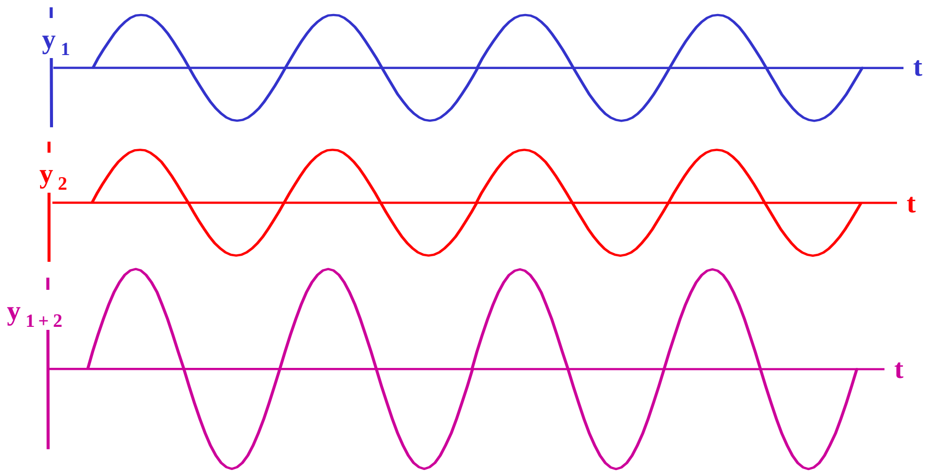
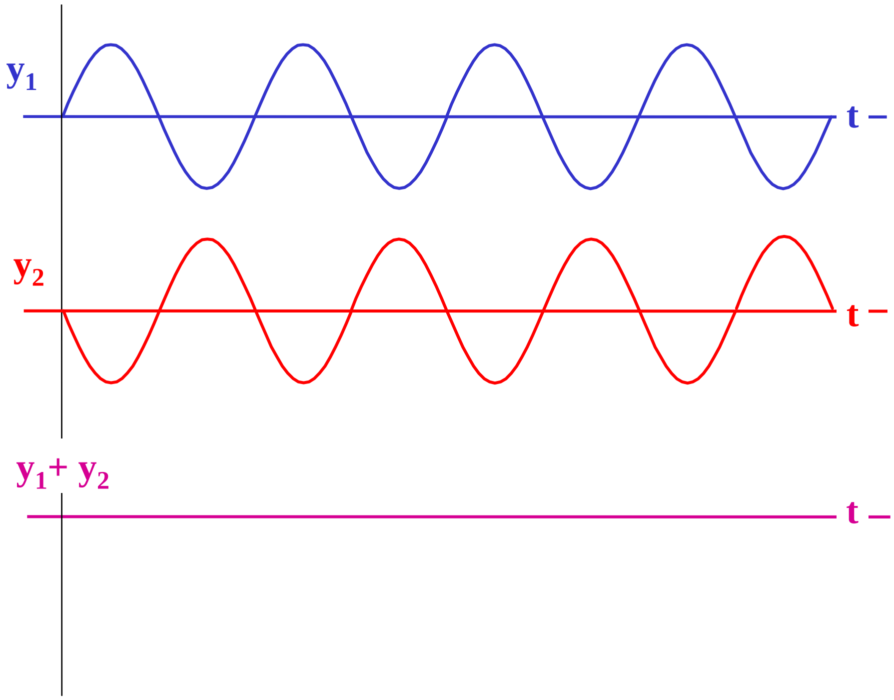
PHYSICS 151 – Notes for Online Lecture #27

Reflection of Waves

Waves obey something we call the **principle of linear superposition**. That is, if two waves are in the same region of space at the same time, they will interact with each other. Linear superposition allows us to describe how they interact fairly easily. If we were to plot the two waves as a function of time, they might look like the top two waves in the picture to the right. Linear superposition means that we just add the value of each wave and plot that as the sum. So the result of this combination of waves is another sine wave, but with an amplitude that is the sum of the amplitudes of the two starting waves.



Let's repeat this, but shift one of the waves by 180°



This time, the maximum of the first wave is at the minimum of the second and vice-versa, so when you add them up, you get zero.

When the waves interact so that the sum is larger than the original waves, we call that **constructive interference**. When they interact so that the sum is smaller, we call that **destructive interference**. You can have everything in between – partially destructive and partially constructive interference.

One of the reasons we care about how the waves interact with each other is because there are a number of places where waves travel into an object – like an organ pipe – and travel out again. Use the rope as an example. If I shake the rope, a pulse traveling down the rope will reach the fixed end and will reflect back inverted. If I keep shaking the rope, I set up a wave train such that, when one pulse reaches the end of the line and turns around, it will interfere with one of the pulses still heading toward the wall. There will be some points along the rope where the waves interact constructively and some points where they interact destructively. The result is that there are some points on the rope that are always standing still. We call these **nodes**. There are other points at which the wave has maximum values, which we call **anti-nodes**. The waves that result from this are called standing waves.

If I move my hand faster up and down, you see that I can change the number of nodes and antinodes. The length of the rope limits the configurations I can set up. The **fundamental** is the configuration in which there are no nodes (except the two at the end). When you pluck a guitar string, for example, you are exciting the fundamental. If you change the length of the string by holding it at one of the frets, you change the wavelength and thus the frequency heard.

Some nomenclature:

Any frequency that is an integral multiple of the fundamental is called a harmonic.

The first harmonic is the frequency, which we'll denote as f_1.

The second harmonic has a frequency exactly twice the fundamental, so $f_2 = 2f_1$

The first harmonic is the situation in which there is one node. Two nodes denote the second harmonic, etc.

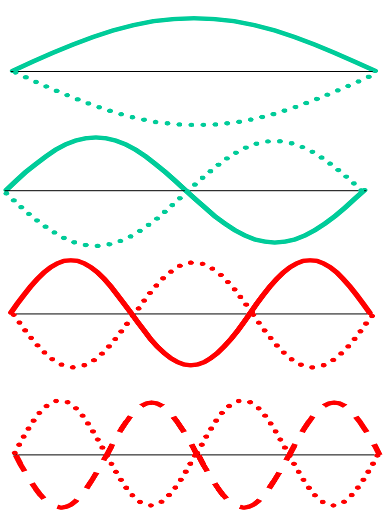
The other piece of nomenclature is the idea of an overtone. Overtones are the harmonics above the fundamental frequency. The first overtone for a wave on a string is the second harmonic. The second overtone for a wave on a string is the third harmonic

You'll notice that we don't have many options here. There are either one, two, three, etc. nodes on our string. This limits the number of patterns we can have. Let's investigate how many patterns are possible and the conditions under which they are produced. The chart at left shows that there is a pattern. The n^{th} harmonic is related to the length as

$$L = \frac{n\lambda_n}{2}$$

where n = 1 for the first harmonic, 2 for the second harmonic, etc.

The wavelengths for each harmonic are given by:



$\lambda_1/2 = L$	**First Harmonic (fundamental)**
$\lambda_2 = L$	**Second Harmonic (first overtone)**
$3\lambda_3/2 = L$	**Third Harmonic (second overtone)**
$2\lambda_4 = L$	**Fourth Harmonic (third overtone)**

$$L = \frac{n\lambda_n}{2}$$
$$\frac{2L}{n} = \lambda_n$$

The n^{th} harmonic will always have n loops in the wave pattern.

Note that the frequency and the wavelength of each wave on the string is different, but that the all the waves have the same velocity.

$$v = f_1\lambda_1 = f_2\lambda_2 = f_3\lambda_3$$

and so on.

We can related the harmonics to the fundamental as follows:

$$f_n = \frac{v}{\lambda_n}; \text{substitute in } \lambda_n = \frac{2L}{n}$$
$$f_n = \frac{v}{\frac{2L}{n}}$$
$$f_n = n\left(\frac{v}{2L}\right) = nf_1$$

EXAMPLE 27-1: A guitar string has a fundamental frequency of 440 Hz and a length of 0.50 m.

a) Draw the picture of the first five overtones and find their frequencies.

b) What are the wavelengths of the waves on the string?

c) What is the velocity of waves on the string?

d) What is the velocity of the sound waves produced by the string?

Solution a: The first three overtones are given by the picture above. The fourth overtone (which is the same as the fifth harmonic) will have four nodes/five loops. The fifth overtone (which is the same as the sixth harmonic) will have five nodes and six loops

A harmonic is an integral multiple of the fundamental. We will always have that $f_n = nf_1$, so

$$f_2 = 2f_1 = 2(440\,\text{Hz}) = 880\,\text{Hz}$$
$$f_3 = 3f_1 = 3(440\,\text{Hz}) = 1320\,\text{Hz}$$
$$f_4 = 4f_1 = 4(440\,\text{Hz}) = 1760\,\text{Hz}$$
$$f_5 = 5f_1 = 5(440\,\text{Hz}) = 2200\,\text{Hz}$$
$$f_6 = 6f_1 = 6(440\,\text{Hz}) = 2640\,\text{Hz}$$

Notice that the difference between any two harmonics that differ by one will always be equal to the fundamental frequency.

$$f_4 - f_3 = 4f_1 - 3f_1 = f_1$$

$$f_3 - f_2 = 3f_1 - 2f_1 = f_1$$

Solution b: The wavelength of the waves can be found from

$$\lambda_n = \frac{2L}{n}$$

$$\lambda_1 = \frac{2L}{1} = 2(0.50\,\text{m}) = 1.0\,\text{m}$$

$$\lambda_2 = \frac{2L}{2} = 0.50\,\text{m}$$

$$\lambda_3 = \frac{2L}{3} = \frac{2}{3}0.50\,\text{m} = 0.33\,\text{m}$$

$$\lambda_4 = \frac{2L}{4} = \frac{1}{2}0.50\,\text{m} = 0.25\,\text{m}$$

$$\lambda_5 = \frac{2L}{5} = \frac{2}{5}0.50\,\text{m} = 0.20\,\text{m}$$

Solution c: The velocity of waves on the string is given by

$$v = f_1\lambda_1 = (440\,\text{Hz})(1.00\,\text{m}) = 440\,\tfrac{\text{m}}{\text{s}}$$

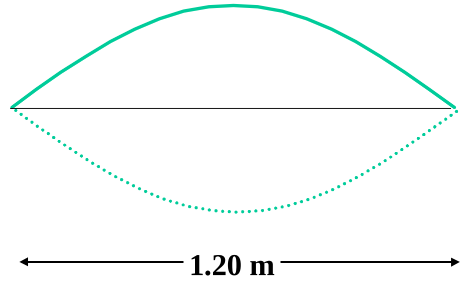
Note that you get the same thing if you multiply any f_n and λ_n!

Solution d: The velocity of the sound waves produced will be 340 m/s, which is the general speed of sound at 15ºC. Don't confuse the two velocities!

EXAMPLE 27-2 A nylon string is stretched between supports 1.20 m apart.

a) what is the wavelength of waves on this string?

b) If the speed of transverse waves on a string is 850 m/s, what is the frequency of the first harmonic and the first two overtones?



a) To determine the wavelength, draw the fundamental.

The fundamental is one half of a wavelength. The wavelength is therefore twice the length of the string, or 2.40 m.

b) The frequency of the first harmonic is given by

$$f_1 = \frac{v}{\lambda_1}$$

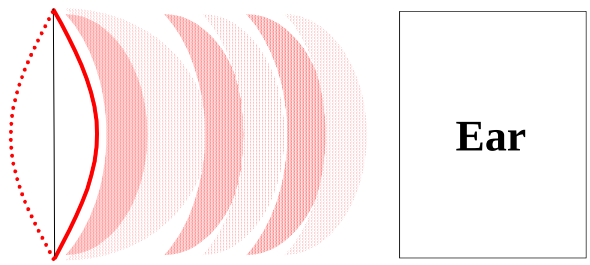
$$f_1 = \frac{850\,\tfrac{\text{m}}{\text{s}}}{2.40\,\text{m}} = 350\,\text{Hz}$$

The frequency of the first two overtones are given by $f_2 = 2f_1 = 700$ Hz and $f_3 = 3f_1 = 1050$ Hz

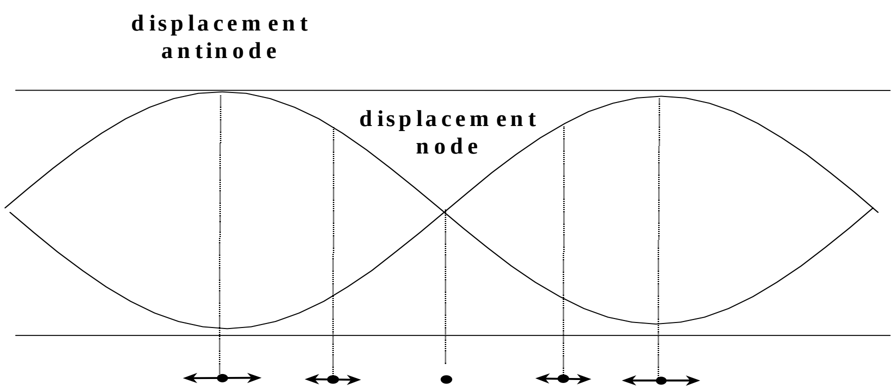
Waves in Tubes

String instruments produce sound by causing vibrations in the string. These vibrations excite the air around the string, causing the air around the strings to be alternately compressed and rarefied creating sound waves. Note that the velocity of waves on a string is not the same thing as the velocity of sound waves.

Making Sound with Strings



In wind instruments, the pressure variations are controlled by using a column. Consider a tube of length L that is open at both ends. When you blow into the tube, you create a longitudinal wave. The sound wave is thus created directly. In a string instrument, you create a transverse wave on the string, which then excites the air surrounding the string and creates the sound wave (which is longitudinal). The sound wave is created directly by wind instruments.

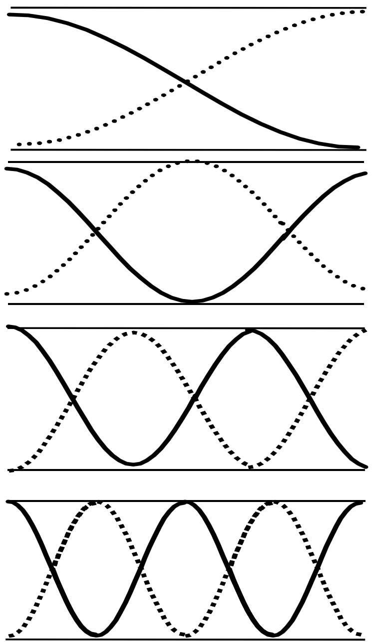


Longitudinal waves are variations in the density of the air in a given part of the tube. If you set up longitudinal standing waves, we find an analogous situation to the transverse standing waves seen on a string. If we could take a picture of the movement of the air molecules in each part of the standing wave, we would find the following: at some points along the standing wave, there is no motion of the molecules at that position. This is called displacement node, exactly like the node along a string when the string doesn’t move. Similarly, there are points along the tube where the molecules oscillate at their maximum amplitudes. These are displacement antinodes. We can plot the amplitude of the motion of the molecules to illustrate this. Note that the diagrams for the production of sound by wind instruments are different, because we're plotting displacement waves and not the actual shape of the air.

Waves in a pipe open at both ends.

We're going to be working in the limit of the tube length being much greater than the diameter of the tube. This allows us to ignore effects at the ends of the tube that would complicate our description.

At the open end of a column of air, the air molecules can move freely, so there will be a displacement antinode at the open end of a pipe. We can use the same approach to determine the modes of a tube of length L open at both ends are as we did in finding the waves on a string - draw the possibilities.



For the first harmonic (fundamental), we have half of a wavelength in the tube

$$L = \frac{\lambda}{2}$$

For the second mode, we have a full wavelength in the tube

$$L = \lambda = \frac{2\lambda}{2}$$

For the third mode,

$$L = \frac{3\lambda}{2}$$

so in general, we can extrapolate this to:

$$L = n\frac{\lambda_n}{2}$$

$$f_n = n\frac{v}{2L}$$

These formulas are good for waves in a **tube open on both ends.**

Examples of instruments with pipes open at both ends:

- flute
- trumpet
- organ pipes

You change the length of the tube by pressing keys. In a flute, closing a key elongates the tube. In a trumpet or French horn, pressing keys adds additional lengths of tubing to the pipe.

EXAMPLE 27-3: a) Calculate the fundamental frequency and the first three overtones of a hollow pipe open at both ends having length 30.0 cm. b) Calculate the wavelength of each wave.

We have

$$f_n = n\frac{v}{2L}$$

so

$$f_1 = \frac{v}{2L}$$

$$f_1 = \frac{340\,\frac{m}{s}}{2(0.30\,m)}$$

$$f_1 = 570\,Hz$$

We have the same relationship between the frequencies: $f_n = nf_1$. So $f_2 = 2f_1 = 1140$ Hz and $f_3 = 3f_1 = 1710$ Hz.

b) The wavelength is given by

$$v = f_n \lambda_n$$

$$\lambda_n = \frac{v}{f_n}$$

$$\lambda_1 = \frac{340\,\frac{m}{s}}{570\,\text{Hz}} = 0.60\,\text{m}$$

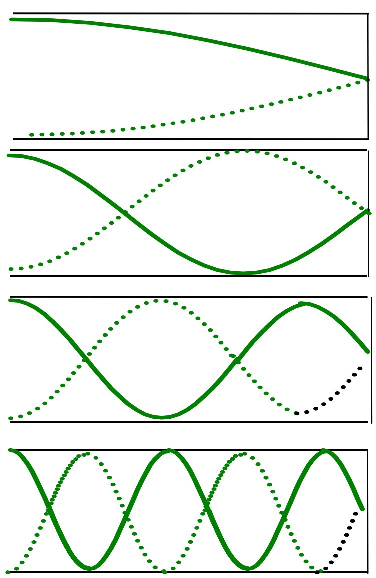
$$\lambda_2 = \frac{340\,\frac{m}{s}}{1140\,\text{Hz}} = 0.30\,\text{m}$$

$$\lambda_3 = \frac{340\,\frac{m}{s}}{1570\,\text{Hz}} = 0.22\,\text{m}$$

Standing Waves in a Pipe Open on One End

We can also have pipes that are closed on one end and open on the other. (Closed on two ends wouldn't make any sense.) This is a slightly different case, because at the closed end, we're required to have a displacement **node**, which will change the wave patterns allowed. Although the frequencies of waves in a pipe open at two ends are the same as those of a string with the same length, the case of waves in a pipe open at only one end will be quite different. We can draw the allowed patterns as shown.

Waves in Pipes Open at One End



In general,

$$L = n\frac{\lambda_n}{4}$$

$$f_n = n\frac{v}{4L}$$

but n can only be odd! Therefore, we talk about this case having only odd harmonics. There are only λ_1, λ_3, λ_5…We call λ_3 the first overtone, λ_5 the second overtone, etc.

Examples of instruments with pipes closed at one end include organ pipes

EXAMPLE 36-4: a) Calculate the fundamental frequency and the first three overtones of a hollow pipe open at one end having length 30.0 cm. b) Calculate the wavelength of each wave.

We have

$$f_n = n\frac{v}{4L}, \text{ but we are restricted to n odd}$$

so

$$f_1 = \frac{v}{4L} = \frac{340\,\frac{m}{s}}{4(0.30\,m)} = 283\,Hz$$

$$f_3 = 3\frac{v}{4L} = 3\frac{340\,\frac{m}{s}}{4(0.30\,m)} = 850\,Hz$$

$$f_5 = 5\frac{v}{4L} = 5\frac{340\,\frac{m}{s}}{4(0.30\,m)} = 1420\,Hz$$

$$f_7 = 7\frac{v}{4L} = 7\frac{340\,\frac{m}{s}}{4(0.30\,m)} = 1980\,Hz$$

c) The wavelength is given by

$$v = f_n \lambda_n$$

$$\lambda_n = \frac{v}{f_n}$$

$$\lambda_1 = \frac{340\,\frac{m}{s}}{283\,Hz} = 1.20\,m$$

$$\lambda_3 = \frac{340\,\frac{m}{s}}{850\,Hz} = 0.40\,m$$

$$\lambda_5 = \frac{340\,\frac{m}{s}}{1420\,Hz} = 0.24\,m$$

$$\lambda_5 = \frac{340\,\frac{m}{s}}{1980\,Hz} = 0.18\,m$$