Determining the Mueller Matrix of an Arbitrary Optical Array Element

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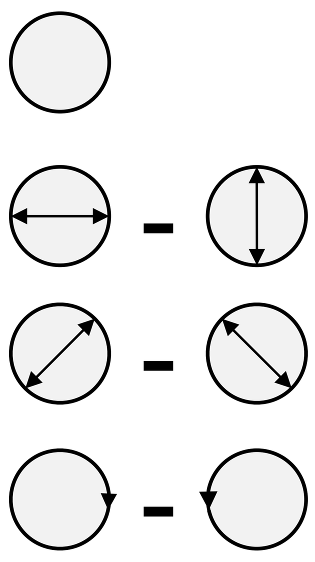
Introduction

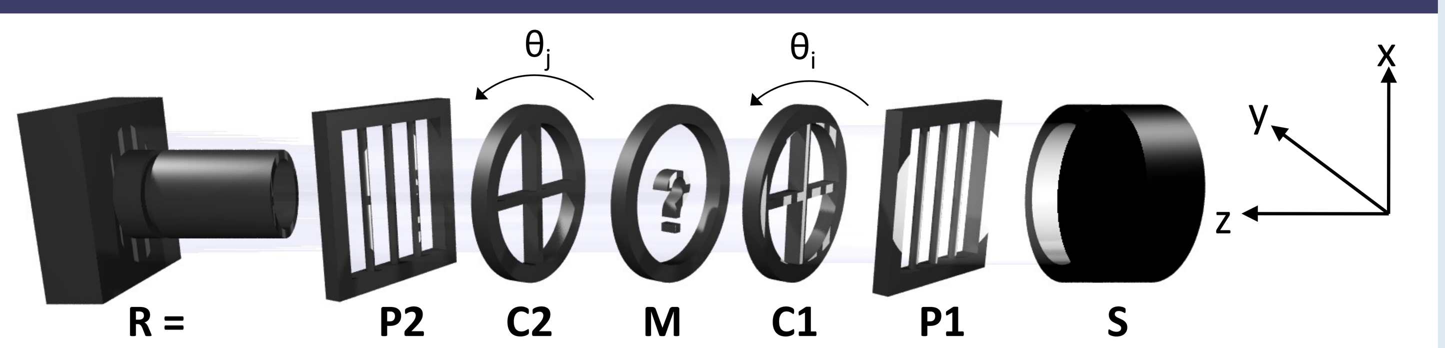
A precise understanding of how optical elements affect the polarization of incident light is important when those elements are used to gather data as part of a more complex experiment [1]. A mathematical tool for characterizing these elements is the Mueller matrix, which represents polarization-dependent effects through action on a Stokes vector describing the incoming light. In order to measure Mueller matrices, a Mueller matrix polarimeter, based on a design from [2], was built and is still undergoing development.

$$\vec{S} = \begin{pmatrix} I_0 \\ I_H - I_V \\ I_{45} - I_{-45} \\ I_R - I_L \end{pmatrix}$$



The Stokes vector is capable of representing polarized, partially polarized, or unpolarized light, and any polarization direction. The first parameter gives the total intensity of the light, while the other three give the intensity difference between two orthogonal polarizations. Terms are assumed to combine incoherently – that is, there is no constant phase difference between any two terms, or between any two Stokes vectors.

Polarimeter Layout



White light from the source with Stokes vector **S** passes through an initial polarizer **P1** and retarder **C1**, the unknown element **M**, then a final retarder **C2** and polarizer **P2** before passing through an interference filter (not shown) and reaching the detector. The polarizers are fixed at 0° relative to the x-axis, while the retarders are independently rotated to 0°, 22.5°, 45°, and 67.5°. Detector response **R** for retarder angles θ_i and θ_j measures the first parameter of the final Stokes vector and is given by the Mueller matrix equation

$$\boldsymbol{R} = [1 \quad 0 \quad 0 \quad 0]\boldsymbol{P2}\,\boldsymbol{C2}_j\,\boldsymbol{M}\,\boldsymbol{C1}_i\,\boldsymbol{P1}\,\boldsymbol{S}$$

Mueller Matrices

$$\begin{bmatrix} p_x^2+p_y^2 & p_x^2-p_y^2 & 0 & 0 \\ p_x^2-p_y^2 & p_x^2+p_y^2 & 0 & 0 \\ 0 & 0 & p_x^2+p_y^2 & 0 \\ 0 & 0 & 0 & p_x^2+p_y^2 \end{bmatrix}$$

Linear Polarizer

$$\alpha \begin{bmatrix} 1 & \frac{1-T^2}{1+T^2} & 0 & 0 \\ \frac{1-T^2}{1+T^2} & 1 & 0 & 0 \\ 0 & 0 & \frac{2T}{1-T^2}\cos\delta_c & \frac{2T}{1-T^2}\sin\delta_c \\ 0 & 0 & -\frac{2T}{1-T^2}\sin\delta_c & \frac{2T}{1-T^2}\cos\delta_c \end{bmatrix}$$

Retarder

Mueller matrix representations for the non-ideal linear polarizers and non-ideal retarders used in the polarimeter. For the polarizer, which polarizes along the x axis, p_x^2 and p_y^2 represent the transmission along the x and y axis respectively. The retarder, which here has its fast axis along the x axis, has a phase shift of δ_c, overall attenuation α, and slow-axis relative attenuation T.

Data

Both polarizers, which were assumed identical, had transmission through both axes measured at three different wavelengths, both as a pair and separately with a polarizing beam splitter. The two retarders were calibrated for retardance δ_c, overall attenuation α, and slow-axis relative attenuation T.

Polarizer	p_x^2	p_y^2
460 nm	0.775 (9)	0.0069 (5)
530 nm	0.76 (1)	0.0000 (3)
640 nm	0.743 (8)	0.0.0060 (1)

C1 Retarder	δ_c	α	T
460 nm	1.393 (2)	0.84 (2)	0.995 (4)
530 nm	1.215 (5)	0.893 (3)	0.999 (3)
640 nm	1.026 (1)	0.87 (2)	0.998 (3)

C2 Retarder	δ_c	α	T
460 nm	2.576 (2)	0.892 (4)	1.004 (1)
530 nm	2.23 (2)	0.89 (5)	1.0032 (2)
640 nm	1.750 (1)	0.95 (1)	0.999 (3)

Measurement

After calibrations were completed, the polarimeter was tested by taking measurements with no unknown element present, theoretically yielding a measurement of the identity matrix. Five measurements per wavelength were averaged together to yield mean and standard deviation Mueller matrices. The results at all three wavelengths were comparable; 460 nm yielded slightly better results, which are reproduced below.

$$\begin{bmatrix} 0.95 & -0.03 & 0.02 & -0.02 \\ -0.10 & 1.07 & -0.06 & 0.09 \\ -0.02 & 0.04 & 0.97 & -0.02 \\ -0.04 & 0.03 & 0.01 & -0.82 \end{bmatrix}$$

460 nm "identity" matrix

460 nm "identity" standard deviation

$$\begin{bmatrix} 0.08 & 0.33 & 0.03 & 0.08 \\ 0.09 & 0.32 & 0.03 & 0.07 \\ 0.02 & 0.02 & 0.23 & 0.01 \\ 0.14 & 0.14 & 0.07 & 0.15 \end{bmatrix}$$

Results

The Mueller matrix polarimeter currently has both high systematic and random error. The negative entry in the (4,4) component of the calculated Mueller matrix shows some sort of problem relating to circularly polarized light, while the high standard deviations show that the polarimeter is sensitive to small changes in measurements. Uncertainties on the order of 1% or smaller in measurements became uncertainties on the order of 10%-1000% in the final Mueller matrix. Unfortunately, the light source intensity goes through long-term variations of about 0.6%, so future development should involve a switch to laser light and/or a modified experimental setup that can detect these fluctuations and take Mueller matrix data simultaneously.

Acknowledgments

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Reference: [1] J. Feeks, E. Litaker, and T. Gay, Measurement and Reduction of Instrumental Asymmetries in an Electron Circular Dichroism Apparatus
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