

PHYSICS 151 – Notes for Online Lecture #26

Acoustics - Hearing Sound

When you hear something, there are two primary characteristics you notice: how loud the sound is, and the pitch of the sound.

Loudness is related to the intensity of the sound

Pitch is related to the frequency of the sound

Because people are sort of complex, we can't say that these two things are totally unrelated to each other; for example, if the same frequency is sounded, but at different intensities, you may perceive them to be different pitches. So there is some subjective component pitch and loudness.

People can hear from about 20 Hz to 20,000 Hz, with the upper range decreasing with age. We call frequencies above 20,000 ultrasonic - meaning only that they are sounds above the range of our hearing. You may have heard this in terms of ultrasonic jewelry cleaners, or ultrasonic toothbrushes. When you think about how sound waves work. Dogs can hear much higher frequencies – up to about 50,000 Hz, and bats, which emit ultrasonic waves to locate food and object they'd like to avoid running into – can hear up to 100,000 Hz.

Loudness

Loudness is related to the intensity of the wave. Intensity is the amount of energy that passes through a given area in a given time.

$$I = \frac{E}{At}$$

Thus intensity has units of W/m^2 . The energy emitted per time by a source spreads out over a larger area– so that we experience a diminishing intensity as we move away from the source.

$$I = \frac{P}{4\pi r^2}$$

The intensity of a wave is proportional to the square of its amplitude. If we call the amplitude of the sound wave A , the intensity I is given by:

$$I = 2\pi^2 v \rho f^2 A^2$$
$$I = CA^2$$

The important thing here to realize is that if you have two waves and one is twice the amplitude of the other, the intensities will differ by a factor of 4.

The human ear can hear a very broad range of intensities – from 10^{-12} W/m^2 to 1 W/m^2 (although this would be painful.)

Experimentally, we find, though, that to produce a sound that is twice as loud as another sound, the louder sound has to have an amplitude about 10 x higher – this indicates that what we hear as loudness is definitely not the same thing as the intensity. For this reason, we use a quantity

called an **intensity level**, which is defined as β and is more in line with what we hear as loudness. β is defined as:

$$\beta(\text{in dB}) = 10 \log\left(\frac{I}{I_o}\right)$$

where I_o is a reference intensity, taken to be $1.0 \times 10^{-12} \text{ W/m}^2$, which is theoretically the lowest sound we can hear. So the lowest value for β occurs when $I = I_o$;

$$\beta(\text{in dB}) = 10 \log\left(\frac{I_o}{I_o}\right)$$

$$\beta(\text{in dB}) = 10 \log(1) = 0 \text{ dB}$$

If we find the sound level for an intensity of $1 \times 10^{-11} \text{ W/m}^2$ – which is 10 times louder than I_o ;

$$\beta(\text{in dB}) = 10 \log\left(\frac{I}{I_o}\right)$$

$$\beta(\text{in dB}) = 10 \log\left(\frac{1 \times 10^{-11} \frac{\text{W}}{\text{m}^2}}{1 \times 10^{-12} \frac{\text{W}}{\text{m}^2}}\right) = 10 \text{ dB}$$

So if the intensity increases 10 times, the intensity level changes from 0 to 10. What if we look at an intensity of 10^{-10} W/m^2 ?

$$\beta(\text{in dB}) = 10 \log\left(\frac{I}{I_o}\right)$$

$$\beta(\text{in dB}) = 10 \log\left(\frac{1 \times 10^{-10} \frac{\text{W}}{\text{m}^2}}{1 \times 10^{-12} \frac{\text{W}}{\text{m}^2}}\right) = 20 \text{ dB}$$

So another increase of 10 times the previous level gets us another 10 dB rise in intensity level. This continues to be true: every time the intensity goes up by a factor of 10, the sound level rises by 10 dB.

The threshold of pain is 1 W/m^2 ,

$$\beta(\text{in dB}) = 10 \log\left(\frac{I}{I_o}\right)$$

$$\beta(\text{in dB}) = 10 \log\left(\frac{1 \frac{\text{W}}{\text{m}^2}}{1 \times 10^{-12} \frac{\text{W}}{\text{m}^2}}\right) = 120 \text{ dB}$$

So 120 dB actually means 10^{12} more intensity than the threshold of hearing.

Source	dB	Intensity (W/m ²)
Jet plane at 30 m	140 dB	100
Threshold of pain	120 dB	1
Siren at 30 m	100 dB	1 x 10 ⁻²
Ordinary conversation	65 dB	3 x 10 ⁻⁶
Whisper	20 dB	1 x 10 ⁻¹⁰
Threshold of hearing	10 dB	1 x 10 ⁻¹²

If we want to compare two sound levels, say β_2 and β_1 , we can write

$$\beta_1 \text{ (in dB)} = 10 \log \left(\frac{I_1}{I_o} \right)$$

$$\beta_2 \text{ (in dB)} = 10 \log \left(\frac{I_2}{I_o} \right)$$

$$\beta_2 - \beta_1 = 10 \log \left(\frac{I_2}{I_o} \right) - 10 \log \left(\frac{I_1}{I_o} \right)$$

Recall that $\log (a) - \log (b) = \log (a/b)$

$$\beta_2 - \beta_1 = 10 \log \left(\frac{I_2}{I_o} \right) - 10 \log \left(\frac{I_1}{I_o} \right)$$

$$\beta_2 - \beta_1 = 10 \left[\log \left(\frac{I_2}{I_o} \right) - \log \left(\frac{I_1}{I_o} \right) \right]$$

$$\beta_2 - \beta_1 = 10 \log \left(\frac{I_2}{I_1} \right)$$

EXAMPLE 26-1: The specifications of stereo equipment often cite a signal-to-noise ratio. A signal-to-noise ratio of 60 dB means that the signal has an intensity level 60 dB higher than that of the noise. What does this figure mean about the intensities of the two sounds?

$$\beta_2 - \beta_1 = 10 \log \left(\frac{I_2}{I_1} \right)$$

$$60 \text{dB} = 10 \log \left(\frac{I_2}{I_1} \right)$$

now use: if $x = \log(y)$ then $y = 10^x$

$$6 = \log \left(\frac{I_2}{I_1} \right)$$

$$10^6 = \frac{I_2}{I_1}$$

This means that I_2 is a million times I_1

EXAMPLE 26-2: A 50 dB sound waves strikes an eardrum whose area is $5.0 \times 10^{-5} \text{ m}^2$.

- (a) How much energy is absorbed by the eardrum per second?
- (b) At this rate, how long would it take your eardrum to receive a total energy of 1J?

$$\beta = 10 \log \frac{I}{I_0}$$

$$\beta = 10 [\log I - \log I_0]$$

$$\frac{\beta}{10} + \log I_0 = \log I$$

$$\frac{50}{10} + (-12) = \log I$$

$$10^{-7} \frac{W}{m^2} = I$$

$$\text{Power} = \text{Intensity} * \text{Area}$$

$$= \left(10^{-7} \frac{W}{m^2} \right) (5 \times 10^{-5} m^2) = 5 \times 10^{-12} W$$

$$\text{time} = \frac{\text{energy}}{\text{power}} = \frac{1J}{5 \times 10^{-12} W} = 2 \times 10^{11} \text{ s} = 6,350 \text{ years}$$

EXAMPLE 26-3: At a recent rock concert, a dB meter registered 130 dB when placed 2.5 m in front of the loudspeaker on stage.

(a) What was the power output of the speaker?

$$\begin{aligned}
 130 &= 10 \log \frac{I}{I_0} \\
 13 &= \log I - \log I_0 \\
 13 &= \log I + 12 \\
 1 &= \log I \\
 10 \frac{W}{m^2} &= I \\
 I &= \frac{P}{4\pi r^2} \\
 P &= I(4\pi r^2) = \left(10 \frac{W}{m^2}\right) 4\pi (2.5m)^2 = 785W
 \end{aligned}$$

(b) How far away would the intensity level be a reasonable 90 dB?

$$\begin{aligned}
 90 &= 10 \log \frac{I}{I_0} \\
 9 &= \log I - \log I_0 = \log I + 12 \\
 -3 &= \log I \\
 10^{-3} \frac{W}{m^2} &= I \\
 I &= \frac{P}{4\pi r^2} \\
 r &= \sqrt{\frac{P}{4\pi I}} = \sqrt{\frac{785W}{4\pi \left(10^{-3} \frac{W}{m^2}\right)}} = 250m
 \end{aligned}$$