

Ohm's Law

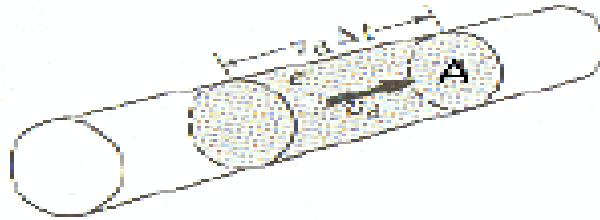
Definition of electric current:

The current in a wire is the amount of charge passing plane per unit time.

$$\mathbf{I = \Delta Q / \Delta t \text{ [coulomb/second]}}$$

$$\mathbf{1 \text{ ampere} = 1 \text{ coulomb per second}}$$

We can relate the current \mathbf{I} to the number of electrons per unit volume \mathbf{n} , the drift velocity $\mathbf{v_d}$, the charge on the electron \mathbf{e} and the cross-sectional area \mathbf{A} . We will assume that every electron moves with a drift velocity $\mathbf{v_d}$. In a time Δt all the particles in the volume $\mathbf{A v_d \Delta t}$, shaded in the figure, will pass through the plane.



Since the number of electrons per unit volume is \mathbf{n} , the number of particles in this volume is $\mathbf{n A v_d \Delta t}$, and the total charge is $\mathbf{\Delta Q = e n A v_d \Delta t}$

The current is thus

$$\mathbf{I = \Delta Q / \Delta t = n e A v_d}$$

Now we can relate the drift velocity to the applied electric field, using our understanding of electric forces and Newton's second law.

Since the drift velocity is the average velocity of an electron picked at random and since the acceleration of that electron caused by the electric force on it is given by

$$\mathbf{a = e E / m_e},$$

where \mathbf{E} is the electric field and $\mathbf{m_e}$ is the effective mass of the electron

Then the drift velocity

$$\mathbf{v_d = a \tau = e E \tau / m_e}$$

where τ is the average time between collisions

$$\mathbf{I = \Delta Q / \Delta t = n e A v_d = n e A e E \tau / m_e = (n e^2 A \tau / m_e) E}$$

For a uniform wire, the electric field is the electric potential divided by the length,

$$\mathbf{E = V / L}$$

$$\mathbf{I = (n e^2 A \tau / m_e) V / L, \text{ or rearranging}}$$

$$\mathbf{V = \{m_e / (n e^2 \tau)\} \{L/A\} I}$$

$$\mathbf{V = R I}$$

$$\text{where } \mathbf{R = \{m_e / (n e^2 \tau)\} \{L/A\} = \rho L/A}$$

where \mathbf{R} is the resistance in ohms and ρ is the resistivity in ohm meters.

Note: 1 ohm = 1 volt/ampere