Astronomy 204: Homework #3 Solutions

2-17:
(a) What is the semi-major axis of the least-energy elliptical orbit of a space probe from Earth to Venus?

The least energy orbit will be one with aphelion coinciding with the Earth’s orbit and perihelion coinciding with Venus’ orbit (on the other side of the sun). Thus, the semi-major axis is:

\[
a = \frac{1}{2} (a_{\text{Venus}} + a_{\text{Earth}}) = \frac{1}{2} (0.723 + 1.000) = 0.862 \text{ AU}
\]

\[
r_{\text{aphelion}} = a (1 + e)
\]
\[
e = \frac{r_{\text{aphelion}}}{a} - 1 = \frac{1.000}{0.862} - 1 = 0.16
\]

(b) Relative to the Earth, what is the velocity of such a probe at the Earth’s orbit?

Using the vis-viva equation:

\[
v^2 = G \left( m_{\text{Sun}} + m_{\text{probe}} \right) \left[ \frac{2}{r} - \frac{1}{a} \right]
\]
\[
v^2 = \left( 6.67 \times 10^{-11} \frac{N \cdot m^2}{kg^2} \right) \left( 1.99 \times 10^{30} \text{ kg} \right) \left[ \frac{2}{1.5 \times 10^{11} \text{ m}} - \frac{1}{1.29 \times 10^{11} \text{ m}} \right]
\]
\[v = 27.2 \frac{\text{km}}{\text{s}}
\]

Since the Earth moves at 29.8 km/s, the probe has a speed 2.6 km/s slower.

(c) When the probe reaches Venus (a = 0.723 AU), what is its velocity relative to that planet?

\[
v^2 = G \left( m_{\text{Sun}} + m_{\text{probe}} \right) \left[ \frac{2}{r} - \frac{1}{a} \right]
\]
\[
v^2 = \left( 6.67 \times 10^{-11} \frac{N \cdot m^2}{kg^2} \right) \left( 1.99 \times 10^{30} \text{ kg} \right) \left[ \frac{2}{1.08 \times 10^{11} \text{ m}} - \frac{1}{1.29 \times 10^{11} \text{ m}} \right]
\]
\[v = 37.9 \frac{\text{km}}{\text{s}}
\]

Since Venus moves at 35.2 km/s, the probe has a speed of 2.8 km/s faster relative to Venus at Venus’ orbit.
**2-22:** Comet Halley has an orbital period of 76 years and an orbital eccentricity of 0.967.

(a) What is the comet’s perihelion distance? Aphelion distance?

\[ P^2 = a^3 \]

\[ a = P^2 / (76)^2 = 17.9 \text{AU} \]

\[ r_{\text{perihelion}} = a(1 - e) = (17.9 \text{AU})(1 - 0.967) = 0.59 \text{AU} \]

\[ r_{\text{aphelion}} = a(1 + e) = (17.9 \text{AU})(1 + 0.967) = 35.2 \text{AU} \]

Thus, the perihelion of Comet Halley is inside the orbit of Venus and the aphelion is outside the orbit of Neptune.

(b) What is the subsolar temperature on comet Halley at perihelion? At aphelion?

\[ T_{ss} = \left( \frac{R_{\odot}}{r_p} \right)^{1/4} T_\odot \approx 394 \left( \frac{1}{0.59} \right) \approx 510 \text{K} \]

\[ T_{ss-\text{perihelion}} = 394 \left( \frac{1}{0.59} \right) \approx 510 \text{K} \]

\[ T_{ss-\text{aphelion}} = 394 \left( \frac{1}{35.2} \right) \approx 66 \text{K} \]

(c) The albedo of Comet Halley is 3%. What is the equilibrium blackbody temperature at perihelion? At aphelion?

Use equation 2.5a to take into account the albedo (and the rotation of the comet nucleus).

\[ T_{eq} = (1 - A)^{1/4} \left( \frac{R_{\odot}}{2r_p} \right)^{1/4} \approx 279 (1 - A)^{1/4} \left( \frac{1}{\sqrt{r_p}} \right) \]

\[ T_{eq-\text{perihelion}} = 279 (1 - 0.03)^{1/4} \left( \frac{1}{\sqrt{0.59}} \right) \approx 360 \text{K} \]

\[ T_{eq-\text{aphelion}} = 279 (1 - 0.03)^{1/4} \left( \frac{1}{\sqrt{35.2}} \right) \approx 47 \text{K} \]
3-1:
(a) How much does a sidereal clock gain (or lose) on a mean solar clock in 5 mean solar hours?

The sidereal clock runs faster than the solar clock. After 1 day of mean solar time a sidereal clock would read $24^h03^m56^s$. After 5 hours of solar time the sidereal clock would gain approximately 49s.

(b) What is the approximate sidereal time when it is noon apparent solar time on the following days:
   i. the first day of spring
      Remember that the sidereal time is equal to the RA of the observer’s meridian.
      The sun is at a RA of $0^h$ at the vernal equinox, so at apparent solar noon the local sidereal time is $0^h$.
   ii. the first day of summer
      The RA of the sun is now $6^h$ so at noon when the sun is on the meridian the LST is $6^h$.
   iii. April 21
      The sun is now 1 month or 2 hours past the vernal equinox, so at apparent solar noon, the LST is $2^h$.
   iv. January 2
      The sun is now one-third of a month past the winter solstice when RA = $18^h$, so at apparent solar noon the LST=$18^h40^m$.

3-2: In terms of azimuth and altitude, describe the Sun’s daily path across the sky during every season of the year at the following latitudes. Use such descriptive terms as noon altitude, sunrise azimuth, sunset azimuth, and angle at which Sun meets horizon.

(a) the equator

<table>
<thead>
<tr>
<th>Date</th>
<th>Rising Azimuth</th>
<th>Altitude at Transit</th>
<th>Setting Azimuth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Either Equinox</td>
<td>90°</td>
<td>90°</td>
<td>270°</td>
</tr>
<tr>
<td>Summer Solstice</td>
<td>66.5°</td>
<td>66.5°</td>
<td>293.5°</td>
</tr>
<tr>
<td>Winter Solstice</td>
<td>113.5°</td>
<td>66.5°</td>
<td>246.5°</td>
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</tbody>
</table>

(b) latitude 35°N

<table>
<thead>
<tr>
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<th>Altitude at Transit</th>
<th>Setting Azimuth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Either Equinox</td>
<td>90°</td>
<td>55°</td>
<td>270°</td>
</tr>
<tr>
<td>Summer Solstice</td>
<td>~45°</td>
<td>78.5°</td>
<td>~315°</td>
</tr>
<tr>
<td>Winter Solstice</td>
<td>~135°</td>
<td>31.5°</td>
<td>225°</td>
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</table>
(c) the north pole

<table>
<thead>
<tr>
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<th>Rising Azimuth</th>
<th>Altitude at Transit</th>
<th>Setting Azimuth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Either Equinox</td>
<td>NA</td>
<td>Sits on horizon all day</td>
<td>NA</td>
</tr>
<tr>
<td>Summer Solstice</td>
<td>NA</td>
<td>23.5° altitude all day</td>
<td>NA</td>
</tr>
<tr>
<td>Winter Solstice</td>
<td>NA</td>
<td>-23.5° altitude all day</td>
<td>NA</td>
</tr>
</tbody>
</table>

5. The matlab assignment simply required you to edit the temperature in the script.