PHYSICS 151 – Notes for Online Lecture #37

Electric Potential:

The electric potential -or just potential - tells us something about how a charge will move in a given situation. For example, the electric potential due to a point charge of magnitude Q is:

$$V(r) = \frac{kQ}{r}$$

The electric potential will be positive or negative depending on the sign of the charge Q. If Q is positive, V will also be positive. If Q is negative, V will be negative. The potential is a scalar -- not a vector. So there is no direction associated with that minus sign. The minus sign tells you what the charge is. This also makes it easier to manipulate this quantity than a vector quantity.

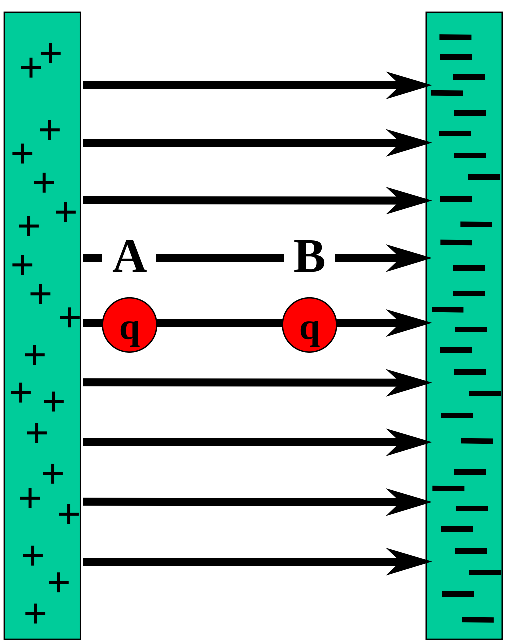
If Q is positive, it will repel other positive charges. The strength of that repulsion is proportional to 1/r.

The unit of potential is called the Volt:

$$V(r) = \frac{kQ}{r} \text{ has units } \frac{\frac{Nm^2}{C^2}C}{m}$$

$$V(r) = \frac{Nm}{C}$$

$$V(r) = \frac{J}{C} = \text{Volt}$$



The potential difference. In most cases, we are most concerned with *differences* in potential energy – not the absolute potential energy. We define the potential difference to be the difference in potential energy of a charge at two different points. For example, if I have a charge, q, that is between two charged plates, its motion will depend on how far it is from each plate. If we compare two positions, A and B, we say that V_{AB} is the potential difference between points A and B.

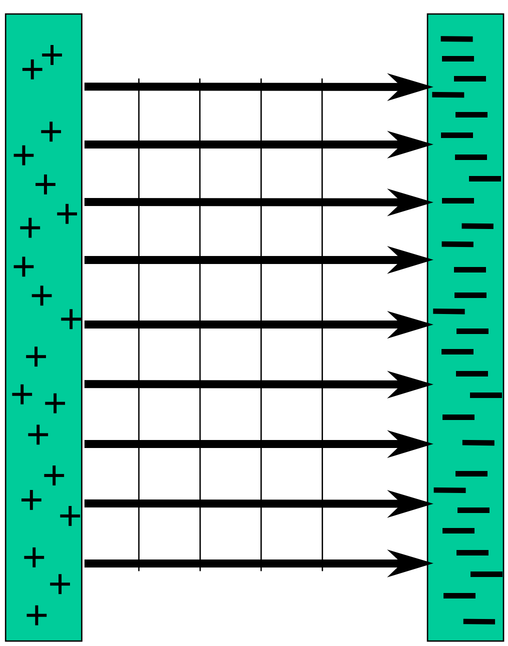
The unit of potential

difference – the volt – was named in honor of Alessandro Volta, who discovered the battery. What I've drawn here is actually a battery. The potential difference is also often referred to as the voltage - but we're going to use potential difference to keep reminding you what this term actually means. The potential is generally measured with respect to ground. Here are the magnitudes of typical potential differences:

	Potential Difference (in Volts)
pn-junction in a transistor	10^{-1}
biological cell	10^{-1}
single-cell battery	1
electrical outlet in your home	10^{2}
TV Set	10^{4}
Thunderclouds	10^{8}

Positive charges move from high potential to low potential. If we look at the two plates, we say that the positive plate is at a high potential and the negative plate is at a low potential. Positive charges naturally move from high potential to low potential and negative charges do the reverse.

Batteries



The history of the battery. In 1791, a man named Luigi Galvani was experimenting with frogs. He was looking for the secret of life. Since electricity was a popular field at the time, he theorized that electricity had something to do with life. To investigate this, he hooked up frog muscles to a static electricity machine that made the frog legs twitch. Strangely, he found that if he attached two different types of metals to either side of the frog leg, it also twitched – even when there was no electricity provided from anywhere else. This led him to suggest that the electricity was coming from the frog and maybe electricity was the secret of life.

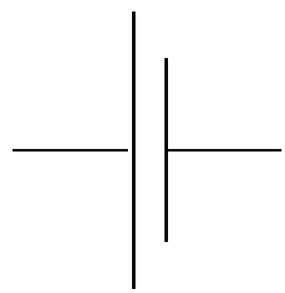
Allesandro Volta was skeptical of this and did some more experiments. He found that if you take two different metals and provide them with some medium to interact, you can produce electricity. He took a silver disc and a zinc disc and put a cloth soaked in a salt solution between them and, sure enough, he produced electricity without needing a frog or any other living thing. He produced a very weak potential difference, but it was enough to establish the principle.

What's in a battery? Batteries are composed of: electrodes (which must be made of two different metals) and an electrolytic solution, such as the salt solution. If you've ever had a battery leak, you've seen the type of solution that is used -- it's acidic and shouldn't be handled.

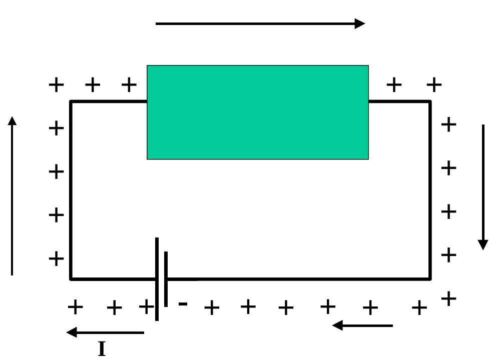
How a battery works. In a battery, the electrolyte attacks one of the metals and causes the ions to dissolve in the solution; however, each ion leaves electrons on the electrode, so that the electrode has a negative charge. The positive ions are still floating around in the solution, travel over to the other electrode and remove electrons from it. This leaves a positive charge on the other electrode, which produces a potential difference.

When the terminals are not connected, the reaction is very slow. The electrodes can't build up charge indefinitely. At some point, no more charges will be pulled off the electrode. When you hook the battery up with a wire, however, electrons will flow through the circuit and more and more of the electrode materials will be used up. A battery "dies" when enough of the electrodes are used up that the reaction can't continue to occur. You may have seen potato or battery clocks - the acid in the orange or potato is providing the electrolytic fluid.

When we connect the terminals of the battery, we provide a path for charges to move. We represent a battery with the symbol shown at right: The larger side represents the positive side of the terminal and the smaller side represents the negative terminal.



Electromotive force or emf. The electromotive force is defined to be the potential difference between the terminals of a battery <u>when no current is present</u>. Emf is measured in volts. An ordinary battery for a flashlight has an emf of 1.5 V.



Current

When we connect a battery to a load, electrons originate at the negative terminal – they want to move toward the positive terminal, but they can't go through the battery. To get to the positive terminal, they must travel around the circuit. We call the motion of charges a *current*. Current is defined to be how much charge travels in a given time.

$$I = \frac{\Delta Q}{\Delta t}$$

In other words, it is as if you were sitting at a certain point in the circuit and counting how many charges travel through the circuit during a given time. The unit for current is the *ampere*, where

$$A = \frac{C}{s}$$

Direction of current. When we discussed static electricity, I emphasized that, in metals, electrons move and the positive cores stay in place. By convention, however, when we define the direction of current, we define it as the direction that **positive** charges would travel. This is because scientists originally thought that electrons were positive. By the time we realized this wasn't correct, we were so used to the convention that no one wanted to change it.

Effect of current on people. You may have heard that it's not voltage that kills a person – it's current. The reason is that the heart depends on electrical signals to tell it when to beat. If you grab a wire and form a complete circuit (high potential, through you to ground, for example,) the current will travel through your body and if the path is right, can travel across your heart and cause it to go into fibrillation. The paddles that are used to get the heart beating again also supply a current, but it's a specific current, the purpose of which is to get your heart beating correctly again. You will often see electricians holding one hand behind their back (usually their left one) when working so that they are less likely to form a complete circuit.

Ohm's Law

Resistance. A battery supplies a potential difference with the goal of causing a current to flow. How the current flows depends on what it is flowing through. We will discuss in some detail what makes some materials electrical conductors and other materials electrical insulators tomorrow. You can think of current in a wire similarly to how water flows through a pipe. If the pipe is narrow, it's harder for the water to flow because less water can get through the pipe at one time. Similar effects exist in current. We quantify these effects by the resistance of the device.

The resistance of an object is defined as the ratio of the potential difference being passed across the object, divided by the current

$$R = \frac{V}{I}$$

This is called **Ohm's Law**. The unit of resistance is the ohm (Ω), where

$$\Omega = \frac{V}{A}$$

Note that the resistance of an object is a property of that object. The amount of current and voltage that the object draws can vary. If you change the potential difference across the object, the current going through that object also changes.

Ex. 1: A 110 V potential difference is applied to a blender with a resistance of 40 Ω.

a) What is the current going through the blender?

b) How much charge will flow through a given point in 2 minutes?

c) How many electrons is this charge equivalent to?

a)

$$I = \frac{V}{R} = \frac{110V}{40\Omega} = 2.8A$$

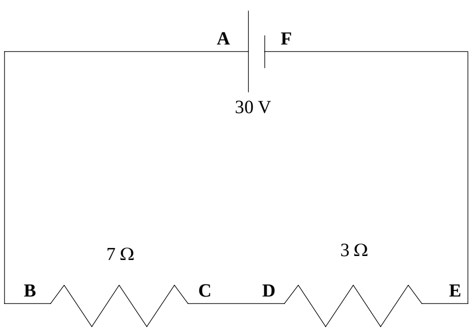
b)

$$I = \frac{\Delta Q}{\Delta t}$$
$$I\Delta t = \Delta Q$$
$$(2.8A)(120s) = \Delta Q$$
$$336C = \Delta Q$$

d) 1 electron has a charge of $1.602x10^{-19}$C. if there is a charge of 336 C, there must be

$$\frac{336C}{1.602x10^{-19}\ \frac{C}{e^-}} = 2.1x10^{21}\,\text{electrons}$$

Ex. 2: In the circuit, a current of 3A flows through the circuit. a) What would a voltmeter read when connected between points A and B? b) What would it read if connected between points B and C? The same current flows through each resistor. c) what is the potential at each point A, B and C?



The current goes in the direction positive to negative, so it goes counterclockwise in the circuit. Note that current isn't "used up" in circuits. At point A, the potential is 30 V. At point B, the potential is still 30 V. At point C, there will be a *potential drop* because of the resistance. The potential drop is given by Ohm's Law

$V_1 = IR_1 = (3\text{ A})(7\Omega) = 21$ V. So the potential difference, which is what is measured by the voltmeter, is 21 V. This means that the absolute potential at C is 30 V – 21 V = 9 V.

Between points D and E, there will be another voltage drop due to the 3Ω resistor. This voltage drop is given by:

$V_2 = IR_2 = (3\text{ A})(3\Omega) = 9$ V. This is what the voltmeter will read.

The potential at point E will be the 9 V at point D minus the 9 V drop, so the potential at point E is 0 V.

Resistivity

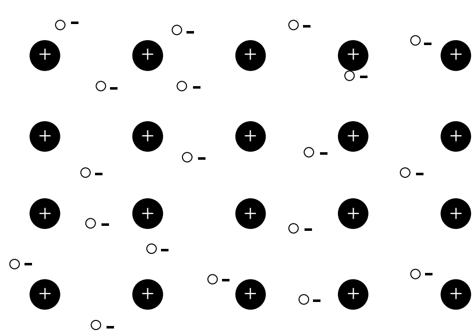
Current is the flow of electrons. You know already that current flows better through some things than through others. Why? First, let's think about why materials might have a resistance. Resistance impedes current. Let's think first about different shapes of materials. We used the analogy of electrons in the current with water flowing through a pipe. Let's say I have a pipe of cross sectional area A and length L. If you have a pipe with a larger area, this makes it easier for the water to flow, so we might guess that the 'resistance' of the pipe to the flow of water is inversely proportional to the area. If the pipe is longer, it will take longer for the water to flow, so a longer pipe has more 'resistance' than a shorter pipe.

So if we think about a wire carrying electrons, we might guess:

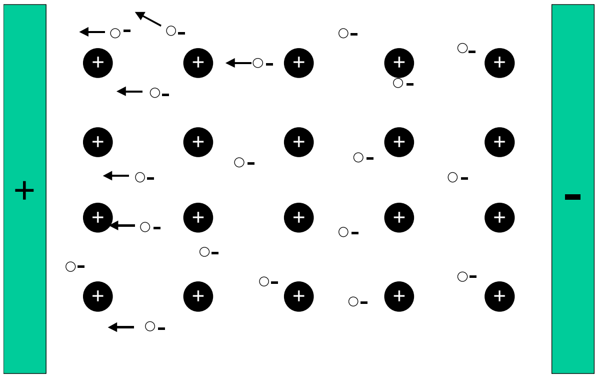
$$R \propto \frac{L}{A}$$

This is a problem, however, because it means that every different shaped piece of material will have a different resistance. So if you had 12 gauge copper wire and 24 gauge copper wire, you would have to give the resistance of both. It would be handy if there were a way to characterize a material according to the type of material and not the specific dimensions of the material. The quantity that accomplishes this is called the resistivity and is represented by the Greek letter rho (ρ). The resistance is a property of a specific piece of material, and the resistivity is a property of the type of material.

So why would the resistivity of different materials be different? Think back to our model of solids. We have a bunch of atoms connected by springs. For now, we're going to assume that there is not very much motion of the atoms - the springs stay still.



I mentioned that some atoms like to give up their electrons. Most metals are like this. We can think of metals as a bunch of ions (atoms that have 'lost' one or more of their valence electrons), surrounded by a sea of valence electrons wandering about the material.



When there is no current flowing, the electrons sort of move around randomly; however, when you apply a potential difference to the material, the electrons now have a preferred direction of travel. However, as they move along, they will bump into the positive ion cores, which causes them to change directions. The potential difference causes most of the motion to be in one direction; however, the scattering of electrons (which is what we call the collisions) causes some of the electrons to take longer in getting there than others. This is the reason that metals have any resistivity at all.

Insulators - things like plastic and wood - have very high resistivities. This is because atoms behave differently in insulators than in metals. Instead of the picture of ions with electrons moving around, you have to think of a picture of a bunch of neutral blobs. The electrons in insulators stay very close to their nucleus, so the electrons can't move as easily when a potential difference is applied. Semiconductors are materials with structures somewhere in between metals and insulators

We characterize different types of materials by their resistivity, ρ. The higher the resistivity, the worse a conductor that material is, so we finally obtain the formula for the resistance of a wire:

$$\boxed{R = \rho \frac{L}{A}}$$

The units of resistivity can be found by solving the above equation for ρ.

$$R = \rho \frac{L}{A}$$

$$R \frac{A}{L} = \rho$$

$$\Omega \frac{m^2}{m} = \rho$$

$$\Omega - m = \rho$$

So the units of resistivity are ohm-meters. (Ω-m).

Some typical values of resistivity are given in the table below. You will notice that I grouped the materials into three categories: conductors, semiconductors and insulators

metals	**ρ (x 10^{-8} Ω-m)**
silver	1.59
copper	1.68
aluminum	2.65
tungsten	5.6
iron	9.71
platinum	10.6
Nichrome (alloy of Ni, Fe, Cr)	100

Semiconductors	ρ (x 10^{-3} Ω-m)
carbon	300-6000
germanium	1 - 500
silicon	.0001 - .06
Insulators	**ρ (Ω-m)**
glass	10^{9} - 10^{12}
hard rubber	10^{13} - 10^{15}

Ex. 3: What is the resistance of a 33 m length of copper wire if the wire has a circular cross section of 0.05 m?

$$R = \rho \frac{L}{A}$$

$$R = 1.68\,x\,10^{-8}\,\Omega - m\left(\frac{33\,m}{\pi(0.05m)^2}\right)$$

$$R = 7.1x10^{-5}\,\Omega$$

One of the other things to remember is that resistance changes with temperature. We find that, for most metals,

$$\rho(T) = \rho_o(1 + \alpha\Delta T)$$

where ρ_0 is the resistivity at some reference temperature, and ρ_T is the resistivity at the temperature that is ΔT above the reference temperature. This comes in handy because we can make a very sensitive thermometer by measuring how the resistance changes with temperature. This is such a sensitive thermometer because the value of α, the temperature coefficient of resistivity is very small. (Don't confuse this α with the α from thermal expansion.) This formula tells you that, for most materials, the resistivity increases with increasing temperature. This is due to a number of factors, but one of the most important is that as temperature increases, the thermal energy of the atoms increases and the springs that hold the atoms together start oscillating more and more. This means that there are more collisions and a higher resistivity. Other effects occur when the temperature increases, but these are more complicated to explain.

Superconductivity

Resistivity is a problem. Let’s say you want to transport power across the state from a power substation. The resistivity of the wire causes a potential drop, so that if the wire goes a long way, you lose some of the potential. One way around this is to find materials that don’t have much resistivity.

Superconductivity is a phenomenon in which the resistivity of a material actually does go to zero. Superconducting materials were first discovered in 1911. So why don’t we use this wonder material? The problem is that we don’t currently know of any materials that are superconducting at room temperature. Most of them have a finite resistance at room temperature, which decreases and decreases and, at some temperature, drops to a number on the order of 10^{-25} Ω, which is for all intents and purposes zero. Until 11 years ago, the highest temperature at which a material was known to superconduct was about 23 K (remember that LN_2 is at a temperature of 77K). Ten years ago at a meeting in New York, some scientists from IBM announced that they had found a material that superconducted at 90 K. This was dubbed high-temperature superconductivity. The significance is that 90 K is higher than liquid nitrogen. Liquid nitrogen is relatively cheap (compared to other refrigerants) and environmentally very sound. Today, materials with temperature up to about 160 K have been found.

The other problem with these materials is that they are ceramics and are very brittle. For this reason, it's very difficult to machine them, or make them into things like wires because they break easily. Add to that the fact that most of these compounds contain things like mercury and/or thallium, so that machining the materials can release decidedly unfriendly things into the environment.

Ex. 4: You have a piece of platinum wire from a car you are repairing. The platinum wire is expensive, so you would like to replace it with cheaper copper wire. The hardware store has copper wire that has the same cross-sectional area as the platinum wire. How much copper wire do you need so that the two wires have the same total resistance?

Start by writing the resistance of each wire. Use P as the subscript for platinum and C as the subscript for Copper

$$R_P = \rho_P \frac{L_P}{A_P} \qquad\qquad R_C = \rho_C \frac{L_C}{A_C}$$

We want the resistance to be the same in each case, so $R_P = R_c$

$$R_C = R_P$$
$$\rho_C \frac{L_C}{A_C} = \rho_P \frac{L_P}{A_P}$$

We also know that the wires have the same cross-sectional area, so $A_C = A_P$.

$$\rho_C \frac{L_C}{A_C} = \rho_P \frac{L_P}{A_P}$$
$$\rho_C L_C = \rho_P L_P$$
$$\frac{L_C}{L_P} = \frac{\rho_P}{\rho_C}$$
$$\frac{L_C}{L_P} = \frac{10.6 \text{ x } 10^{-8}\ \Omega\text{m}}{1.68 \text{ x } 10^{-8}\ \Omega\text{m}}$$
$$\frac{L_C}{L_P} = 6.3$$

So we need to buy a piece of wire that is 6.3 times the length of the platinum wire we have.