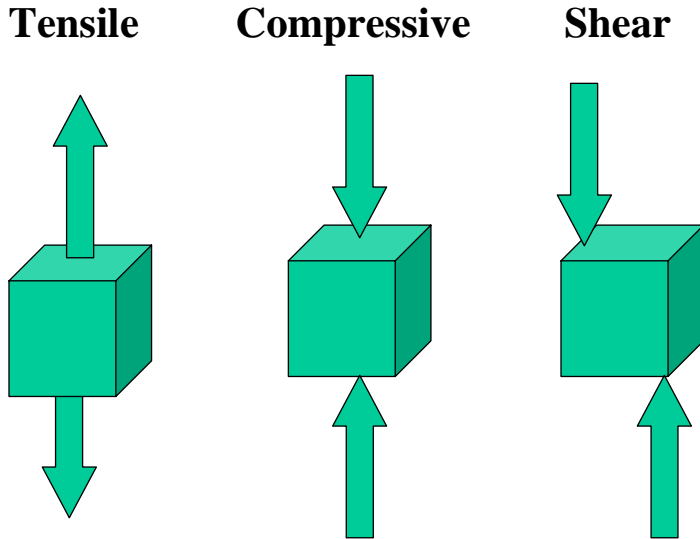


PHYSICS 151 – Notes for Online Lecture #34

Stress, Strain and Elasticity



In many applications, a beam or other object must be loaded – that is, some force must be exerted on it.

If we pull on the ends of a bar, we say that we exert a **tensile** stress – the bar is in tension

If we push on the ends of a bar, we say that we exert a **compressive** stress.

You can also exert a type of stress called **shear**, in which you push on opposite sides of an object. This is best illustrated by a telephone

book. If I place the book on the table, and push, the frictional force causes a force toward the left, while I push toward the right. What you see is that I have deformed the object in a different way than with tensile or compressive stress.

We define **stress** as the ratio of the force applied to a material to the area over which the force is applied.

$$\text{stress} = \frac{\text{force}}{\text{area}} = \frac{F}{A} \quad \text{The units of stress are N/m}^2$$

When you apply a stress, you will find that the object will experience a length change: if you hang a weight from a wire, the object will get longer. If you have a beam support a roof, the beam will get shorter.

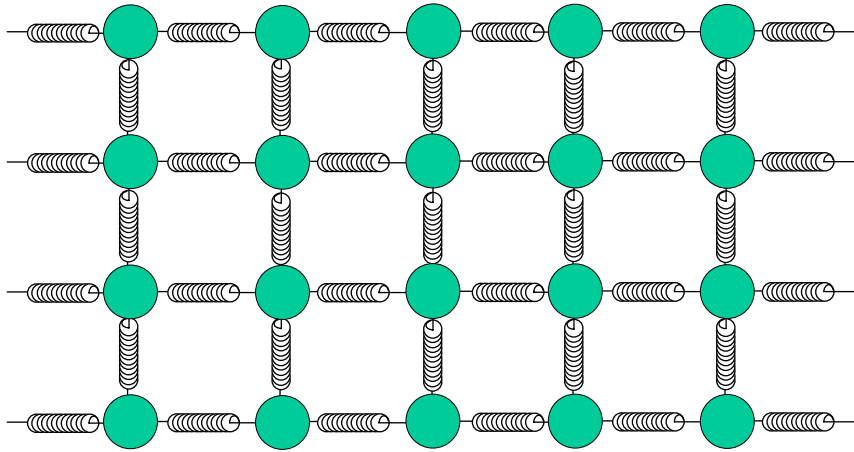
The **strain** is defined as the ratio of the change in length divided by the original length. If we say that the object originally had length L_0 and that it changed by an amount ΔL ,

$$\text{strain} = \frac{\Delta L}{L_0} \quad \text{Strain has no units}$$

The Relationship between Stress and Strain

If I take something sort of stretchy, like a piece of saran wrap, you'll notice that I can stretch it a little ways and it pretty much returns to its original shape. If I stretch it further, some of the materials may return to its original shape, but there is some of the material that doesn't. If I stretch it even further, it breaks.

All materials have some amount of elasticity - they give a little bit when they are stretched or compressed. In that way, materials are like springs. We can model a solid like this:



Where each molecule is connected to the next by a spring.

If you take a spring and pull (or push) it a distance Δx from its resting position, the force we need to pull (or push) is

$$F = k \Delta x$$

This is called Hooke's law and it applies to springs in the limit of small displacements.

	<p>The Saran Wrap is like a spring in that, if I pull a little bit, I cause a deformation, but when I let go, the material 'springs' back into place. We call this elastic deformation, because it is reversible.</p>
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We know, however, that an object can't just stretch forever. As you saw with the Saran wrap, at some point, you start doing some permanent damage to the material.

	<p>This is called plastic deformation. Now, the relationship between the force and the displacement is no longer linear.</p>
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$$F = ?\Delta x^n$$

In Hooke's law, the 'k' is material dependent, but also geometry dependent. That is, if I give you a piano wire and a 2" thick cable, both made of the same material, and you're going to use one of them to support your weight, you'd pick the thick cable - it has a stronger spring constant, k.

Stress is proportional to Strain with a constant of proportionality called **Young's modulus Y** (or E in some books). It must have units N/m². E depends only on the type of material. Young's modulus is a lot more useful than a spring constant, because we've taken all of the geometry out of it.

$$Y = \frac{\text{Stress}}{\text{Strain}}$$

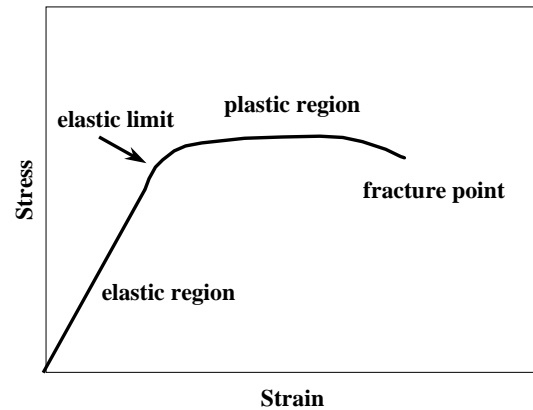
$$Y = \frac{F L_o}{A \Delta L}$$

$$\Delta L = \frac{1}{Y} \frac{F}{A} L_o$$

In most cases, the Young's modulus is the same for both tensile and compressive stresses.

Some Values of Young's Modulus	
Material	Y (in 10 ¹⁰ N/m ²)
Granite:	45
Tungsten	36
Steel	20
Copper	11
Brass	9.1
Aluminum	7.0
Bone	1.5
Pine parallel to the grain	1
Nylon	0.5
Pine perpendicular to the grain	0.1
Rubber	4.0 x 10 ⁻⁴

If you plot stress vs. strain for a real material, your graph will look like this:



FRACTURE:

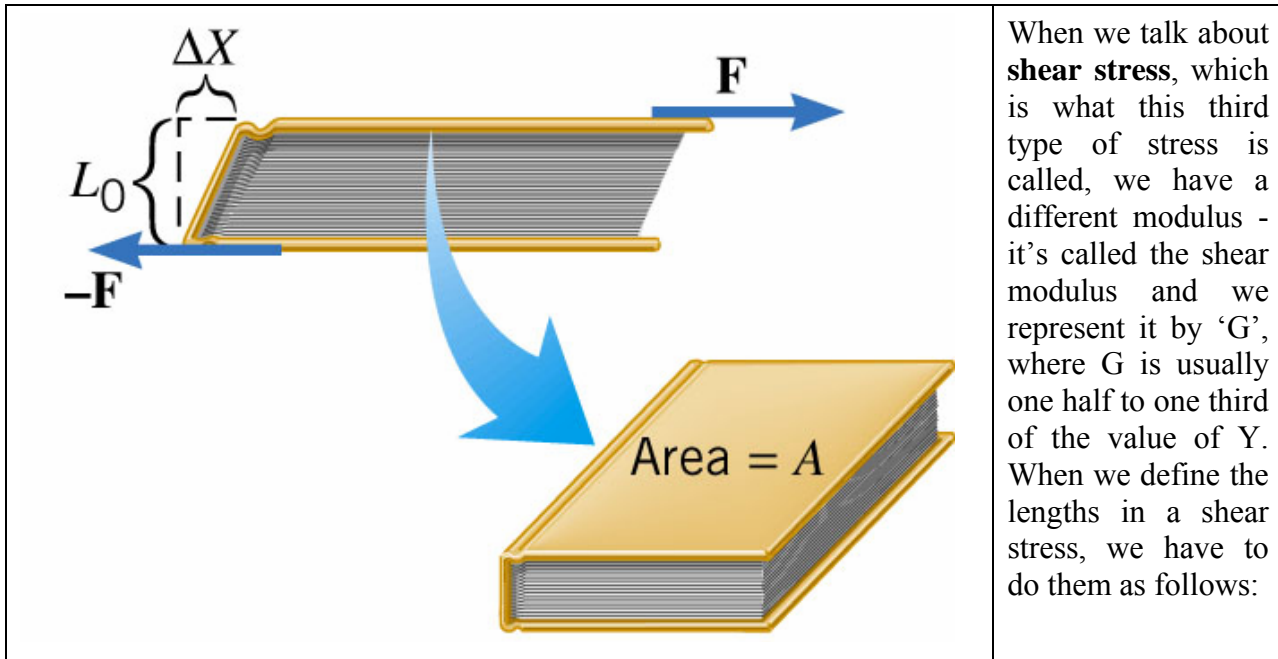
The largest stress that can be applied to the material without causing irreparable damage is called the elastic limit. When you're designing something, you usually don't push a material to its ultimate strength. Instead, you use a formula that says the maximum force that can be applied is equal to the elastic limit divided by some safety factor. For static loading, the safety factor is in the area of 2 and for dynamic loading, on the order of 4. This means that you only design to 25% - 50% of the actual strength of the material, just to be on the safe side.

This means that you're usually within the elastic limit in your design; however, you have to account for the stretching or compressing that will be caused by the application of a load when you're designing something.

If you put too much stress on something, it will eventually break. Regardless of the type of stress, compressive, tensile or shear, all materials have some upper limit, which we called the ultimate strength of the material.

Some materials are better at dealing with one type of stress than another. For instance, materials like concrete, granite, brick, etc. are very good at withstanding compressive stress, but terrible for withstanding tensile stress.

If you think back to our cantilever problem, we had two supports, one of which is under tensile stress and one under compressive stress. For this reason, we would have to use either two different materials, or one material that could withstand both types of stress.



When we talk about **shear stress**, which is what this third type of stress is called, we have a different modulus - it's called the shear modulus and we represent it by 'G', where G is usually one half to one third of the value of Y. When we define the lengths in a shear stress, we have to do them as follows:

$$\Delta L = \frac{1}{G} \frac{F}{A} L_0$$

In this case, A is the area parallel to the applied force, not perpendicular! L_0 and ΔL must be perpendicular to each other. We're not going to do anything more with shear in this class, but it is a very important type of force.

If you are designing things that will go underwater or into the air, where the water pressure, or the air pressure will acts on them from all sides, you have to worry about changes in length in all three dimensions, which is actually a change in volume. Here, we define a bulk modulus, B, such that

$$\Delta V = -\frac{1}{B} \Delta P V_0$$

Note carefully the minus sign – this indicates that, if the pressure increases, the volume decreases.

$$\frac{\Delta V}{V_0} = -\frac{1}{B} \Delta P$$

Ex. 34-1: A round nylon string on a tennis racquet is 1.00 mm in diameter and under a tension of 2.50×10^2 N.

- What is the strain in the string?
- What is the stress in the string?
- How much will the string be lengthened if its original length is 3×10^{-2} m?

$$F = 2.50 \times 10^2 \text{ N}$$

$$A = \pi r^2 = \pi(0.5\text{mm})^2 = \pi(0.5 \times 10^{-3}\text{m})^2 = 7.85 \times 10^{-7} \text{ m}^2$$

$$L_o = 3 \times 10^{-2} \text{ m}$$

$$Y = 5 \times 10^9 \text{ N/m}^2$$

$$\text{STRAIN} = \frac{\Delta L}{L_o}$$

$$\text{STRAIN} = \frac{1.91 \times 10^{-2} \text{ m}}{30.0 \times 10^{-2} \text{ m}} = 0.64$$

$$\text{STRESS} = \frac{F}{A}$$

$$\text{STRESS} = \frac{250\text{N}}{\pi(.5 \times 10^{-3} \text{ m})^2} = 3.18 \times 10^8 \frac{\text{N}}{\text{m}^2}$$

To find the length change:

$$Y \frac{\Delta L}{L_o} = \frac{F}{A}$$

$$\Delta L = \frac{1}{Y} \frac{F}{A} L_o$$

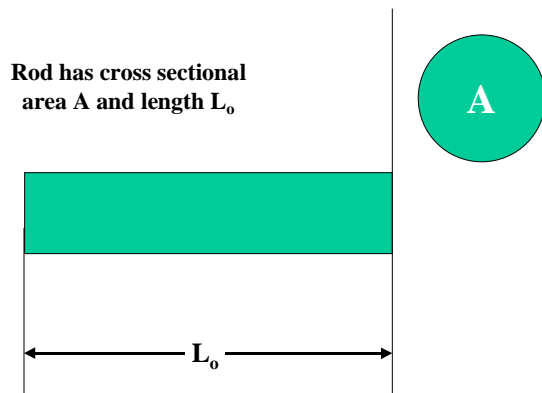
$$\Delta L = \frac{1}{5 \times 10^9 \frac{\text{N}}{\text{m}^2}} \frac{2.50 \times 10^2 \text{ N}}{7.85 \times 10^{-7} \text{ m}^2} (3 \times 10^{-2} \text{ m})$$

$$\Delta L = 1.9 \times 10^{-3} \text{ m}$$

Ex. 34-2: The Mariana trench is located in the Pacific Ocean, and at one place it is nearly seven miles beneath the surface of the water. The water pressure at the bottom of the trench is enormous, being about $\Delta P = 1.1 \times 10^8$ Pa greater than the pressure at the surface of the ocean. A solid steel ball of volume $V_o = 0.20 \text{ m}^3$ is dropped into the ocean and falls to the bottom of the trench. What is the change ΔV in the volume of the ball when it reaches the bottom?

$$\Delta V = -\frac{V_o \Delta P}{B} = \frac{-(0.20 \text{ m}^3)(1.1 \times 10^8 \text{ Pa})}{\left(1.4 \times 10^{11} \frac{\text{N}}{\text{m}^2}\right)} = -1.6 \times 10^{-4} \text{ m}^3$$

Stress due to Thermal Expansion



We have talked about stresses and strains. If I have a rod of length L_0 rigidly fixed in place, say, against a wall, an increase in temperature will cause the rod to exert a force on the wall. (This can also apply to liquids – think what happens when ice freezes in your water pipes.) Hooke's Law says:

We know that we can calculate the length change due to temperature. Combining this with the Young's Modulus equation yields:

This tells you how much force, is exerted on the wall due to the expansion of the rod. The quantity F/A is called the **thermal stress**.

$$\Delta L = \frac{1}{Y} \frac{F}{A} L_0 = \alpha \Delta T L_0$$

$$\frac{1}{Y} \frac{F}{A} = \alpha \Delta T$$

$$\frac{F}{A} = Y \alpha \Delta T$$

$$F = Y A \alpha \Delta T$$

Ex. 34-3: A cylindrical steel beam is placed in contact with a concrete wall. If the beam was placed on a very cold day (0°C), will there be a problem when the temperature rises to 40°C ?

E for Steel = $200 \times 10^9 \text{ N/m}^2$

$$\frac{F}{A} = Y \alpha \Delta T \text{ – Thermal Stress}$$

$$\frac{F}{A} = Y \alpha \Delta T$$

$$\frac{F}{A} = \left(200 \times 10^9 \frac{\text{N}}{\text{m}^2} \right) \left(12 \times 10^{-6} \frac{1}{^\circ\text{C}} \right) (40^\circ\text{C} - 0^\circ\text{C})$$

$$\frac{F}{A} = 9.6 \times 10^7 \frac{\text{N}}{\text{m}^2}$$

The tensile strength of concrete is only $2 \times 10^6 \text{ N/m}^2$, the compressive strength is only $20 \times 10^6 \text{ N/m}^2$ and the shear strength is only $2 \times 10^6 \text{ N/m}^2$. The thermal stress exceeds all of these.