Whenever we measure a quantity, that measurement is reported as a number and a unit of measurement. We say such a quantity has **dimensions**.

Units are a necessity – they are part of any answer and the answer is wrong without them.

We will preferentially use metric units, although some of the problems in the book use English units. Conversions are given in the inside front cover of your book.

**SI BASE UNITS** (Metric System)

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Metric</th>
<th>American</th>
<th>Conversion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>meter (m)</td>
<td>foot (ft)</td>
<td>1 ft = .305 m</td>
</tr>
<tr>
<td>Mass</td>
<td>kilogram (kg)</td>
<td>slug</td>
<td>1 slug = 14.6 kg</td>
</tr>
<tr>
<td>Time</td>
<td>seconds</td>
<td>seconds</td>
<td></td>
</tr>
</tbody>
</table>

There are other base units that we will get to later in the course. We will also use many derived units that are combinations of base units such as the unit for energy called a joule which is a kg-m²/s².

**How many meters are in a mile?**

**Dimensional Analysis** – Any valid physics formula must be dimensionally consistent meaning that each term must have the same units.

\[
v = v_0 + at
\]

\[
\begin{align*}
\left[ \frac{L}{T} \right] &= \left[ \frac{L}{T} \right] + \left[ \frac{L}{T} \right]^2 [T] \\
\left[ \frac{L}{T} \right] &= \left[ \frac{L}{T} \right] + \left[ \frac{L}{T} \right] \\
\left[ \frac{L}{T} \right] &= \left[ \frac{L}{T} \right]
\end{align*}
\]

**Is the equation** \( x = x_0 + at \) **Dimensionally consistent?**
**Scientific Notation** - Mathematical Shorthand for expressing very large and very small numbers. The number is written with one digit to the left of the decimal place and then multiplied by a power of ten.

**Examples:**
the speed of light is \( c = 300,000,000 \text{ m/s} = 3.0 \times 10^8 \text{ m/s} \)

the size of a human hair is \( 0.000070 \text{ m} = 7.0 \times 10^{-5} \text{ m} \)

- The nearest star is around \( 41,000,000,000,000,000 \text{ m} \) from the sun,

\[ 41,000,000,000,000,000 \text{ m} = \text{________________________}_\text{ m} \]

The exponent is 16 since the decimal place was moved 16 places to the left.
- The wavelength of visible light is \( 0.0000005 \text{ m} \)

\[ 0.0000005 = \text{________________________}_\text{ m} \]

The exponent is -7 since the decimal place was moved 7 places to the right.

---

**Metric Prefixes** - We often use prefixes to simplify the notation. You’re already used to using prefixes – we use them in talking about ‘millions’ ‘trillions’, etc.

<table>
<thead>
<tr>
<th>Metric Prefix</th>
<th>Symbol</th>
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<tbody>
<tr>
<td>Peta</td>
<td>P</td>
<td>(10^{15})</td>
</tr>
<tr>
<td>Tera</td>
<td>T</td>
<td>(10^{12})</td>
</tr>
<tr>
<td>Giga</td>
<td>G</td>
<td>(10^9)</td>
</tr>
<tr>
<td>Mega</td>
<td>M</td>
<td>(10^6)</td>
</tr>
<tr>
<td>Kilo</td>
<td>k</td>
<td>(10^3)</td>
</tr>
<tr>
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<td>c</td>
<td>(10^{-2})</td>
</tr>
<tr>
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</tr>
<tr>
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</tr>
<tr>
<td>Femto</td>
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<td>(10^{-15})</td>
</tr>
</tbody>
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---

**You Try It!**

What is 360,000 in Scientific Notation?

**Metric Prefixes** - We often use prefixes to simplify the notation. You’re already used to using prefixes – we use them in talking about ‘millions’ ‘trillions’, etc.

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<td>Femto</td>
<td>f</td>
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</tr>
</tbody>
</table>

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Can you express 0.024 in terms of a convenient metric prefix?

**You Try It!**
Examples:

- the speed of light is \( c = 300,000,000 \text{ m/s} = 3.0 \times 10^8 \text{ m/s} = 3.0 \text{ Gm/s} \)

- the size of a human hair is \( 0.000070 \text{ m} = 70 \times 10^{-6} \text{ m} = 70 \text{ µm} \)

- The nearest star is around \( 41,000,000,000,000,000 \text{ m} \) from the sun,

\[
41,000,000,000,000,000 \text{ m} = _____________________
\]

- The wavelength of visible light is \( 0.0000005 \text{ m} \)

\[
0.0000005 = _____________________
\]

Scientific Notation is also very convenient for doing arithmetic in that you do math on the numbers first and then on the powers of ten.

Rules for exponents:

If you multiply two numbers, the exponents add:

\[
10^1 \times 10^3 = 10^4
\]

If you divide two numbers, the bottom exponent is subtracted from the top exponent:

\[
\frac{10^7}{10^3} = 10^4
\]

For example, if you want to divide \( 1,000,000 \) by \( 4,000 \):

\[
1,000,000 = 1 \times 10^6
\]

\[
4,000 = 4 \times 10^3
\]

\[
\frac{1,000,000}{4,000} = \frac{1 \times 10^6}{4 \times 10^3} = \frac{1}{4} \times 10^3 = 0.25 \times 10^3 = 2.5 \times 10^2
\]

\[
(2 \times 10^4) \times (4 \times 10^6) =
\]
**Unit Conversion**

For example, there are 60 minutes in one hour. If the answer to a problem were 1.65 hours, and I wanted the answer in minutes, you could do the following:

\[
\frac{60 \text{ minutes}}{1 \text{ hour}} = 1
\]

This means that the quantity \( \frac{60 \text{ minutes}}{1 \text{ hour}} \) equals 1.

\[
\text{Answer} = 1.65 \text{ hour} \left( \frac{60 \text{ minutes}}{1 \text{ hour}} \right) = 99 \text{ minutes}
\]

Note that the unwanted units must cancel. There is a ‘hour’ in the numerator of the answer and an ‘hour’ in the denominator of the conversion factor, so they cancel, which leaves only the desired minutes.

Note that these factors are reversible.

\[
\frac{60 \text{ minutes}}{1 \text{ hour}} = 1
\]

so the same factor will take you from minutes to hours.

**EXAMPLE:** The Eiffel Tower is 301 m high. What is its height in feet?

We can look up the conversion from meters to feet. 1 meter is 3.281 feet.

\[
1 \text{ m} = 3.281 \text{ ft}
\]

\[
\frac{3.281 \text{ ft}}{1 \text{ m}}
\]

\[
(301 \text{ m}) \left( \frac{3.281 \text{ ft}}{1 \text{ m}} \right) = 988 \text{ ft}
\]

**EXAMPLE:** Kangaroos have been clocked at speeds of 65 km/hr. What is their speed in mi/h?

\[
1 \text{ mi} = 1.609 \text{ km}
\]

\[
\left( \frac{65 \text{ km}}{\text{h}} \right) \left( \frac{\text{mi}}{1.61 \text{ km}} \right) = 40 \text{ mi/h}
\]
EXAMPLE: A field measures 20 km by 30 km. What is the area in m²?

\[ A = (20 \text{ km}) \times (30 \text{ km}) = 600 \text{ km}^2 \]

\[
600 \text{ km}^2 = 600 \text{ km}^2 \left( \frac{1000 \text{ m}}{1 \text{ km}} \right)^2 = 6 \times 10^8 \left[ \frac{\text{km}^2}{\text{m}^2} \right] = 6 \times 10^8 \text{ m}^2
\]

This example illustrates why it is so important for you to include units when doing your calculations. If you accidentally use the wrong conversion factor, you should be able to catch yourself at the end when the units don’t work out correctly.

EXAMPLE : How old are you in seconds?

Solution:

\[
\text{age in years} \times \left( \frac{365 \text{ days}}{1 \text{ yr}} \right) \left( \frac{24 \text{ hr}}{1 \text{ day}} \right) \left( \frac{60 \text{ min}}{1 \text{ hr}} \right) \left( \frac{60 \text{ s}}{1 \text{ min}} \right)
\]

\[
\text{age in years} \times \left( 3.2 \times 10^7 \frac{s}{\text{yr}} \right)
\]

<table>
<thead>
<tr>
<th>Age in years</th>
<th>Age in seconds</th>
</tr>
</thead>
<tbody>
<tr>
<td>18</td>
<td>5.8 \times 10^8</td>
</tr>
<tr>
<td>19</td>
<td>6.1 \times 10^8</td>
</tr>
<tr>
<td>20</td>
<td>6.4 \times 10^8</td>
</tr>
<tr>
<td>21</td>
<td>6.7 \times 10^8</td>
</tr>
<tr>
<td>22</td>
<td>7.1 \times 10^8</td>
</tr>
</tbody>
</table>

You Try It!

A tile store sells tile at the rate of $2.69 per square foot. How much does the tile cost per square meter?
Measurement and Significant Figures

No measurement is exact. Although we have atomic clocks that are highly accurate, the accuracy of the watch on your wrist is probably good enough for you to make it to class on time. The accuracy you need depends on what you’re going to use the measurements for.

Let’s return to the example of measuring the lengths of the metal rod. The ruler in question has as its smallest markings a tenth of an inch. You measure the length and find that it falls about halfway 4.1 and 4.2 cm. You estimate that the length is 4.15 cm, but the 0.05 in is a guess, so you would report that the length of the rod as $4.15 \pm 0.05$ mm. The smallest marking on the measuring device represents the precision of your measurements.

The accuracy of a measurement can be determined by repeating it more than once. The accuracy of a measurement is reflected in the way the number is written. When a number is reported, assume that the number of digits reported is the number known with any certainty. The uncertainty is generally assumed to be one or two units of the last digits. When counting the number of significant figures:

- All digits 1 through 9 count as significant figures
- Zeroes to the left of all of the other digits are not significant
- Zeroes between digits are significant
- Zeroes to the right of all other digits are significant if after the decimal point and may or may not be significant if before the decimal point.

For example

<table>
<thead>
<tr>
<th>Number</th>
<th>Number of Significant Figures</th>
<th>Possible Range of the Real Measurement</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.2</td>
<td>2</td>
<td>1.1 - 1.3</td>
</tr>
<tr>
<td>3.61</td>
<td>3</td>
<td>3.60 – 3.62</td>
</tr>
<tr>
<td>19.61</td>
<td>4</td>
<td>19.60 - 19.62</td>
</tr>
<tr>
<td>0.017</td>
<td>2</td>
<td>.0016 - .0018</td>
</tr>
<tr>
<td>10.25</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.1020</td>
<td></td>
<td></td>
</tr>
<tr>
<td>80</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Sometimes, the number of significant digits can be unclear. For example, if we write \(80\) it is not clear whether the zero is significant or not. Is the measurement between 70 and 90 or between 79 and 81?

If you write \(80.0\), there are 3 significant figures, because zeroes to the right of the decimal point are significant.

Exponential notation removes this ambiguity. We write all of the significant figures out front and then show what power of ten the significant figures should be multiplied by.

For example (assuming that there is one unit of uncertainty in the last significant figure):

- \(8 \times 10^1\) means \(8 (\pm 1) \times 10^1 = \) measurement is between 70 and 90
- \(8.0 \times 10^1\) means \(8.0 (\pm 0.1) \times 10^1 = \) measurement is between 79 and 81

Rules for manipulating significant figures

- Addition or subtraction: keep the place of the digit which is the same as the least significant place of the numbers you are adding/subtracting.
- Multiplication or division: keep the same number of digits as the multiplicand with the least number of significant figures.

**EXAMPLE 2-6:** A room is measured to be 5.5 feet wide and 6.75 feet long. What are the area and perimeter of the room?

\[ A = \text{width} \times \text{length} = 5.5 \text{ feet} \times 6.75 \text{ feet} = 37.125 \text{ feet}^2. \]

Do we know this to 5 significant figures? Nope.

5.5 feet = 2 s.f.
6.75 feet = 3 s.f.

We keep the same number of significant figures as the multiplicand with the smallest number of significant figures, so we can only use 2 s.f.

\[ A = \text{width} \times \text{length} = 5.5 \text{ feet} \times 6.75 \text{ feet} = 37.125 \text{ feet}^2 = 37 \text{ feet}^2. \]

\[ P = 2(l + w) = 2(5.5 + 6.75) = 24.50 \text{ ft} = 24.5 \text{ ft} \]

Calculate the volume of a long cylinder that has a radius of \(r = 4.2\) cm and a length of 26.52 cm. After making the calculation convert your volume to cubic inches.