Astronomy 204 – Homework #7 Solutions

Problem 11-1: Astronomers living on Mars would define their astronomical unit in terms of the orbit of Mars. If they defined parsec in the same manner as we do, how many Martian astronomical units would such a parsec contain? How many Earth astronomical units would equal a Martian parsec? How many Earth parsecs are there in a Martian parsec?

We define a parsec as the distance at which 1 AU subtends 1 arcsecond. Thus, the number of AU in a parsec depends only on the number of arcseconds in a radian, 206,265. Thus, 1 pcMars = 206,265 AU.Mars.

Since 1 AU.Mars = 1.524 AU.Earth
Thus, 1 pc.Mars = 1.524 pc.Earth = 1.524 * (206,265 AU.Earth) = 3.14 x 10^5 AU.Earth

Note that a Martian astronomer could determine parallax with 1.5 times better accuracy than an Earth based astronomer ignoring atmospheric effects, or a Martian astronomer could determine parallaxes for stars 1.5 times farther away with comparable accuracy (determined by atmospheric effects). However, since the Martian year is 2.1 Earth years, it would take 2.1 times longer.

Problem 11-3: What is the combined apparent magnitude of a binary system consisting of two stars of apparent magnitudes 3.0 and 4.0?

First figure out the combined flux of the two stars compared to the brighter star.

\[ m_1 - m_2 = 2.5 \log_{10} \left( \frac{f_1}{f_2} \right) \]

\[ \log_{10} \left( \frac{f_1}{f_2} \right) = \frac{3.0 - 4.0}{2.5} = -0.4 \]

\[ \left( \frac{f_1}{f_2} \right) = 0.4 \]

\[ f_1 = 0.4 f_2 \]

\[ f_1 + f_2 = 1.4 f_1 \]

Now compare the combined magnitude to the magnitude of the 3.0 star.

\[ m_1 - m_{1+2} = 2.5 \log_{10} \left( \frac{f_{1+2}}{f_1} \right) \]

\[ 3.0 - m_{1+2} = 2.5 \log_{10} (1.4) = 0.37 \]

\[ m_{1+2} = 2.63 \]
Problem 11-6: What are the absolute magnitudes of the following stars:

Make use of \( M = m + 5 - 5 \log_{10} (d) = m + 5 + 5 \log_{10} (\pi) \)

(a) \( m = 5.0, d = 100 \text{ pc} \)
\[ M = 5.0 + 5 - 5 \log_{10} (100) = 0.0 \]

(b) \( m = 10.0, d = 1 \text{ pc} \)
\[ M = 10.0 + 5 - 5 \log_{10} (1) = 15.0 \]

Note that there is no star this close to the sun.

(c) \( m = 6.5, d = 250 \text{ pc} \)
\[ M = 6.5 + 5 - 5 \log_{10} (250) = -0.5 \]

(d) \( m = -3.0, d = 5 \text{ pc} \)
\[ M = -3.0 + 5 - 5 \log_{10} (5) = -1.5 \]

(e) \( m = -1.0, d = 500 \text{ pc} \)
\[ M = -1.0 + 5 - 5 \log_{10} (500) = -9.5 \]

(f) \( m = 6.5, \text{ parallax} = 0.0004" \)
\[ M = 6.5 + 5 + 5 \log_{10} (0.0004") = -0.5 \]

Problem 11-8: The Sun has an apparent magnitude of -26.75.

(a) Calculate its absolute visual magnitude.
\[ M = m + 5 - 5 \log_{10} (d) = -26.75 + 5 + 5 \log_{10} (206,265) = 4.8 \]

(b) Calculate its magnitude at the distance of Alpha Centauri (1.3 pc).
\[ m = M - 5 + 5 \log_{10} (d) = 4.8 - 5 + 5 \log_{10} (1.3) = 0.37 \]

(c) The Palomar Sky Survey is complete to magnitudes as faint as +19. How far away (in parsecs) would a star identical to the Sun have to be in order to just barely be bright enough to be visible on Sky Survey photographs?
\[ d = 10^{\frac{m-M+5}{5}} = 10^{\frac{19-4.8+5}{5}} = 6900 \text{ pc} \]
Problem 11-10: A certain globular cluster has a total of $10^4$ stars; 100 of them have $M_V = 0.0$ and the rest have $M_V = +5.0$. What is the integrated visual magnitude of the cluster?

We should first calculate the combined magnitude of the 0.0 stars, then the combined magnitude of the 5.0 stars, and then finally combine those two numbers.

$$M_{1 \text{star}} - M_{100 \text{stars}} = 2.5 \log_{10} \left( \frac{L_{100 \text{stars}}}{L_{1 \text{star}}} \right)$$

$$M_{100 \text{stars}} = 0.0 - 2.5 \log_{10} (100) = -5.0$$

$$M_{1 \text{star}} - M_{9900 \text{stars}} = 2.5 \log_{10} \left( \frac{L_{9900 \text{stars}}}{L_{1 \text{star}}} \right)$$

$$M_{9900 \text{stars}} = 5.0 - 2.5 \log_{10} (9900) = -5.0$$

Now, combining these two values by comparing the magnitude of the 100 stars to the combined magnitude.

$$M_{100 \text{stars}} - M_{\text{all}} = 2.5 \log_{10} \left( \frac{L_{\text{all}}}{L_{100 \text{stars}}} \right)$$

$$M_{\text{all}} = -5.0 - 2.5 \log_{10} (2) = -5.75$$

Problem 11-11: The $V$ magnitudes of two stars are both observed to be 7.5, but their blue magnitudes are $B_1 = 7.2$ and $B_2 = 8.7$.

(a) What is the color index of each star?

Star 1: $B-V = 7.2 - 7.5 = -0.3$ a hot blue star

Star 2: $B-V = 8.7 - 7.5 = 1.2$ a cool red star

(b) Which star is bluer and by what factor is it brighter at blue wavelengths that the other stars?

Star 1 is bluer by 1.5 magnitudes. This corresponds to $2.512^{1.5} = 4.0$ times as bright in the blue.

Problem 11-18: What is the largest distance that a star of absolute magnitude -6 could be detected by the Palomar 5-m telescope? By the HST?

The limiting magnitude of the Palomar is +23.5.

$$d = 10^{\frac{m-M+5}{5}} = 10^{\frac{23.5-(6.0)+5}{5}} = 7.9 \text{ Mpc}$$

The limiting magnitude of the HST is +25. (according to your text. Other references say 28 and the Hubble Deep Field supposedly has objects at +30).

$$d = 10^{\frac{m-M+5}{5}} = 10^{\frac{25-(6.0)+5}{5}} = 16 \text{ Mpc}$$