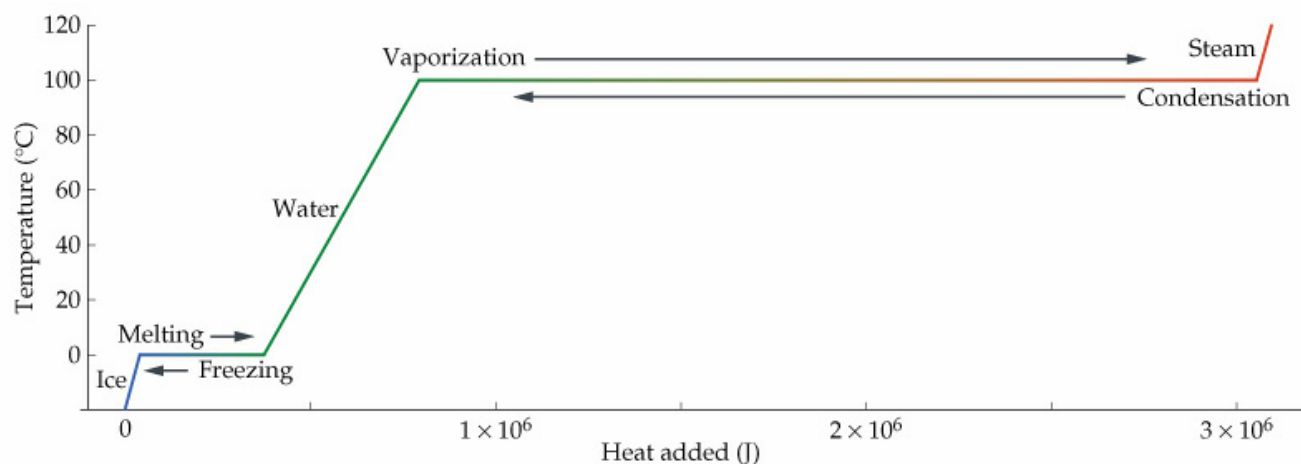# PHYSICS 151 – Notes for Online Lecture #35

Latent Heat

In talking about the heats involved in changing temperatures, one thing we've neglected is what happens when a material changes phase. We talked about the three phases - gas, liquid and water. When a material changes from one phase to another, we call this a **phase transition** or **change of phase**.

We can explore this phenomenon by looking at an example. Let's imagine that we have a 100 g piece of ice at -40°C, and we want to heat it to a temperature of 110°C. How much heat (energy) does that take?

We know that when we heat the water from 0°C to 100°C, we can calculate how much heat is necessary to add in order to accomplish this by using $Q = mc\Delta T$. However, if we plot the heat added to the system against the temperature increase over the entire -40°C to 110°C range, we would not find as linear relationship. We would actually find that there are certain regions where we put heat in, but we don't get any temperature change. These are the regions surrounding the phase transitions.
If we start with ice at -40°C, we find the linear increase with ΔT that we would expect from the relationship between Q and ΔT. Upon reaching 0°C, we find that we put in energy, see no temperature change, put in more energy, see no temperature change, until finally we see that the temperature again starts increasing. The slope is different because the specific heat, c, is different for different phases.
For ice, c = 0.5 kcal/kg-°C, for water, c =1.0 kcal/kg-°C and for steam, c = 0.48 kcal/kg-°C.
The amount of heat that must be added to accomplish the phase transition from solid to liquid is called the **heat of fusion** and the symbol is L_F. For water, the heat of fusion is 80 kcal/kg or 335 kJ/kg.
The **heat of vaporization**, L_v, is the amount of heat necessary to accomplish the phase transition from liquid to solid. For water, the heat of vaporization is 540 kcal/kg or 2260 kJ/kg. A large amount of heat must therefore be added to vaporize water. Additional heat is then necessary to raise the temperature of the steam to 110°C. The heat of vaporization and of fusion are collectively referred to as latent heats. The total amount of heat resulting from a phase transition is thus $Q = mL$
where L is the latent heat for the appropriate process (fusion or vaporization) and m is the mass of whatever is changing phase.

Thus, the total energy needed is:

$$Q_{Total} = mc_{ice}\Delta T + mL_f + mc_{water}\Delta T + mL_V + mc_{steam}\Delta T$$

$$= (0.1kg)\left(2090\frac{J}{kg - C°}\right)\left[(0C°)-(-40C°)\right] + (0.1kg)\left(3.35\times10^5\frac{J}{kg}\right)+$$

$$(0.1kg)\left(4186\frac{J}{kg - C°}\right)\left[(100C°)-(0C°)\right] + (0.1kg)\left(2.26\times10^6\frac{J}{kg}\right)+$$

$$(0.1kg)\left(2010\frac{J}{kg - C°}\right)\left[(110C°)-(100C°)\right] = 3.12\times10^5 J$$

The process also works in reverse: when water changes from steam to water, it releases heat in the same amount. Note that some materials can change from liquid to gas at temperatures other than the boiling point – for example, when water evaporates from your skin. When water evaporates, the latent heat must come from the water itself – therefore, the water must cool as it gives up heat. We call this evaporative cooling.

If you think back to our molecular picture of solids, liquids and gases, you remember that melting involves breaking the forces acting between the molecules that hold them together. When we start at the lower temperature, all of the heat that we put into the solid goes into increasing the kinetic energy of the molecules, which in turn raises their temperature. When you get to the melting temperature, the heat stops going toward increasing the temperature and instead starts working to break down the forces that hold the molecules together. There is no temperature rise during this time. At some point, sufficient energy to overcome the forces is present, and the material melts. In a liquid, the molecules are further apart from each other, so breaking the forces that hold them together as a liquid is harder – thus the heat of vaporization is larger than the heat of fusion.

We have to account for these heats in the conservation of energy equation: the latent heat must be included if a phase change is involved.

Ex. 35-1: How much heat is necessary to raise a 50.0 g block of ice from -20°C to 20°C?
In raising the temperature moving from -20°C to 0°C, we need an amount of heat

$$Q = mc\Delta T = (5.00 \times 10^{-2}\, kg)\left(0.5\frac{kcal}{kg-°C}\right)(0°C - (-20°C)) = 0.5\, kcal$$

Then we must melt the ice, so we need an additional amount of heat:

$$Q = mL_f = (5.00 \times 10^{-2}\, kg)(79.7\, kcal/kg) = 4.0\, kcal$$

Then we have to raise the temperature of the resulting water from 0°C to 20°C:

$$Q = mc\Delta T = (5.00 \times 10^{-2}\, kg)\left(1.0\frac{kcal}{kg-°C}\right)(20°C - 0°C) = 1.0\, kcal$$

The total is thus 0.5 kcal + 4.0 kcal + 1.0 kcal = 5.5 kcal.

Ex. 35-2: 1.5 g of ice at a temperature of -21 °C is placed in a calorimeter made of aluminum with a mass 1.5 x 10² g at a temperature of 75°C. The system is allowed to come to thermal equilibrium. What is the final temperature of the mixture?

A reminder that c_{Al} = .22 cal/g °C c_{ice} = .5 cal /g-°C c_{water} = 1.0 cal/g °C $L_{f\text{-water}}$= 80 cal/g

Before you can begin writing a conservation of energy equation for the heat involved, you need to be sure that you know what happens? Does any of the ice melt? Does all of the ice melt? You need to do a back of envelope calculation to determine the final state.

What happens:
1) The ice will change temperature from T_{iice} =-21°C to 0°C
2) The ice will change into water at 0°C
3) The water will change temperature from 0°C to T_f.
4) At the same time, the calorimeter will change temperature from its initial temperature T_{ic} = 75 °C to the final temperature T_f.

Heat change of calorimeter is $Q = m_{Al}c_{Al}\left(T_f - T_{iAl}\right)$

Heat change of ice from T_{ii} to 0°C: $Q = m_{ice}c_{ice}\left(0°C - T_{iice}\right) = -m_{ice}c_{ice}T_{iice}$

Ice change to water $Q = m_{ice}L_w$

Heat change of water from 0 to T_f $Q = m_{ice}c_{water}\left(T_f - 0°C\right) = m_{ice}c_{water}T_f$

The total heat gained by the calorimeter will have to equal the total heat lost by the water. Another way of saying this is that the two heats have to add to zero.

In specific heat terms the signs are taken care by the temperature difference – In latent heat terms you need to add the sign yourself – positive for heat gained, negative for heat lost.

$$Q_{lost} + Q_{gained} = 0$$

$$m_{Al}c_{Al}\left(T_f - T_{iAl}\right) + m_{ice}c_{ice}\left(0°C - T_{iice}\right) + m_{ice}L_w + m_{ice}c_w\left(T_f - 0°C\right) = 0$$

$$m_{Al}c_{Al}\left(T_f - T_{iAl}\right) - m_{ice}c_{ice}T_{iice} + m_{ice}L_w + m_{ice}c_wT_f = 0$$

$$m_{Al}c_{Al}T_f + m_{ice}c_wT_f = m_c c_{Al}T_{iAl} + m_{ice}c_{ice}T_{iice} - m_wL_w$$

$$\left(m_{Al}c_{Al} + m_{ice}c_w\right)T_f = m_c c_{Al}T_{ic} + m_{ice}c_{ice}T_{ii} - m_wL_w$$

$$T_f = \frac{m_c c_{Al}T_{ic} + m_{ice}c_{ice}T_{ii} - m_wL_w}{\left(m_{Al}c_{Al} + m_{ice}c_w\right)}$$

$$T_f = \frac{\left(150g\right)\left(0.22\frac{cal}{g-°C}\right)\left(75°C\right) + \left(1.5g\right)\left(0.5\frac{cal}{g-°C}\right)\left(-21°C\right) - \left(1.5g\right)\left(80\frac{cal}{g}\right)}{\left(\left(150g\right)\left(0.22\frac{cal}{g-°C}\right) + \left(1.5g\right)\left(1.0\frac{cal}{g-°C}\right)\right)}$$

$$T_f = 67.8°C$$

How high would the amount of ice have to increase before our conclusion about the final temperature being greater than 0°C would change?

$$693\,cal = m_{ice}\left(c_{ice}\Delta T + L_f\right)$$

$$m_{ice} = \frac{693\,cal}{\left(c_{ice}\Delta T + L_f\right)} = \frac{693\,cal}{\left(0.5\dfrac{cal}{g-{}^\circ C}\right)\left(21{}^\circ C\right)+\left(79.7\dfrac{cal}{g}\right)}$$

$$m_{ice} = 7.7\,g$$

How much ice would there need to be before none of the ice would melt (meaning that the final answer would be between -21℃ and 0℃?