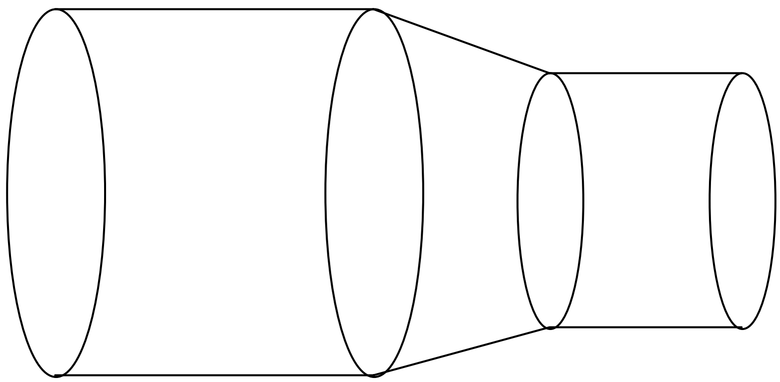
PHYSICS 151 – Notes for Online Lecture #30

FLUID FLOW

In this section we want to consider the flow of fluids. We will deal with flows where the paths of each small element of fluid (called **streamlines**) do not become tangled with other streamlines. This type of ordered flow is called **laminar**, as opposed to irregular, complex flow known as **turbulent**.

We need to derive something called the equation of continuity. This describes how fluid flow changes through pipes of changing diameter.



Since the flow is constant and a fluid is incompressible, the same amount of fluid leaves each region from the right as enters each region from the left during the same time interval.

$$\rho A_1 v_1 \Delta t = \rho A_2 v_2 \Delta t$$
$$v_1 A_1 = v_2 A_2$$

The equation of continuity states that the flow of material through a tube of changing cross section is constant. It is really an expression of conservation of mass. The product Av is the volume rate of flow and has units of m^3/s.

Thus one can see that when pipes are narrow (have small cross sectional areas) the velocity must be large and when pipes are wide (have large cross sectional areas) the velocity must be small.

Ex. 30-1 - The aorta has an inside diameter of approximately 0.50 cm, compared to that of a capillary, which as about 1.0 x 10^{-5} m. In addition, the average speed of flow is approximately 1.0 m/s in the aorta and 1.0 cm/s in a capillary. Assuming that all the blood that flows through the aorta also flows through the capillaries, how many capillaries does the circulatory system have?

$$
\begin{aligned}
A_\mathrm{a} v_\mathrm{a} &= n A_\mathrm{c} v_\mathrm{c} \\
n &= \frac{A_\mathrm{a} v_\mathrm{a}}{A_\mathrm{c} v_\mathrm{c}} \\
&= \frac{\frac{\pi D_\mathrm{a}^2}{4} v_\mathrm{a}}{\frac{\pi D_\mathrm{c}^2}{4} v_\mathrm{c}} \\
&= \left(\frac{D_\mathrm{a}}{D_\mathrm{c}}\right)^2 \frac{v_\mathrm{a}}{v_\mathrm{c}} \\
&= \left(\frac{0.0050 \text{ m}}{1.0 \times 10^{-5} \text{ m}}\right)^2 \left(\frac{1.0 \ \frac{\text{m}}{\text{s}}}{0.01 \ \frac{\text{m}}{\text{s}}}\right) \\
&= 2.5 \times 10^7
\end{aligned}
$$

You Try It!

To water the yard, you use a hose with a diameter of 3.2 cm. Water flows from the hose with a speed of 1.3 m/s. If you partially block the end of the hose so the effective diameter is now 0.55 cm, with what speed does water spray from the hose?

Bernoulli's Equation

Another important equation describing fluid flow is Bernoulli's Equation. It is basically an expression of the conservation of energy and is useful for bringing pressure into the picture of fluid flow. It is derived in your textbook, but will be presented here without justification.

$$P_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2$$

If we consider fluid flow through a horizontal pipe where $h_1 = h_2$, then

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

$$P_1 = P_2 + \frac{1}{2}\rho\left(v_2^2 - v_1^2\right)$$

$$P_1 = P_2 + \frac{1}{2}\rho\left(v_2^2 - v_2^2\frac{A_2^2}{A_1^2}\right)$$

$$P_1 = P_2 + \frac{\rho v_2^2}{2A_1^2}\left(A_1^2 - A_2^2\right)$$

Where we have used the equation of continuity to eliminate v_2. If A_1 is greater than A_2 than the second term on the right hand side is positive and P_1 is greater than P_2. Thus, when a fluid enters a narrower section of pipe (or artery), its speed increases but the pressure on the fluid decreases. These values can be measured with a Venturi meter.

Some examples of Bernoulli's Equation at work include:

- Perfume Atomizers
- Railway Sway
- Curveballs

Ex. 30-2 - Water is flowing in a horizontal pipe of variable cross section. Where the cross-sectional area is 1.0 x 10^{-2} m^2, the pressure is 5.0 x 10^5 Pa and the velocity is 0.5 m/s. In a constricted region where the area is 4.0 x 10^{-4} m^2, what are the pressure and velocity?

We can get the velocity from the equation of continuity.

$$A_1 v_1 = A_2 v_2$$

$$v_2 = \frac{A_1}{A_2}v_1 = \frac{1.0\times 10^{-2}\,m^2}{4.0\times 10^{-4}\,m^2}\left(0.5\frac{m}{s}\right) = 12.5\frac{m}{s}$$

We will need to apply Bernoulli's Equation to get the pressure.

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

$$P_2 = P_1 + \frac{1}{2}\rho\left(v_1^2 - v_2^2\right)$$

$$P_2 = \left(5.0\times 10^5\,Pa\right) + \frac{1}{2}\left(998\frac{kg}{m^3}\right)\left[\left(0.5\frac{m}{s}\right)^2 - \left(12.5\frac{m}{s}\right)^2\right] = 4.2\times 10^5\,Pa$$

Ex. 30-3 A large storage tank is filled with water. Neglecting viscosity, show that the speed of water emerging through a hole in the side a distance h below the surface is $v = \sqrt{2gh}$. This result is known as Toricelli's Theorem.

$$P_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2$$

$$\rho g h_1 + \frac{1}{2}\rho v_1^2 = \rho g h_2 + \frac{1}{2}\rho v_2^2$$

Since both locations are open to the atmosphere $P_1 = P_2$. At the top, v_1 is approximately zero. We can define h to be equal to the difference in height h_1-h_2.

$$v = \sqrt{2g\left(h_1 - h_2\right)} = \sqrt{2gh}$$