PHYSICS 151 – Notes for Online Lecture #38

Power

Power is defined as the energy transformed/time.

$$P = \frac{\text{Energy}}{\text{time}}$$

When a charge, q, passes across a potential difference, V, it acquires an energy qV. If it takes a time t to pass across the potential difference, the power is

$$P = \frac{qV}{t} = \left(\frac{q}{t}\right)V = IV$$

Where we've used that q/t is the current. The power is thus given by IV. The unit of Power, the Watt, is equivalent to an A-V

We can also write the power in terms of the resistance, by using Ohm's law:

$$P = IV$$

$$P = I(IR) = I^2R$$

$$P = IV = \left(\frac{V}{R}\right)V = \frac{V^2}{R}$$

These two latter forms apply to things that have resistors and the first one applies in general.

Ex. 1: What is the resistance of a 100-W light bulb operating at 120 V?

$$P = \frac{V^2}{R}$$

$$R = \frac{V^2}{P}$$

$$R = \frac{(120V)^2}{100W} = 144\,\Omega$$

Ex. 2: What is the current passing through a 1200 W hair dryer operating at 120 V?

Here, we use the P=IV form:

$$P = IV$$

$$I = \frac{P}{V}$$

$$I = \frac{1200\,W}{120\,V} = 10\,A$$

Ex. 3: If electricity is \$0.05 per kW-hr, how much does it cost to run a 100 W light bulb for 1 hour?

First, what is a kWh –it's not a SI unit, so it's not obvious whether it is an energy or power or what. Using 1 kW = 1000 W and 1 h = 3600 s,

$$1\ \text{kWh} = (1000\text{W})(3600\text{s}) = 3.6\text{x}10^6\text{J}$$

A kW-hr is thus a unit of energy. From Power = Energy/time,

$$E = Pt$$

$$E = \left(100\frac{J}{s}\right)(3600s) = 360000J = 3.6x10^5\,J$$

We now have to convert this to kWh

$$E = 3.6x10^5\,J\left(\frac{1\,kWh}{3.6\,x\,10^6\,J}\right)$$

$$E = 0.1\,kWh$$

so the total energy used for the light bulb above will be 0.1 kWh. At \$0.05 per kWh, it costs about \$0.005 to run the light bulb for an hour.

Current produces heat. A current passing through an object with a resistance will produce heat. Light bulb filaments are made of tungsten. As you increase the amount of current passing through the bulb, the filament glows brighter and brighter. Although some of the power goes to producing light, most of the power goes to producing *heat*. This is the principle upon which your hair dryer works. The power dissipated within the tungsten filament is

$$P = I^2R$$

If the resistance is very large, the power is large and the wire will heat up and could melt or start a fire.

Kirchoff's Rules and Simple Resistive Circuits

If you buy cheap Christmas tree lights, you will find that when one lamp goes out, all of the lights go out. This has to do with how the current is delivered to the different lights. In order to understand this phenomena, we need to understand how current behaves when confronted with multiple resistors. There are two fundamental ideas that have to be followed.

1) energy must be conserved in the circuit
2) charge must be conserved

These two ideas are expressed in **Kirchoff's Rules**. The first rule – conservation of energy – is expressed as:

The algebraic sum of the potential differences around a closed conducting loop must be zero.

This is sometimes known as Kirchoff's voltage rule or the "loop rule".

The second rule – conservation of charge – says

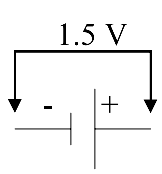
The net current into any junction must be zero

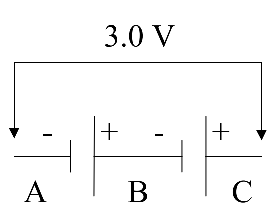
In other words, all of the current (electrons) that comes into a junction must leave the junction. This is sometimes called Kirchoff's current rule, or the "junction rule".

We're going to look at several situations to discover how combinations of resistance and batteries can be explained.

Batteries in Series

In some situations, we might want to put multiple batteries together (say, for instance, in a flashlight or radio.). If I take a single flashlight battery, the potential difference across its terminals is 1.5 V. Now imagine that I hook two batteries together, with the positive terminal of one battery hooked to the negative of the other. What is the voltage then? In this situation, the voltages of the individual batteries add, so that the total voltage is 1.5 V + 1.5 V = 3.0 V. The potential at point A is 0 V, at point B is 1.5 V and at point C is 3.0 V.

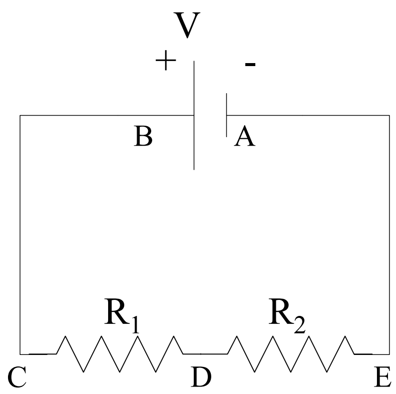




Note that if I hook the batteries up in the same way, but put the positive terminals together, I don't get any potential difference.

Note that batteries placed in parallel do not increase the voltage, however they allow current to be produced for a longer time.

Resistors in Series



Consider a circuit consisting of two resistors, R_1 and R_2 and a battery with a voltage V.

The current through resistors in series is the same. If you think about the current traveling from the positive terminal of the battery, there is only one path for the current to take. This means that the current through both of the resistors is the same. We'll call it I.

The potential drop across the first resistor is given by Ohm's Law:

$$V_1 = IR_1$$

and the potential across the second resistor is

$$V_2 = IR_2$$

Kirchoff's voltage rule states that the algebraic sum of all of the potential differences must be zero around a closed loop.

If you start from the negative terminal of the battery, the potential there is zero (ground). Going across the positive terminal, you get a positive potential difference of V. The battery makes a positive contribution to the overall potential difference.

When you move across the resistor R_1, there is a potential drop, which means that the potential on the right side of the resistor is lower than the potential on the left side of the resistor.

Going around the circle, we can make the following table:

	Potential
A	0
B	V
C	V
D	$V - IR_1$
E	$V - IR_1 - IR_2$

If we write Kirchoff's voltage rule, we get:

$$V - IR_1 - IR_2 = 0$$

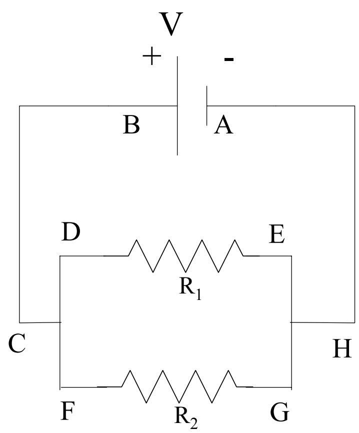
We can solve this equation for V

$$V - IR_1 - IR_2 = 0$$
$$V = I(R_1 + R_2)$$
$$V = IR_s$$

where R_s is the resistance of the two resistors in series. Two resistors in series can thus be considered as a single resistance with a magnitude equal to the sum of the two. This works for as many resistors as you have, provided they are all hooked up in series.

$$\boxed{R_s = R_1 + R_2 + R_3 + \ldots}$$

Resistors in Parallel

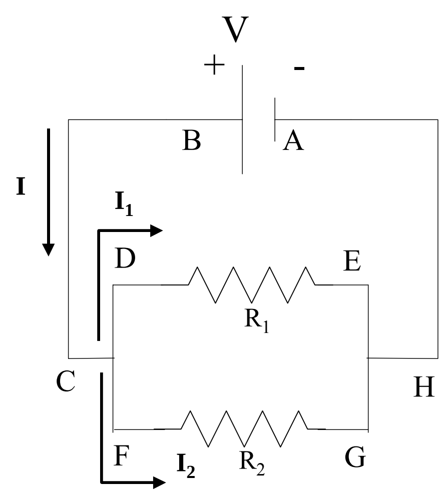


In some cases, we want to hook up electrical resistances in parallel, as shown to the left. Let the battery again have voltage V.

This time, if you think about the current, you see that when the current reaches point C, it has an option. It can either go toward point D or toward point F. You can use the water through a pipe analogy again. The larger the resistance, the smaller diameter pipe. More water will go through the pipe having less-resistance (the wider one). Similarly, more current will go through the less-resistive resistor.

When the current reaches point C, it will split. Kirchoff's second rule tells us that the sum of the current going into a junction plus the current leaving the junction must equal zero. If we assign the different currents names as we approach junction C, as shown below, Kirchoff's second rule requires that

$$I = I_1 + I_2$$



There will be a potential drop across each resistor. The potential drop across R_1 will be given by V_1 where

$$V_1 = I_1 R_1$$

Similarly,

$$V_2 = I_2 R_2$$

The voltage across each branch of a parallel circuit is the same. Compare the potential differences across points C to H. Regardless of which path you choose, point C starts out with some potential (in this case, V) and must have some other potential at point H. This means that the potential drop across R_1 is the same as the potential drop across R_2.

$$V_1 = V_2 = V$$

This means that the equations representing the potential drop across the two resistors are:

$$V = I_1 R_1 \text{ and } V = I_2 R_2$$

we can solve each of these for the current:

$$I_1 = \frac{V}{R_1} \qquad\qquad I_2 = \frac{V}{R_2}$$

Plug these values into Kirchoff's current rule:

$$I = I_1 + I_2$$

$$I = \frac{V}{R_1} + \frac{V}{R_2}$$

$$I = V\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

Solving for V,

$$V = \frac{I}{\left(\frac{1}{R_1} + \frac{1}{R_2}\right)}$$

$$V = IR_p$$

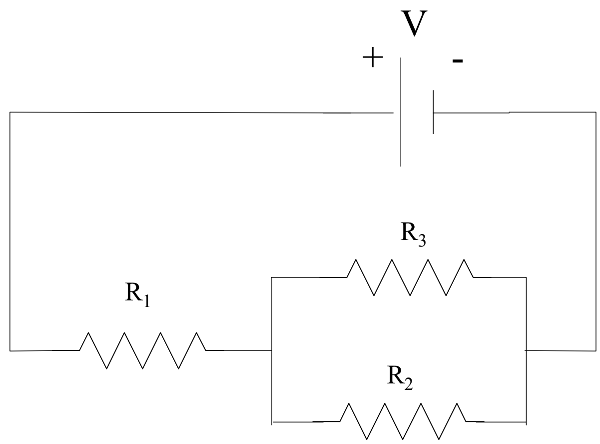
where R_p is the equivalent resistance for the two resistors in parallel.

In the general case, the equivalent resistance of resistors connected in parallel is:

$$\boxed{\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots}$$

1. When resistors are in series, the effective resistance is greater than any of the individual resistors
2. When resistors are in parallel, the effective resistance is less than any of the individual resistances

Ex. 4: Analyze the following circuit if $R_1 = 8\Omega$, $R_2 = 6\Omega$, $R_3 = 3\ \Omega$, and $V = 20V$.



First, find the effective resistance of the parallel circuit

$$\frac{1}{R_p} = \frac{1}{R_2} + \frac{1}{R_3}$$

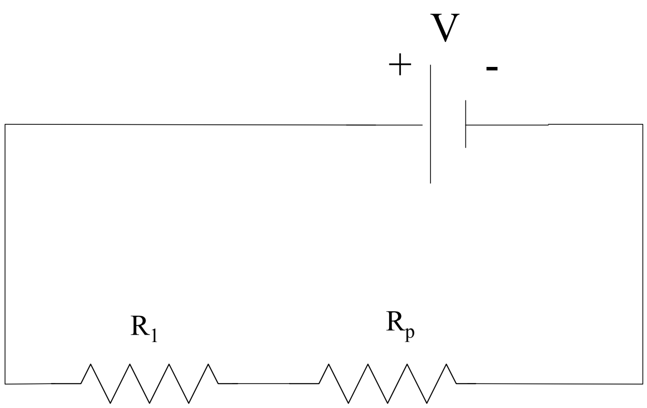
$$\frac{1}{R_p} = \frac{1}{6\Omega} + \frac{1}{3\Omega}$$

$$\frac{1}{R_p} = \frac{3}{6}\frac{1}{\Omega}$$

$$R_p = \frac{6}{3}\Omega$$

$$R_p = 2\Omega$$

Now what you have is R_1 in series with R_p



Add these two resistors in series:

$$R_{TOTAL} = R_1 + R_p = 8\Omega + 2\Omega = 10\Omega$$

So the current leaving the battery is 2A and this must also be the current through R_1. However, when the current gets to the junction where the two paths lead to R_2 and R_3 it must divide.

Note that when the current passes through R_1 there is a voltage drop of $V = IR_1 = (2A)(8\Omega) = 16V$. So the voltage drop across both R_2 and R_3 must be 4V.

So

$(4V) = I_2 R_2 = I_2\ (6\ \Omega)$ $\qquad I_2 = 0.67$ A

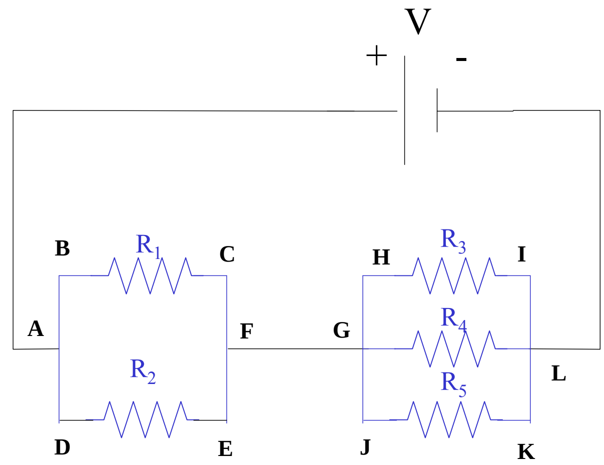
$(4V) = I_3 R_3 = I_3\ (3\ \Omega)$ $\qquad I_3 = 1.33$ A

Note that $I_1 = I_2 + I_3$ which we know must be true.

Ex. 5: For the circuit shown to the right, find:

a) the current passing through each resistor

b) the voltage drop across each resistor



Take:

$R_1 = 2/3\ \Omega$

$R_2 = 2\ \Omega$

$R_3 = 3\ \Omega$

$R_4 = 4\ \Omega$

$R_5 = 12\ \Omega$

$V = 12\ V$

Solution:

First, find the equivalent resistances for the parallel combination involving R_1 and R_2

$$\frac{1}{R_{pa}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\frac{1}{R_{pa}} = \frac{1}{\frac{2}{3}\Omega} + \frac{1}{2\Omega}$$

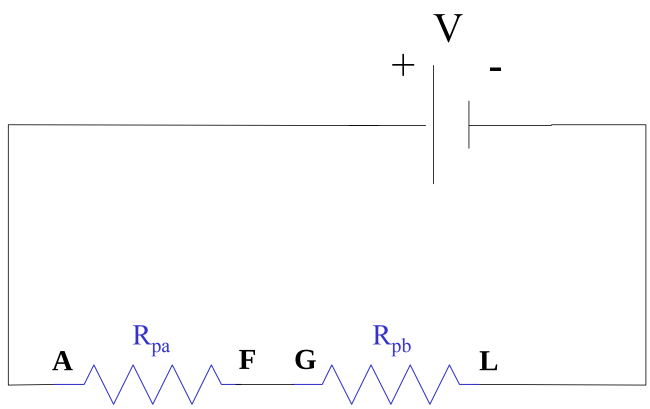
$$\frac{1}{R_{pa}} = \frac{3}{2\Omega} + \frac{1}{2\Omega}$$

$$\frac{1}{R_{pa}} = \frac{4}{2}\frac{1}{\Omega}$$

$$R_{pa} = \frac{2}{4}\Omega$$

$$R_{pa} = \frac{1}{2}\Omega$$

Now, find the equivalent resistance for the parallel combination involving R_3, R_4 and R_5:



$$\frac{1}{R_{pb}} = \frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5}$$
$$\frac{1}{R_{pb}} = \frac{1}{3\Omega} + \frac{1}{4\Omega} + \frac{1}{12\Omega}$$
$$\frac{1}{R_{pb}} = \frac{4}{12\Omega} + \frac{3}{12\Omega} + \frac{1}{12\Omega}$$
$$\frac{1}{R_{pb}} = \frac{8}{12}\frac{1}{\Omega}$$
$$R_{pb} = \frac{12}{8}\Omega$$

The circuit can now be replaced by its equivalent

the total resistance is thus the sum of the two resistances in series

$$R_T = R_{pa} + R_{pb}$$
$$R_T = \frac{1}{2}\Omega + \frac{12}{8}\Omega$$
$$R_T = \frac{4}{8}\Omega + \frac{12}{8}\Omega$$
$$R_T = \frac{16}{8}\Omega$$
$$R_T = 2\Omega$$

From this, we can find the current that flows through the circuit.

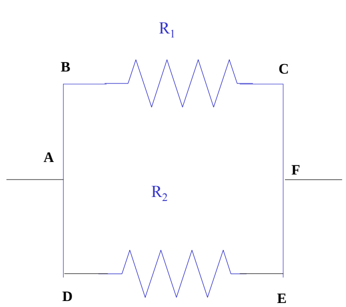
$$I = \frac{V}{R} = \frac{12V}{2\Omega} = 6A$$

Note that this is the current going into each of the combinations of resistors in parallel: i.e. 6 A flows from A to F and from G to L.

The potential drop over each combination can now be calculated:

$$V_{AF} = IR_{pa}$$
$$V_{AF} = (6\,A)(\tfrac{1}{2}\Omega) = 3\,V$$

We can calculate V_{GL} one of two ways. First, we know that the sum of the voltage drops across the circuit must algebraically add to zero, so if the total voltage is 12 V and the potential drop from A to F is 3V, the drop from G to L must be 9 V. We can also calculate it directly:



$$V_{GL} = IR_{pb}$$

$$V_{GL} = (6\,\text{A})\left(\tfrac{12}{8}\,\Omega\right) = 9\,\text{V}$$

But these calculations don't tell us how much current and voltage are going through each resistor. To do that, we have to look more carefully at the parallel combinations.

We know from our above calculation that $V_{AF} = 3$ V. We know that the same voltage drop will occur over each branch of the parallel circuit, so

$V_{BC} = V_{DE} = V_{AF} = 3$ V

The currents across R_1 and R_2 are then given by Ohm's Law:

$$I_{R_1} = \frac{V_{BC}}{R_1} = \frac{3\,\text{V}}{\frac{2}{3}\,\Omega} = 4.5\,\text{A}$$

$$I_{R_2} = \frac{V_{DE}}{R_2} = \frac{3\,\text{V}}{2\,\Omega} = 1.5\,\text{A}$$

We can check this. We know that 6 A of current passes from point A to point F, so the sum of the currents through R_1 and R_2 must be equal to 6 A.

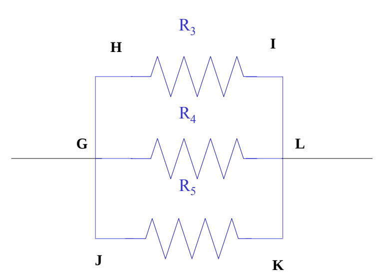
$$I_{R_1} + I_{R_2} = 6\,\text{A}$$

$$4.5\,\text{A} + 1.5\,\text{A} = 6\,\text{A}$$

The same type of calculation can be performed for the combination that makes up R_{pb}. The same voltage drop will occur over each branch of the parallel circuit, so

$V_{HI} = V_{GL} = V_{JK} = 9$ V

The currents across R_3, R_4 and R_5 are then given by Ohm's Law:



$$I_{R_3} = \frac{V_{HI}}{R_3} = \frac{9\,\text{V}}{3\Omega} = 3\,\text{A}$$

$$I_{R_4} = \frac{V_{GL}}{R_4} = \frac{9\,\text{V}}{4\,\Omega} = 2.25\,\text{A}$$

$$I_{R_5} = \frac{V_{JK}}{R_5} = \frac{9\,\text{V}}{12\,\Omega} = 0.75\,\text{A}$$

We can check this. We know that 6 A of current passes from point A to point F, so the sum of the currents through R_1 and R_2 must be equal to 6 A.

$$I_{R_1} + I_{R_2} = 6\,\text{A}$$

$$3\,\text{A} + 2.25\,\text{A} + 0.75\,\text{A} = 6\,\text{A}$$

In summary

Resistor	Current (A)	Voltage (V)

$R_1 = 2/3\ \Omega$	4.50	3
$R_2 = 2\ \Omega$	1.50	3
$R_3 = 3\ \Omega$	3.00	9
$R_4 = 4\ \Omega$	2.25	9
$R_5 = 12\ \Omega$	0.75	9