2+B^2)^{1/2}\, b$

From either 1 or 2 you can obtain

$$\frac{a}{b} = \frac{iA}{B-(A^2+B^2)^{1/2}}$$

4 points

B 2

Then the eigen vector for + Eigen value is

$$\chi_+ = N_{\text{normalization}} \begin{bmatrix} iA \\ B - (A^2+B^2)^{1/2} \end{bmatrix}$$

For N need $[\quad][\quad] = N^2$

6 points

i.e. $(-iA)(iA) + [B-(A^2+B^2)^{1/2}][B-(A^2+B^2)^{1/2}] = 1$

$$A^2 + [B-(A^2+B^2)^{1/2}]^2$$

$$\therefore N = \left[\frac{1}{A^2+(B-(A^2+B^2)^{1/2})^2}\right]^{1/2}$$

$$\therefore \chi_+ = \left[\frac{1}{A^2+[(B-(A^2+B^2)^{1/2}]^2}\right]^{1/2} \begin{bmatrix} iA \\ B-(A^2+B^2)^{1/2} \end{bmatrix}$$

For the NEGATIVE Eigen value, repeat same steps to obtain

$$\chi_- = N \begin{bmatrix} iA \\ B + (A^2+B^2)^{1/2} \end{bmatrix}$$

$$\& \; N = \left[\frac{1}{A^2+(B+(A^2+B^2)^{1/2})^2}\right]^{1/2}$$

$$\chi_- = \left[\frac{1}{A^2+(B+(A^2+B^2)^{1/2})^2}\right]^{1/2} \begin{bmatrix} iA \\ B+(A^2+B^2)^{1/2} \end{bmatrix}$$

B2 Part (b) 8 points

Let us choose $\chi_+ = \left[\dfrac{1}{A^2+(B-(A^2+B^2)^{1/2})^2}\right]^{1/2} \begin{bmatrix} iA \\ B-(A^2+B^2)^{1/2} \end{bmatrix}$

4pts. What is the probability that a measurement of S_z gives $+\hbar/2$?

$$\chi_+ = c_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

1pt The probability is $|c_1|^2$ + we can get it either

i) by inspection

i.e. $c_1 = \left[\dfrac{1}{}\right]^{1/2} iA$

$\therefore |c_1|^2 = \left[\dfrac{1}{A^2+(B-(A^2+B^2)^{1/2}}\right] A^2$

3 pts.

OR

ii) $\begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \chi_+ \end{bmatrix} = c_1$

which gives us the same!

$$\therefore \boxed{P_{+\hbar/2} = \dfrac{A^2}{A^2+(B-(A^2+B^2)^{1/2})}}$$

B3 Hamiltonian

$$\frac{p^2}{2m} - q\bar{E}.\bar{r}$$

5 points

a) Time dep Sch equation

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\Psi(x,t) - qE x\,\Psi(x,t) = i\hbar\frac{\partial \Psi}{\partial t}(x,t)$$

b) Show that the expectation value of the position operator obeys Newton's 2nd law.

$$\frac{dp}{dt} = m\frac{d^2x}{dt^2}$$

5pts.

12 pts.

$$\frac{d\langle p\rangle}{dt} = \frac{i}{\hbar}[\hat{H},\hat{p}]$$

$$= \frac{i}{\hbar}\underbrace{\left[\frac{p^2}{2m},\hat{p}\right]}_{=0} + \frac{i}{\hbar}[-qE\hat{x},\hat{p}]$$

$$\frac{d\langle p\rangle}{dt} = \frac{i}{\hbar}(-qE)[\hat{x}\hat{p} - \hat{p}\hat{x}] \qquad \Delta x\Delta p \geq \frac{\hbar}{2}$$

$$= \frac{i}{\hbar}(-qE)[i\hbar] = qE = \text{force!}$$

$$\frac{d\langle x\rangle}{dt} = \frac{i}{\hbar}\left[\frac{\hat{p}^2}{2m}, x\right] + \frac{i}{\hbar}(-qE)\underbrace{[\hat{x},\hat{x}]}_{=0}$$

$$= \frac{i}{\hbar}\frac{1}{2m}[\hat{p}\hat{p}\hat{x} - \hat{x}\hat{p}\hat{p}]$$

See over

$$= \frac{i}{\hbar}\frac{1}{2m}[\hat{p}\hat{p}\hat{x} - \cancel{\hat{p}\hat{x}\hat{p} + i\hbar}]$$

$$= \frac{i}{\hbar}\frac{1}{2m}[\hat{p}[px - xp] + i\hbar] = \frac{i}{\hbar}\frac{1}{2m}(\hat{p}(-i\hbar) + i\hbar)$$

B3 $[\hat{x}\hat{p} - \hat{p}\hat{x}] = i\hbar$

5pts.

$$\frac{d\langle x\rangle}{dt} = \frac{i}{\hbar}\left[\frac{\hat{p}^2}{2m}, x\right] + \frac{i}{\hbar}(-qE)\underbrace{[\hat{x},\hat{x}]}_{=0}$$

$$= \frac{i}{2m\hbar}\left[\hat{p}\hat{p}\hat{x} - \hat{x}\hat{p}\hat{p}\right] \qquad \hat{x}\hat{p} = \hat{p}\hat{x} + i\hbar$$

$$= \frac{i}{\hbar}\frac{1}{2m}\left[\hat{p}\hat{p}x - \hat{p}\hat{x}\hat{p} - i\hbar\hat{p}\right]$$

$$= \frac{i}{\hbar}\frac{1}{2m}\hat{p}\left[-i\hbar - i\hbar\right] = -2i\cancel{\hbar}\frac{i}{\cancel{\hbar}}\frac{1}{2m}\hat{p}$$

$$= \frac{\hat{p}}{m}$$

2 pts.

$$\frac{d\langle p\rangle}{dt} = qE \qquad + \quad \frac{d\langle x\rangle}{dt} = \frac{\hat{p}}{m}$$

$$\therefore \left[\frac{d}{dt}\frac{d\langle x\rangle}{dt} = \frac{1}{m}qE\right]$$

$$\therefore \left[m\frac{d^2\langle x\rangle}{dt^2} = qE\right]$$

c) Generalize to an arbitrary potential $V(x)$

8pts.

Then $\frac{d\langle p\rangle}{dt} = \frac{i}{\hbar}\left[V(x)\hat{p}\right] = \frac{i}{\hbar}\left[V(x)\hat{p} - \hat{p}V(x)\right]$

6pts. $= \frac{i}{\hbar}\left[V(x)\frac{\hbar}{i}\frac{\partial\Psi}{\partial x} - \frac{\hbar}{i}\frac{\partial}{\partial x}V(x)\Psi\right]$

$$= \frac{i}{\hbar}\left[\cancel{V(x)\frac{\hbar}{i}\frac{\partial\Psi}{\partial x}} - \cancel{\frac{\hbar}{i}V(x)\frac{\partial\Psi}{\partial x}} - \frac{\hbar}{i}\Psi\frac{\partial V}{\partial x}\right]$$

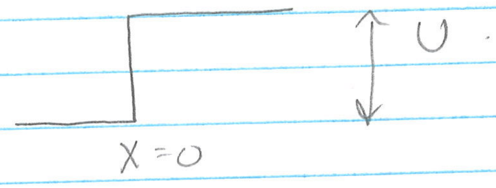
$$\frac{d\langle p\rangle}{dt} = -\frac{\partial V}{\partial x}$$

$$\frac{d\langle x\rangle}{dt} = \frac{i}{\hbar}\left[\hat{x}, \frac{\hat{p}^2}{2m}\right] + \frac{i}{\hbar}\underbrace{[\hat{x}, V(x)]}_{=0}$$

2 pts. $= \frac{\hat{p}}{m}$ $\quad\therefore$ SAME

B4

$E < U$



5 pts

for $x < 0$
$$\psi = Ae^{ikx} + Be^{-ikx} \qquad k = \frac{\sqrt{2mE}}{\hbar}$$

for $x > 0$
$$\psi = Ce^{-Kx} \qquad K = \frac{\sqrt{2m(U-E)}}{\hbar}$$

15 points

(a) Match boundary conditions

5 pts.

① $A + B = C$

② $ikA - ikB = -KC$

Reflection coefficient $= \left|\frac{B}{A}\right|^2$

① $1 + B/A = C/A$

② $ik - ik\,B/A = -K\,C/A$

Substitute for C/A in ②

5 pts

$$ik(1 - B/A) = -K(1 + B/A)$$

$$ik - ik\frac{B}{A} = -K - K\frac{B}{A}$$

$$ik + K = \frac{B}{A}(ik - K)$$

$$\frac{B}{A} = \frac{ik+K}{ik-K} \quad + \quad \left|\frac{B}{A}\right|^2 = \frac{(ik+K)(-ik+K)}{(ik-K)(-ik-K)}$$

$$= \frac{k^2+K^2}{k^2+K^2} = 1$$

(13)

B4

(c) Find the penetration depth for protons $E = 10$ MeV $U = 20$ MeV.

10pts $\delta = \dfrac{1}{K} = \dfrac{\hbar}{\sqrt{2m(U-E)}}$ 6 points

$= \dfrac{1.055 \times 10^{-34}}{\sqrt{1.673 \times 10^{-27} \times 2 \times 10^{7} \times 1.6 \times 10^{-19}}}$ 2 pts

10^{-21-19}

$= 10^{-40}$

10^{-20}

$\delta = 0.144 \times 10^{-14}$ m 2 pts.