PHYSICS 151 – Notes for Online Lecture #29

Archimedes' Principle

A body, whether completely or partially submerged in a fluid, is buoyed upward by a force that is equal to the weight of the displaced fluid.

Archimedes' Principle can easily be demonstrated by hanging a mass from a spring scale and noting its weight. If you then submerge the weight in water the reading on the spring scale will decrease due to the buoyant force.

Ex. 29-1 Apply Archimedes' Principle to an iceberg. Why couldn't the Titanic lookouts spot the iceberg in time?

$$m_{ice}g - F_B = 0$$

$$\rho_{ice}V_{ice-total}\,g - \rho_{seawater}V_{ice-submerged}\,g = 0$$

$$\frac{V_{ice-submerged}}{V_{ice-total}} = \frac{\rho_{ice}}{\rho_{seawater}} = \frac{917\,\frac{kg}{m^3}}{1025\,\frac{kg}{m^3}} = 0.89$$

Thus, it is very difficult to spot icebergs since 89% of the iceberg is below water.

You Try It!

A cube of aluminum 0.1 m on a side is submerged in water. What does it weigh submerged?

Ex. 29-2 A king's crown is said to be solid gold but may be made of lead and covered with gold. When it is weighed in air, the scale reads 0.475 kg. When it is submerged in water, the scale reads 0.437 kg. (a) Is it solid gold? (b) If not, what percentage by mass is gold? We can determine the volume of the crown from its weight when submerged in water.

$$m_{crown}g - F_B = (0.437\,kg)\,g$$
$$(0.475\,kg)\,g - \rho_{water}V_{crown}g = (0.437\,kg)\,g$$
$$V_{crown} = \frac{0.475\,kg - 0.437\,kg}{\rho_{water}} = \frac{0.038\,kg}{1000\,\frac{kg}{m^3}} = 3.8\times10^{-5}\,m^3$$

Thus, the density of the crown is

$$\rho_{crown} = \frac{m}{V} = \frac{0.475\,kg}{3.8\times10^{-5}\,m^3} = 12500\,\frac{kg}{m^3}$$

Since the density of gold is 19300 kg/m^3, the crown is certainly not solid gold!

Let’s try and determine the percentage of the crown that is gold. We can write an equation that states that the total mass of the crown is equal to the sum of the gold and the lead.

$$\rho_{average}V_{tot} = \rho_{gold}V_{gold} + \rho_{lead}\left(V_{tot} - V_{gold}\right)$$
$$V_{gold}\left(\rho_{gold} - \rho_{lead}\right) = \left(\rho_{average} - \rho_{lead}\right)V_{tot}$$
$$\frac{V_{gold}}{V_{tot}} = \frac{\left(\rho_{average} - \rho_{lead}\right)}{\left(\rho_{gold} - \rho_{lead}\right)}$$
$$\frac{V_{gold}}{V_{tot}}\left(\frac{\rho_{gold}}{\rho_{average}}\right) = \left(\frac{\rho_{gold}}{\rho_{average}}\right)\frac{\left(\rho_{average} - \rho_{lead}\right)}{\left(\rho_{gold} - \rho_{lead}\right)}$$
$$\frac{m_{gold}}{m} = \left(\frac{\rho_{gold}}{\rho_{average}}\right)\left(\frac{\rho_{average} - \rho_{lead}}{\rho_{gold} - \rho_{lead}}\right)$$
$$\frac{m_{gold}}{m} = \left(\frac{19.3}{12.5}\right)\left(\frac{12.5 - 11.4}{19.3 - 11.4}\right) = 0.215$$

So only 21.5% of the crown is gold by weight.

Ex. 29-3 A block of wood floats on water. A layer of oil is now poured on top of the water to a depth that more than covers the block as shown in the figure below.

(a) Is the volume of wood submerged in water greater than, less than, or the same as before?

(b) If 90% of the wood is submerged in water before the oil is added, find the fraction submerged when oil with a density of 875 kg/m^3 covers the block.

(a) The block is submerged in less water than before because the oil provides additional buoyant force.

(b) Before oil is added:

$$F_b = \rho_w V_{submerged} g = \rho_w (0.9\,V_{total}) g = 0.9 \rho_w V_{total} g$$

Note that this is the weight of the wood!

After oil is added:

Let f_w = fraction of block in water
f_{oil} = fraction of block in oil
Substitute.

$$F_{b,w} + F_{b,oil} = F_b$$

$$\rho_w V_{in\ water} g + \rho_{oil} V_{in\ oil} g = 0.9\,\rho_w V_{total} g$$

$$\rho_w f_w V_{total} + \rho_{oil} f_{oil} V_{total} = 0.9 \rho_w V_{total}$$

$$\rho_w f_w + \rho_{oil} (1 - f_w) = 0.9 \rho_w$$

$$f_w (\rho_w - \rho_{oil}) = 0.9 \rho_w - \rho_{oil}$$

$$f_w = \frac{0.9 \rho_w - \rho_{oil}}{\rho_w - \rho_{oil}} = \frac{0.9\left(1000\ \frac{kg}{m^3}\right) - 875\ \frac{kg}{m^3}}{1000\ \frac{kg}{m^3} - 875\ \frac{kg}{m^3}} = 0.2$$