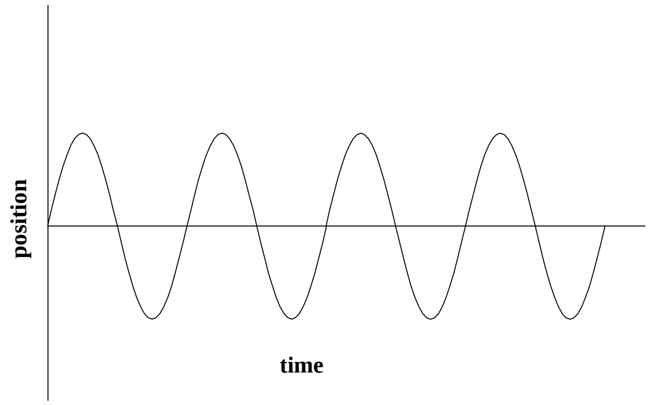
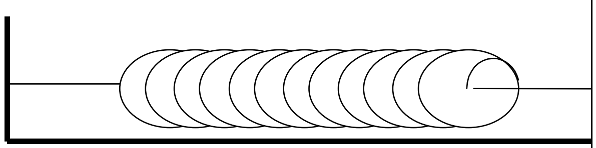
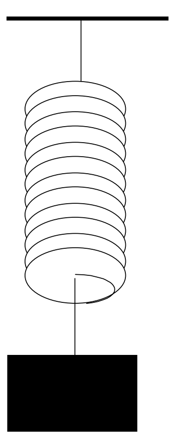
PHYSICS 151 – Notes for Online Lecture #25

WAVES

Transverse Waves

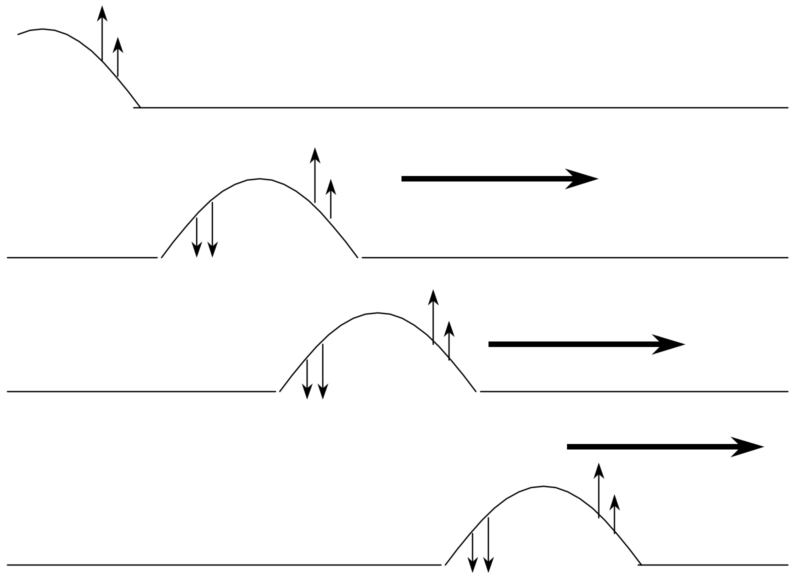
We've shown that we can describe the motion of a SHO using sines and cosines. So if I wanted to plot the position of a mass on a spring of a pendulum bob as a function of time, the result would be a sine or cosine wave. We're now going to make a switch from thinking about waves as a function of time to thinking about waves as a function of space. That is, the pictures that we draw now will actually look like the wave.



	$x(t) = x_o \cos\left(\frac{2\pi t}{T}\right)$
	$y(t) = y_o \cos\left(\frac{2\pi t}{T}\right)$

Now, we're going to plot waves as if we were taking a snapshot of the wave itself.

If I give the end of the rope a quick movement up and down, I create a pulse that travels along the rope. Note that the individual parts of the rope aren't moving from my hand to the end of the wall – it's the pulse that is moving. If we took a series of snapshots of the pulse as it moved from one end of the rope to the other, it would look like this:



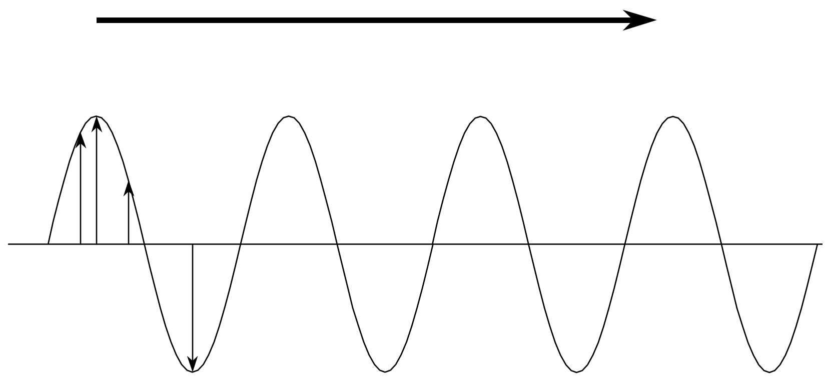
At each point, some parts of the rope are moving up and some are moving down. The connection between parts of the rope causes the next part of the rope to move upward as the preceding part moves down.

In this type of wave, the pieces of the rope move up and down, while the pulse moves across. This type of pulse is called a **transverse** pulse.

Reflection of Waves

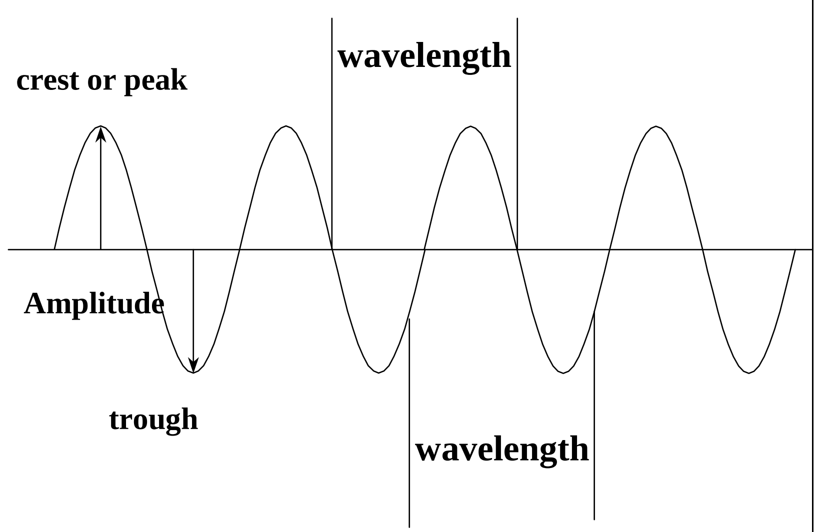
If you are on campus on a football day, you might notice that the sound appears to come from places other than the stadium. Outside the physics building, it sounds like the noise is coming from the Lied Center. This is because waves can **reflect** from surfaces. We can investigate reflection by looking at a wave on a string and what happens to the pulse when it hits a surface, like the wall.

The pulse reflects from the wall, as we might except, but it reflects upside down. When the pulse reaches the end of the support, it is pushing upward, just as it would if it were hitting another part of the string and trying to cause that part of the string to move up. Because the wall is fixed, the support exerts an equal and opposite force down, causing the reflection. Note that some of the energy of the wave is absorbed by the wall or transformed into thermal energy, which is why the amplitude of the wave decreases after it hits the wall.



If I shake the rope with a constant up-and-down motion, I form a continuous wave. Continuous waves are always due to some type of vibration or oscillation. If the oscillation is sinusoidal in nature (i.e. SHM), the resulting wave will also be sinusoidal. When the individual parts of the rope are moving up and down, but the pulse is propagating perpendicularly to the motion of the rope, the wave is called a **transverse** wave.

We have special names for different parts of waves.



A **crest** is the maximum displacement of the wave from zero. A **trough** is the maximum displacement in the negative direction.

The **amplitude** is the maximum height of a crest or a trough. The **wavelength** (λ) is the repeat unit. The wavelength is measured from any one point on the wave to the next time the identical point occurs.

The **frequency** is the number of waves that pass a given point per second.

A wave travels a distance of one wavelength during one period. The **velocity** of the wave is thus

$$v = \lambda f$$

Note that this is the velocity of the wave and *not* the velocity of the parts of the string moving up and down. You can think of this as if I start by raising the rope and measuring how fast that initial pulse travels.

EXAMPLE 25-1: A transverse wave has a velocity of 25 m/s. Successive wave crests are 4.0 m apart. What is the frequency of the wave?

Using v =fλ,

$$f = \frac{v}{\lambda} = \frac{25\,\frac{m}{s}}{4.0\,m} = 6.3\,\frac{1}{s}$$

EXAMPLE 25-2: Radio waves travel at the speed of light. What is the wavelength of KFRX radio (92.7 MHz)? The speed of light is 3.00 x 10^8 m/s.

Using v =fλ,

$$\lambda = \frac{v}{f} = \frac{3x10^8\,\frac{m}{s}}{92.7\,x10^6\,Hz} = 3.2\,m$$

For the simple harmonic motion of a mass on a spring or a pendulum, we could express the displacement using a sine or a cosine. We can do the same thing here, but in this case, the displacement of the individual parts of the rope will depend on both how far the part of the rope is from the source of the pulse and the time. We will call up and down the y-direction and across the x-direction. If we freeze the wave at t=0 and just describe the displacement of the parts of the rope as a function of their position, we would have

$$y(x, t = 0) = y_o \sin\left(\frac{2\pi x}{\lambda}\right)$$

If, on the other hand, we look at a single position (x =0) and write the position of that piece of rope as a function of time, we get:

$$y(x = 0, t) = -y_o \sin\left(\frac{2\pi t}{T}\right)$$

which is the same thing we had for the SHM cases before. Mathematically, we can combine these two expressions to get:

$$y(x, t) = -y_o \sin\left(2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)\right)$$

This equation is valid when y = 0 at x = 0 when t = 0.

Speed of Waves

For transverse waves on a string, we can relate the wave velocity to the tension in the string, T, and the linear density - that is, the mass per unit length - or the string.

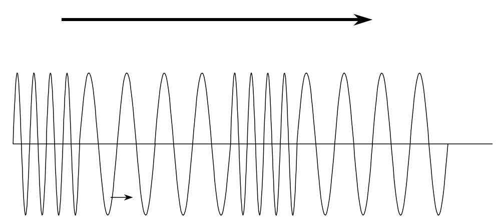
$$v = \sqrt{\frac{T}{\mu}} \text{ where } \mu = \frac{m}{l}$$

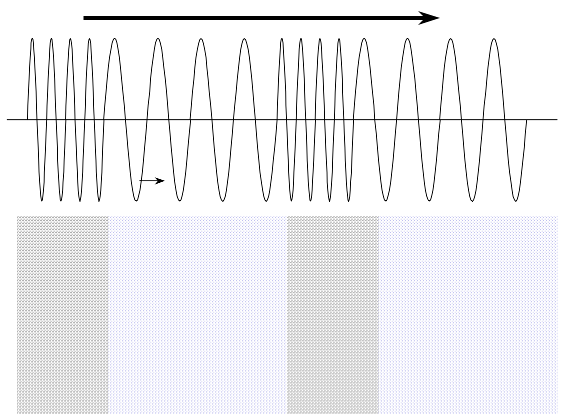
When the tension gets larger, the velocity will increase. For a heavy rope (m/l large), the velocity will be smaller.

Longitudinal Waves

When I make a wave on a rope, each part of the rope moves perpendicular to the direction that the wave moves in. This is a transverse wave.

If the individual parts of the media in which the wave travels move in the *same* direction as the direction in which the wave moves, we call it a **longitudinal** wave. A longitudinal wave looks like a slinky.

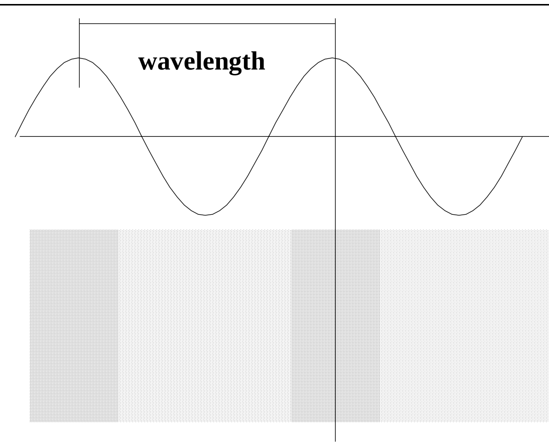




If I quickly push the slinky forward, I initiate a pulse that travels down the length of the slinky. This is different that the case of the rope. In the case of the rope, my initial displacement is in one direction and the motion of the pulse is in a different direction. In the case of the slinky, my initial displacement is in the same direction as the direction the pulse travels.

Sound waves are a good example of a longitudinal type of wave. When we speak, we make air molecules move back and forth, causing regions of compression (dense areas) and regions of rarefaction (not-so-dense regions). Again - individual air molecules don't move along with the wave – they are alternately in dense and not-so-dense regions.

Longitudinal and transverse waves follow the same rules in terms of identifying the parts of the wave. We can define an amplitude, wavelength, etc. For a longitudinal wave, a crest is the highest density that the air molecules achieve. A trough is the lowest density. The wavelength is the distance from a place in the wave with one density to the next place where that same density is found.



Just like transverse waves, longitudinal waves obey $v = \lambda f$

The Speed of Longitudinal Waves

The speed of the wave can be calculated using

$$v = \sqrt{\frac{\text{Elastic Force Factor}}{\text{Inertial Factor}}}$$

For a longitudinal wave traveling down a long solid

$$v = \sqrt{\frac{\text{E}}{\rho}}$$

where E is the elastic modulus and ρ is the density of the material.

For a liquid or gas,

where B is the bulk modulus of the liquid or gas.

$$v = \sqrt{\frac{\text{B}}{\rho}}$$

EXAMPLE 25-3: A wave travels along a rope of mass 3.0 kg and length 20.0 m at a velocity of 14 m/s. What is the tension in the rope?

This is a wave on a string, which is a transverse wave, so the formula is

$$\lambda = \frac{v}{f} = \frac{345\,\frac{m}{s}}{20\,Hz} = 17\,m$$

$$v = \sqrt{\frac{T}{\mu}}$$

$$v^2\mu = T$$

$$\left(14\,\tfrac{m}{s}\right)^2\left(\tfrac{3.0\,kg}{20.0\,m}\right) = T$$

$$29N = T$$

The Speed of Sound Waves

Note that, for sound waves, the speed depends on the temperature, pressure and humidity. At sea-level pressure and 0° C, the speed of sound in air is 331.5 m/s. For other temperatures where T is in Celsius:

$$v(T) = (331.5 + 0.6T)\,\tfrac{m}{s}$$

EXAMPLE 25-4: The range of sound that humans can hear is from 20 to 20,000 Hz. If the speed of sound at 20°C is 345 m/s, what are the wavelengths of the sounds we can hear?

v = fλ, so

$$\lambda = \frac{\text{v}}{\text{f}} = \frac{345\,\frac{\text{m}}{\text{s}}}{20\,\text{Hz}} = 17\,\text{m}$$

$$\lambda = \frac{\text{v}}{\text{f}} = \frac{345\,\frac{\text{m}}{\text{s}}}{2.0\,\text{x}\,10^4\,\text{Hz}} = 17\,\text{cm}$$