Electromagnetism

There are four fundamental forces in nature:

1) gravity
2) weak nuclear
3) electromagnetic
4) strong nuclear

The latter two operate within the nucleus of an atom and we’re not going to discuss them very much. Gravity, we’ve seen, operates on everything, although we’ve applied it primarily to macroscopic objects.

You’ve probably heard of both electricity and magnetism. These two phenomena are related very closely – they are different aspects of the same phenomenon. We group them together and call the topic electromagnetism.

This force is significant at the level of an atom, so let’s remind ourselves of what an atom looks like.

Structure of the Nucleus. The atom is made up of a nucleus, which in turn is made up of protons and neutrons. The protons have a charge of +1 – we call them positive.

Surrounding the nucleus are a number of electrons – usually the same number of electrons as there are protons. Electrons also have a charge, but they are negatively charged and carry a -1 charge.

<table>
<thead>
<tr>
<th>Neutrons</th>
<th>no charge</th>
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<tbody>
<tr>
<td>Protons</td>
<td>+1</td>
</tr>
<tr>
<td>Electrons</td>
<td>-1</td>
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If we have an atom like, say, Carbon – which is number 6 on the periodic table – we know that there are 6 protons, 6 neutrons and 6 electrons. If we add up the charge that is carried by all of the atomic components, we find

\[
\text{net charge} = 6 (+1)+6(0)+6(-1) = 0
\]

In other words, there is no net charge on the atom. We call this a neutral atom.

Some atoms like grabbing electrons off other atoms. Some atoms like giving up electrons to other atoms. Those of you with some chemistry background know that the electrons fill up shells and that
the electrons left in the outermost shell are called valence electrons and can be donated to other atoms or can receive more electrons.

In our carbon atom, for example, we have two electrons in the 1s shell and four in the 1p shell. Let’s say that the carbon interacts with another atom and loses one of these outer electrons. The net charge now is:

$$6(+1) + 6(0) + 5(-1) = +1$$

Anytime the net charge on an atom is non-zero, we call it an **ion**.

When we put atoms together to form solids, the atoms all line up in a pattern, but typically the whole atom is drawn as a dot. If we look a little deeper, we find that the nuclei stay pretty much in one place, but that some electrons are free to move all over the material. Because the electrons are charged, they will respond to external stimuli. In some cases, you can actually move these electrons from one place to another.

**Transferring Charge.** For example, let’s say that you shuffle your feet along the carpeting. Initially, both the carpeting and your feel are neutral. As we’ll see, things like to be neutral.

As you shuffle along, there is friction between your foot and the carpeting. The carpet rubs electrons off your foot and collects them. This leaves the carpet with excess electrons, thus giving it a net negative charge. Your foot has lost electrons, and it is thus positively charged.

Now – and this is important – you must recognize that

**NO NET CHARGE IS GAINED OR LOST**

Charge is *transferred* from the foot to the carpeting, but no new charges are created, nor are any destroyed. This is a conservation law, just like the conservation of other quantities we’ve studied, such as energy or momentum. The same # of electrons that are rubbed off the foot appear on the carpet.

Note that I’m drawing only a few charges – in reality, a huge number of charges are transferred – about $10^{10}$ charges at a time.

Another rule that will become important to you, now that you’re walking around with a net positive charge is that
THINGS DON’T LIKE HAVING A NET CHARGE

So somehow, your foot is going to have to attract some extra electrons to make it neutral again. There are two ways this can happen:

1) If it’s humid, the water in the air can help neutralize unbalanced charges. Water is what we call a polar molecule:

   ![Water molecule diagram]

   The net charge on the water molecule is zero - it is indeed a neutral molecule. Why then can it help neutralize charge? The answer is because of its polar character. It turns out that the electrons like to hang out around the oxygen atom more than they like to hang out around the H atoms. This gives the oxygen end of the molecule a negative charge and the hydrogen ends a positive charge. The contact of water vapor with your foot allows the charge to be carried away. This is the preferred method for ridding your foot of excess charge.

2) If, however, it is not humid, there are not sufficient water molecules in the air to take away excess charge. The charge has to get out some other way. If you touch something metal with your foot, for example, the difference in charge will be so large that electrons will jump from the metal to your foot, giving you a shock.

The type of electricity produced by friction is called static electricity. In your dryer, clothes rub against each other continuously. Dryer sheets, like Bounce, contain lots of polar molecules to help keep the charge of your clothes neutral, thus avoiding static electricity.

When you’re thinking about the charges being transferred, remember that it is the electrons that are transferred – the nuclei stay in place and don’t get transferred. A net positive charge means that there is a lack of electrons. A net negative charge means that there is an excess of electrons.

Another good example of static electricity with which you’re probably familiar is the electricity produced when you rub a balloon on your hair. In this case, the electrons from the balloon are attracted to your hair. After rubbing, your hair has a net negative charge and the balloon has a net positive charge.

Attraction and Repulsion. You’ll note that the balloon and the wall seem to attract each other. This is an illustration of another fundamental property of charged things.

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<th>attract</th>
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<tr>
<td>+</td>
<td>+</td>
<td>repel</td>
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<tr>
<td>-</td>
<td>-</td>
<td>repel</td>
</tr>
</tbody>
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If I rub a PVC rod with a paper towel, electrons leave the PVC and move onto the towel, so the PVC is left with a net positive charge. If I take another PVC rod and do the same thing, both rods are positively charged. If I bring them near each other, they repel. A similar effect is found for negative charges, which leads us to another fundamental property of charges:

LIKE CHARGES REPEL: UNLIKE CHARGES ATTRACT

A comment here that the assignment of the negative charge to electrons is really very arbitrary - Ben Franklin decided on this convention and we’ve stuck with it. There is no reason why we couldn’t have gone the other way and called the charge on an electron positive – but we didn’t.
Going back to my example about shuffling your feet on the carpeting and touching something metal. What would happen if you instead touched something wood? You won’t get a shock, or at least, not much of one. This is because there are some big differences between different types of materials.

**Conductors and Insulators.** In metals, the electrons are not as tightly bound to the nuclei and they are more free to move. If I have a + charge on my finger and I bring it near a metal doorknob, the electrons are attracted to the + charges and move over toward the finger:

This is what allows the electrons to ‘jump’ onto the hand, producing a shock. In an **insulator** – such as wood, plastic, rubber, etc. – electrons are more closely bound to the nucleus, so the electrons can’t move as easily. This results in there being less charges available for neutralization. This is why wire is usually copper (although silver actually conducts electricity better, it is more expensive) and has rubber or plastic on it as an insulator.

There are also materials that are in between conductors and insulators – we call them semiconductors. Under the right conditions, they can conduct electricity, but not as well as metals.

**Conduction and Induction**

Now I want to address the process by which things acquire charge. I’m going to use rods to symbolize whatever objects we’re looking at.

There are two different ways to transfer electrons:

1) **induction**

2) **contact or conduction**
Let’s say that I have a rod I’ve already charged with positive charges. When I bring this rod close to the neutral rod, the electrons in the neutral rod will move toward the positive rod end. This leaves a positive charge at the other end of the rod, where the electrons are deficient. This process is called charging by **induction**. The previously neutral rod is still neutral – no charges have been transferred – but there is now a charge separation so that one end of the rod is negative and the other end is positive.

Now let’s bring the two rods in contact – being in contact allows charges to actually move from one rod to the other. The electrons were already over by the end nearest the positive charge and they now jump over to the positively charged rod. This leaves a lack of electrons, which gives a positive charge to the second rod. This is called charging by **conduction** or **contact**.

You may have heard the term ‘**ground**’ in conjunction with electricity. You also hear ‘ground’ referred to as ‘Earth’ - because a real ground is often a long metal pipe stuck in the ground. The Earth is so massive that it acts like a source or sink of electrons. You can deposit or withdraw many electrons and not change the overall charge of the Earth. The third prong you see on plugs is a ground – it is there because, if there is a need for the electrons that are being carried in the other two wires to leave those wires, you want them to go somewhere other than your body. The ground plug helps to funnel them safely away from people.

The electroscope is a device that can be used to tell if two things have the same or different charges. (it consists of a metal electrode attached to two pieces of foil. The contraption is usually placed in a glass bulb with a rubber stopper (why? - both glass and rubber are good insulators of electricity and isolate the system.) If you charge the electroscope with a known charge (say by touching it with a charged rod), you now have a known charge, let’s say a negative one. You can then take your unknown charged object and bring it near. If the charge is the
same as the one already on the equipment, the leaves move further apart. If it is different, they will move closer together.

Now we want to quantify the interaction between charges. A scientist named Coulomb (1750’s - 1800’s) used a torsion balance to investigate interactions between charges.

By charging a sphere and bringing it near one of the spheres on the torsion balance, a certain deflection is observed. To quantify this, though, you have to know how much charge is on the sphere. What Coulomb reasoned is that if you charge up a sphere with some charge, then touch another conducting sphere, the charge will be split equally between them. Now you at least know the ratio of the charge on the two spheres. By varying the charge ratio, he found that:

If the charge on either of the spheres is doubled, the force is doubled

If the distance between the spheres is doubled, the force decreases by a factor of four.

We abstract the balls as point charges – an imaginary point that contains the same amount of charge as that on the ball. If one of the point charges has charge \( q_1 \) and the other has \( q_2 \), and they are separated by a distance \( r \), the force between them is given by Coulomb’s Law:

\[
F = k \frac{q_1 q_2}{r^2}
\]

If we wanted to, we could choose to work in a system of units where \( k = 1 \) (and some people do) however, in the SI system, we use the coulomb (C) as the unit for charge, so that the constant \( k \) has the value

\[
k = 8.988 \times 10^9 \text{ Nm}^2/\text{C}^2, \text{ or approximately } 9 \times 10^9 \text{ Nm}^2/\text{C}^2
\]

One C is thus the amount of charge that, if placed on two objects a distance of 1 m apart, will cause each object to exert a force of 9 \times 10^9 N on each other.

**Direction of the Coulomb Force:** Remember that force is a vector, so in addition to the magnitude, we also have to think about the direction. For two charges, the force will always lie along the line connecting the two charges. If there are more than two charges, we find the force on a charge due to all of the other charges by doing a vector sum.

Static electricity caused by friction produced about a microcoulomb or less of charge. How many electrons is this?

**Charge on an Electron:** The charge of an electron has been measured. It is a fundamental quantity and has the value \( 1.602 \times 10^{-19} \text{ C} \) - but note that it is a negative number. We define

\[
e = 1.602 \times 10^{-19} \text{ C}
\]

so that the charge on an electron is -e. The charge on a proton is +e.

An interesting consequence of this value is that charge only comes in these little packets. You can have a charge of e, 2e, 3e, 4e, etc. but you can’t have 5/2 e. In general, e is such a small number and we deal with so many electrons at a time, that we don’t notice that there are jumps; however, as electronics get smaller and smaller, this becomes a problem as the structures that carry current
(electrons) get so small that they only let one electron pass through at a time and then you do see the effects of the quantization of charge.

If \( q_1 > 0 \) and \( q_2 > 0 \), then \( F \) will be positive. This corresponds to the two charges trying to repel each other. Note that the force the two charges exert will be equal and opposite in all cases. The same situation applies when the two charges are both positive. When the sign on the force is positive, the force is **repulsive**.

If the two charges have opposite signs, the force will be a negative number, which corresponds to **attraction** between them.

We sometimes write Coulomb’s law slightly differently:

\[
F = k \frac{q_1 q_2}{r^2} = \frac{1}{4\pi \varepsilon_o} \frac{q_1 q_2}{r^2}
\]

where \( \varepsilon_o \) is called the permittivity of free space.

\[
k = \frac{1}{4\pi \varepsilon_o}
\]

\[
\varepsilon_o = \frac{1}{4\pi k} = 8.85 \times 10^{-12} \frac{C^2}{Nm^2}
\]

As usual, there are some restrictions on this law – it only works when the sizes of the objects are much smaller than the distance between them.

**Notation:** We will use the notation \( F_{ab} \) to mean the force on object \( a \) due to object \( b \).

That works for two charges: what if there are more?

**Ex. 1:** A charge of -3 \( \mu \)C and a charge of +7 \( \mu \)C are a distance of 0.5 m apart. Which of the following is true:

- a) \( F_{12} \) is to the right and \( F_{21} \) is to the right
- b) \( F_{12} \) is to the left and \( F_{21} \) is to the left
- c) \( F_{12} \) is to the left and \( F_{21} \) is to the right
- d) \( F_{12} \) is to the right and \( F_{21} \) is to the left
- e) Since \( F_{12} = -F_{21} \), the forces sum to zero
**Ex. 2:** A charge of -3 \( \mu \text{C} \) and a charge of +7 \( \mu \text{C} \) are a distance of 0.5 m apart. Find \( F_{12} \) and \( F_{21} \)

\[
F_{12} = k \frac{q_1 q_2}{r^2}
\]

\[
F_{12} = \left(9 \times 10^9 \text{ Nm}^2/\text{C}^2\right) \frac{(-3 \times 10^{-6} \text{ C})(+7 \times 10^{-6} \text{ C})}{(0.5 \text{ m})^2}
\]

\[
F_{12} = -0.76 \text{ N}
\]

The negative sign means that the forces are attractive. If we draw the picture as shown at left, the force that 1 exerts on 2 (\( F_{21} \)) is to the left. The force that 2 exerts on 1 (\( F_{12} \)) is of the same magnitude and to the right.

**Ex. 3:** A third charge of magnitude -12 \( \mu \text{C} \) is placed 0.7 m to the left of the -3 \( \mu \text{C} \) charge and colinear with both charges. Find the total force on each charge.

Here, we are going to be interested only in the magnitudes of the Coulomb force: the sign tells us the direction of the force.

We already have from above that

\( F_{12} = \) force on 1 due to 2 is 0.76 N to the right

\( F_{21} = \) force on 2 due to 1 is 0.76 N to the left
These two charges are the same, so their interaction will cause
$F_{13} = \text{force on charge 1 due to charge 3 is 0.66 N to the right}$
$F_{31} = \text{force on charge 3 due to charge 1 is 0.66 N to the left}$

Now

$F_{23} = k \frac{q_2q_3}{r^2}$

$F_{23} = \left(9 \times 10^9 \text{ Nm}^2 \text{ C}^{-2}\right) \left(-3 \times 10^{-6} \text{ C}\right) \left(-12 \times 10^{-6} \text{ C}\right) \left(0.7 \text{ m}\right)^2$

$F_{23} = -0.53 \text{ N}$

The sign is negative – the force here is attractive.
$F_{23} = \text{force on charge 2 due to charge 3 is 0.53 N to the left}$
$F_{32} = \text{force on charge 3 due to charge 2 is 0.53 N to the right}$

In sum:
$F_{13} = \text{force on charge 1 due to charge 3 is 0.66 N to the right}$
$F_{31} = \text{force on charge 3 due to charge 1 is 0.66 N to the left}$
$F_{23} = \text{force on charge 2 due to charge 3 is 0.53 N to the left}$
$F_{32} = \text{force on charge 3 due to charge 2 is 0.53 N to the right}$
$F_{12} = \text{force on 1 due to 2 is 0.76 N to the right}$
$F_{21} = \text{force on 2 due to 1 is 0.76 N to the left}$

We now want to assign signs to these forces: let’s take left to be negative

<table>
<thead>
<tr>
<th>Charge</th>
<th>Force</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_{13}$</td>
<td>$+0.66 \text{ N}$</td>
</tr>
<tr>
<td>$F_{31}$</td>
<td>$-0.66 \text{ N}$</td>
</tr>
<tr>
<td>$F_{23}$</td>
<td>$-0.53 \text{ N}$</td>
</tr>
<tr>
<td>$F_{32}$</td>
<td>$+0.53 \text{ N}$</td>
</tr>
<tr>
<td>$F_{12}$</td>
<td>$+0.76 \text{ N}$</td>
</tr>
<tr>
<td>$F_{21}$</td>
<td>$-0.76 \text{ N}$</td>
</tr>
</tbody>
</table>

The total force on charge 1 is thus:
\[ F_{on1} = F_{12} + F_{13} = +0.76 \text{ N} + 0.66 \text{ N} = +1.42 \text{ N} \text{ (to the right)} \]
\[ F_{on2} = F_{21} + F_{23} = -0.76 \text{ N} - 0.53 \text{ N} = -1.29 \text{ N} \text{ (to the left)} \]
\[ F_{on3} = F_{31} + F_{32} = -0.66 \text{ N} + 0.53 \text{ N} = -0.13 \text{ N} \text{ (to the left)} \]

**Ex. 4:** One model of the hydrogen atom (which has one proton and one electron) is called the Bohr model. In this model, the electron orbits the proton similarly to how the earth orbits the sun. The distance between the proton and the electron is \( 5.3 \times 10^{-11} \text{ m} \). What is the electrostatic force between the two particles?

\[
F_{ep} = k \frac{q_e q_p}{r^2}
\]

\[
F_{ep} = \left( 9 \times 10^9 \text{ Nm}^2/\text{C}^2 \right) \frac{(-1.6 \times 10^{-19} \text{ C})(1.6 \times 10^{-19} \text{ C})}{(5.3 \times 10^{-11} \text{ m})^2}
\]

\[ F_{ep} = -4.3 \times 10^{-18} \text{ N} \]