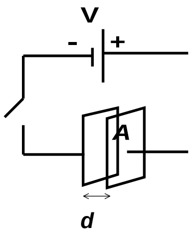
How Do We Sense, Think, and Move? -- Lab #9
A Closer Look at Circuits with Capacitors

Task #1 - Review of the Physics of Capacitors

A capacitor is a device made from two conductors of arbitrary shape. These two conductors are called plates and the most common example is a parallel plate capacitor. A capacitor is said to be charged when its plates carry an equal amount of opposite charges, $+q$ and $-q$.

The typical method for transferring equal and opposite charges to a capacitor is to use a voltage source such as a battery or power supply to create a potential difference between the two conductors. Electrons will then flow off of one conductor (leaving positive charges) and on to the other until the potential difference between the two conductors is the same as that of the voltage source. This is the method you used to charge the parallel plate capacitor in lab #7.

In general, the amount of charge needed to reach the appropriate potential difference will depend on the size, shape, and location of the conductors relative to each other. The capacitance of a given capacitor is defined mathematically as the ratio of the magnitude of the charge, q, on either one of the conductors to the voltage, V, applied across the two conductors. That is, $C = \frac{q}{V}$. Thus, capacitance is defined as a measure of the amount of net charge on either one of the conducting plates per volt.



d = spacing between the plates

A = surface area of the plates

V = voltage applied across the plates

1. Consider two identical metal plates of area A, separated by a nonconducting material (such as air) which has a thickness d. They are connected in a circuit with a battery and a switch, as shown above. When the switch is open, there is no excess charge on either plate. The switch is then closed. What will happen to the amount of charge on the metal plate that is attached to the negative terminal of the battery? What will happen to the amount of charge on the plate that is connected to the positive terminal of the battery? Explain.

2. Can excess charges on one plate of a charged parallel plate capacitor interact with excess charges on the other plate? If so how do they interact?

The unit of capacitance is the farad, F, named after Michael Faraday. One farad is equal to one coulomb/volt. Common capacitance values are often expressed in smaller units. Some common units and notations are listed below.

microfarad:	10^{-6} F	=	1 µF	=	1 UF			
picofarad:	10^{-12} F	=	1 pF	=	1 µµF	=	1 UUF	
nanofarad:	10^{-9} F	=	1 nF	=	1000µµF	=	1000 UUF	

Task #2 - Building a Circuit with Resistors, Capacitors, and a Voltage Sensor

Equipment: Battery set, Plastic circuit block, Two assorted resistors, Two assorted capacitors, Key switch, Banana leads, Digital multimeter (DMM), Capacitance meter, MBL voltage sensor

Experiment #1 – Classifying the circuit components

In preparation for your experiments today, you should measure and record the following quantities. Write a brief description of how you made your measurements.

Resistors

- Color code of each resistor and their predicted resistance values. Identify these resistors as R_1 and R_2, such that $R_1 < R_2$.
- Resistance of each resistor as measured by the DMM.

Capacitors

- Stated capacitance value given on the capacitors. Identify these capacitors as C_1 and C_2, such that $C_1 < C_2$.
- Capacitance of each capacitor as measured by the capacitance meter. Be sure to orient the + and – sides of the capacitor correctly!

Battery set

- Stated potential difference value given for two batteries in series.
- Potential difference value of two batteries in series as measured by the DMM.

Internal resistance of the measuring device(s)

- Internal resistance of the MBL voltage sensor system as measured by the DMM.

3. In general, how good would you say the "given" values are for these different circuit components? Are they always exactly right or do you think it is important to measure them instead of relying on the given stated values? Explain.

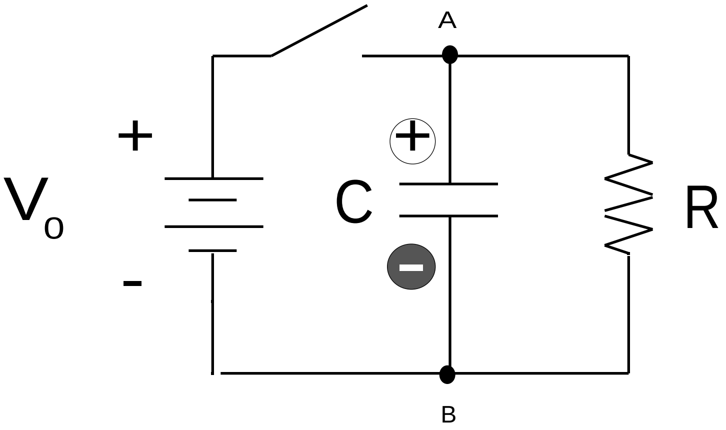
4. Explain why you think it may be important to know the internal resistance of the MBL voltage sensor.

Build the following circuit. This circuit is very similar to the circuit you used in lesson #7. The only difference is that the capacitor has a different shape and the presence of an additional resistor, R.

V_0 = 2 batteries in series
$R = R_1$ (where $R_1 < R_2$)
$C = C_2$ (where $C_1 < C_2$)

***Warnings*!**

- **Be careful to place the capacitor in the circuit with the (-) and (+) in the correct orientation!**
- **Have your instructor verify your circuit before you close the key switch!**



5. Discuss in your group the following questions. Record a summary of your responses and explanations. Do not close the key switch in your circuit until after you answer these questions (and have the circuit inspected by your instructor).

   (a) If you closed the key switch, what potential difference V_{AB} would result on the capacitor after it was "charged up"?

   (b) Suppose you placed the MBL voltage sensor at locations A and B (to measure V_{AB}). Would the internal resistance of the MBL voltage sensor be in parallel or in series with the resistor R?

   (c) Based on your answer to (b), what would be the equivalent resistance (R_P or R_S) of the circuit?

Draw a picture of the circuit in your logbook. Include the voltage sensor (along with its internal resistance) connected to measure V_{AB} in your drawing. Clearly label all relevant quantities.

Task #3 - Discharging a Charged Capacitor

Equipment: Battery set, Plastic circuit block, Resistors, Capacitors, Key switch, Banana leads, MBL voltage sensor

Experiment #2 – Circuit containing V_0, R_1, and C_2 (small R and big C)

In the following experiment, you will use the energy of the batteries to put an excess of charges on the two plates of the capacitor. After the plates are charged up, you will disconnect the battery by opening the switch. The two plates will be connected electrically through a circuit containing resistance. You will measure the resulting potential difference across the plates as a function of time using the MBL voltage sensor.

Data Collection Procedure:

- Be sure your instructor has checked your circuit.
- Open the file: *Lab #9 MBL - Voltage sensor*
- Charge up the capacitor's plates by holding down the key switch.
- One person should start recording data. Once data is being collected, the other person should let go of the key switch.
- The capacitor will now discharge through the circuit's resistor.
- Stop recording data once the electric potential across the capacitor's plates has returned to zero.
- Use the "zoom" tool to inspect the resulting decay curve. If the curve is smooth, then you are ready to move on.
- If there are spikes and bumps in the decay curve, then try repeating the experiment. Be careful to cleanly release the key switch.

Data Saving and Analysis Procedure:

- Once you have a good set of data for this circuit, then save the file in the User Folder.
- Record the name of this data file in your logbook.
- Record the run number for the relevant Experiment #2 data set.
- Using the data-analysis procedure of lab #7, make two estimates of $T_{1/2}$ for this data. Be sure to show all of your data and calculations.

6. Did this capacitor circuit behave in a manner similar to the circuit with the parallel plate capacitor in lab #7? Explain how they were similar and how they were different.

7. If you increased the resistance in this circuit, what do you think would happen to the half-life time, $T_{1/2}$? Discuss this with your partners and record your prediction in your logbook. Explain your reasoning.

Experiment #3 – Circuit containing V_0, R_2, and C_2 (bigger R and big C)

Replace the resistor in your circuit with the provided resistor that has the larger resistance (R_2). Keep the same capacitor as you used in Experiment #2. Repeat your experimental procedure, being sure to save your data when you are done.

Data Collection Procedure:

- Sketch this new circuit in your logbook, carefully labeling all relevant components.
- Charge up the capacitor's plates by holding down the key switch.
- One person should start recording data. Once data is being collected, the other person should let go of the key switch.
- Stop recording data once the capacitor has discharged.
- If there are spikes and bumps in the decay curve, then try repeating the experiment. Be careful to cleanly release the key switch.

Data Saving and Analysis Procedure:

- Once you have a good set of data for this circuit, then resave the file.
- Record the run number for the relevant Experiment #3 data set.
- Make two estimates of $T_{1/2}$ for this data. Be sure to show all of your data and calculations.

8. What is the equivalent resistance of this circuit? Is it more or less than the circuit you used in Experiment #2?

9. Compare your data and results for Experiment #2 and Experiment #3. How were the two similar and how were the two different?

10. What effect does changing the resistance have on the resulting half-life time? Does this effect make sense to you? Explain.

11. Compute the ratio of the half-life time divided by the equivalent resistance for each experiment ($T_{1/2\,\#2} / R_{Eq\,\#2}$ and $T_{1/2\,\#3} / R_{Eq\,\#3}$). How do these ratios compare? Based on this result does $T_{1/2}$ seem to be related to the total resistance of the circuit? For example, are they proportional? Inversely proportional? Or…?

12. If you decreased the capacitance in this circuit, what do you think would happen to the half-life time, $T_{1/2}$? Discuss this with your partners and record your prediction in your logbook. Explain your reasoning.

Experiment #4 – Circuit containing V_0, R_2, and C_1 (bigger R and smaller C)

Replace the capacitor in your circuit with the provided capacitor that has the smaller capacitance (C_1). Be sure to orient the capacitor correctly in the circuit! Keep the same resistor as you used in Experiment #3. Repeat your experimental procedure, being sure to save your data when you are done.

Data Collection Procedure:

- Sketch this new circuit in your logbook, carefully labeling all relevant components.
- Charge up the capacitor's plates by holding down the key switch.
- One person should start recording data. Once data is being collected, the other person should let go of the key switch.
- Stop recording data once the capacitor has discharged.
- If there are spikes and bumps in the decay curve, then try repeating the experiment. Be careful to cleanly release the key switch.

Data Saving and Analysis Procedure:

- Once you have a good set of data for this circuit, then resave the file.
- Record the run number for the relevant Experiment #4 data set.
- Make two estimates of $T_{1/2}$ for this data. Be sure to show all of your data and calculations.

13. What is the equivalent resistance of this circuit? How does it compare to the circuit you used in Experiment #3?

14. Compare your data and results for Experiment #3 and Experiment #4. How were the two similar and how were the two different?

15. What effect does changing the capacitance have on the resulting half-life time? Does this effect make sense to you? Explain.

16. Compute the ratio of the half-life time divided by the capacitance for each experiment ($T_{1/2\,\#3} / C_{\#3}$ and $T_{1/2\,\#4} / C_{\#4}$). How do these ratios compare? Based on this result does $T_{1/2}$ seem to be related to the capacitance of the circuit? For example, are they proportional? Inversely proportional? Or…?

Task #4 - Developing Mathematical Models of Your Data

Answer the following questions to assist in creating a mathematical model for each of your data sets.

17. By measuring $T_{1/2}$ for your data, you have assumed that the measured voltages are changing exponentially as a function of time. Does this seem to be a reasonable assumption? Explain why you do or do not think the data for the discharging capacitor is exponential using the general properties of exponential functions.

18. If the data for the discharging capacitor is exponential, then it should obey a relationship with the form: $y(x) = A_0 e^{-bx}$.

    - What does the variable y represent for this system?
    - What does the variable x represent for this system?
    - What does the quantity A_0 represent for this system?
    - What are the units of y, x, A_0, and b?
    - Rewrite this general equation ($y(x) = A_0 e^{-bx}$) using variables and names appropriate for the system you have been studying (namely RC circuits).

19. The half-life time, $T_{1/2}$, is defined as the time it takes the potential difference across the plates of the capacitor to decrease by 50%. Using this fact and the general equation you wrote in the previous question, derive an algebraic expression for $T_{1.2}$.
    Hint! $V(T_{1/2}) = (1/2)\ V_0 = \ldots$
    Hint! $T_{1/2}$ should be a function of the quantity b.

20. According to theory (See for example Section 20.13 of your textbook), $b = \frac{1}{RC}$.
    Use your algebraic expression for $T_{1/2}$ and values of R and C for each circuit and calculate a predicted half-life time for each circuit (#2, #3, and #4).

21. Compare your predicted values of the half-life time for each circuit with those that you measured experimentally. How well do they compare? Does this theory seem to agree with your data?

22. Look at your data for Experiment #2 in *ScienceWorkshop*. Using all of the information from this page, write in your logbook a mathematical model of the discharging of the capacitor. For simplicity sake, assume that *time = 0 s* occurs at the instant you released the switch (even though that may not be time = 0 s on your graph). Your model should include numerical values (and units) for all constants.

You can use the tools of *ScienceWorkshop* to create a mathematical model of the actual data. To do this, follow the procedure outlined on the next page.

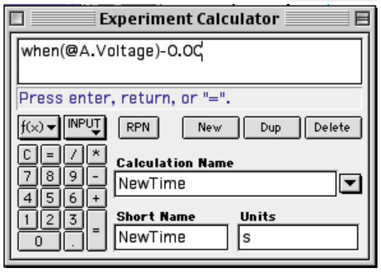
Creating a Model of the Actual Data

- Use the *zoom* tool to display the relevant data.
- Use the *cursor* tool to estimate the time when the switch was released and the capacitor began to discharge.

- You will now use the *Experiment Calculator* to create a variable called *NewTime*. *NewTime* will be the time adjusted so that the time when the switch was released will be *NewTime = 0 s*. That is, *NewTime = Time – (time when the switch was released).*

- Click on the *Experiment Calculator* button to see the window.

- Replace the value 0.00 with the actual time when the switch was opened. Then press "return" on the keyboard. You have now defined the variable *NewTime* for your data set.



when(@A.Voltage)-0.00

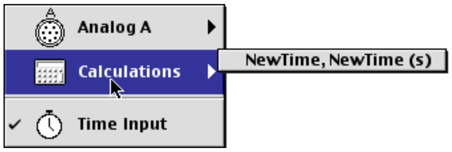
Press enter, return, or "=".

Calculation Name NewTime

Short Name NewTime

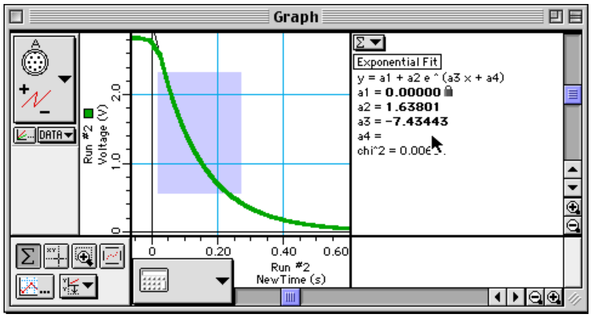
Units s

- Regraph your data by selecting the variable *NewTime* for the horizontal axis.



- Highlight a region of the relevant data with the mouse.
- Press the *statistics* button and select an *Exponential Fit.*
- *ScienceWorkshop* uses the exponential model: $y = a1 + a2\ e^{-}(a3\ \ x + a4)$. However, in our case, we want the model to have a1 = a4 = 0.00. You can lock in these two values by clicking on each of them and entering in the value of 0.00.
- After you have set these two constants to zero, *ScienceWorkshop* will give you a model of the form: $y = a2\ e^{-}(a3\ \ x)$.

- Print a copy of the graph showing the Exponential Fit and place it in your logbook.



Exponential Fit
y = a1 + a2 e ^ (a3 x + a4)
a1 = **0.00000**
a2 = **1.63801**
a3 = **−7.43443**
a4 =
chi^2 = 0.006

23. How did your predicted model (question #22) compare to the experimental model (from *ScienceWorkshop*)? Can you account for any differences?

24. Repeat questions 22 and 23 for your data from Experiment #4.

<u>End of Lab Cleanup</u>

- Turn off the multimeter.
- Unplug all banana leads.
- Return the resistors and capacitors to the box provided at your station.

Reference source: Workshop Physics ©1990-93 Dept. of Physics & Astronomy, Dickinson College