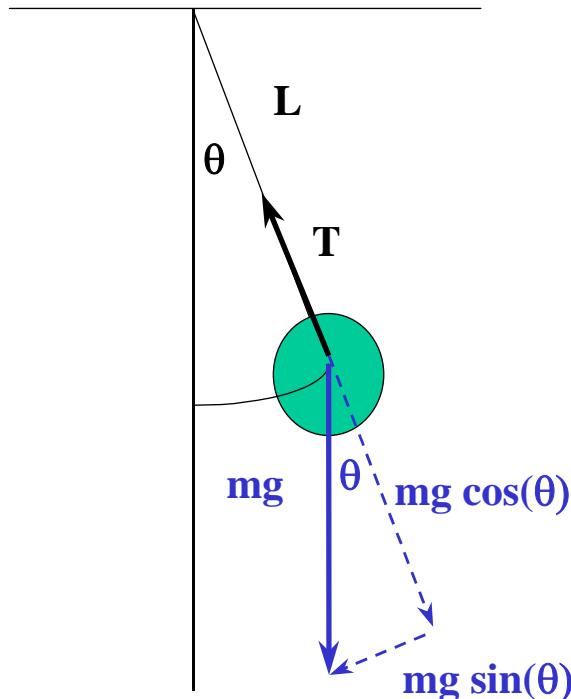


PHYSICS 151 – Notes for Online Lecture #24

Pendula

A simple pendulum consists of an object suspended from a string. The motion of the pendulum swinging back and forth can also be described by simple harmonic motion (under certain conditions).



Let's first draw a free-body diagram for the bob on the pendulum. We can decompose the weight, mg , into a component **along** the direction of the string and a component **perpendicular** to the string.

The component of the weight in the direction of the string will be:

$$mg \cos(\theta)$$

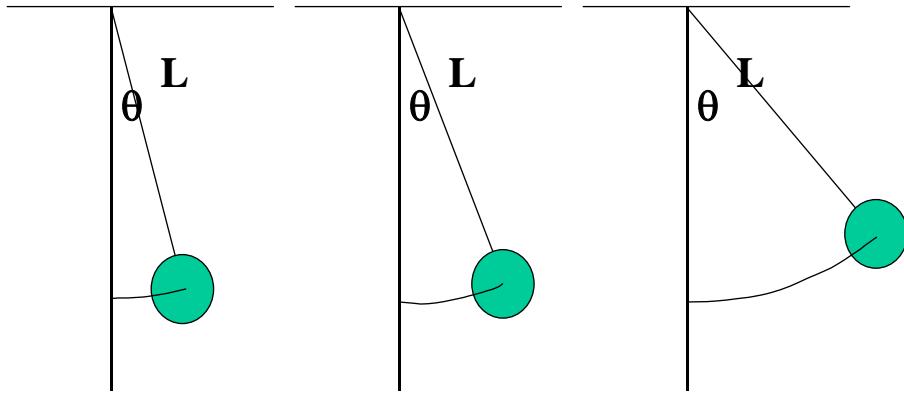
The component of the weight perpendicular to the string is

$$-mg \sin(\theta)$$

Write $\vec{\Sigma F} = 0$. We have to write one equation for each direction.

Along the string	Perpendicular to the string
<p>In the direction along the string, there is no acceleration, so taking toward the pivot as positive,</p> $\vec{\Sigma F}_{\text{along the string}} = 0$ $T - mg \cos(\theta) = 0$ $T = mg \cos(\theta)$	<p>There is, however, acceleration in the direction perpendicular to the string due to the unbalanced force:</p> $\vec{\Sigma F}_{\text{perpendicular to the string}} = ma$ $F = -mg \sin(\theta)$

One of the things we learned in the last lecture, is that object that undergo simple harmonic motion following something that looks like Hooke's law, with the force proportional to the displacement. This equation doesn't look like Hooke's law.



If θ is very small, we can make an approximation. If we let s be the displacement of the pendulum bob, s will be an arc.

If θ is small, we can write that $\sin \theta = \frac{s}{L}$

We can then write

$$F = -mg \sin(\theta)$$

$$F = -mg \frac{s}{L}$$

$$F = -\frac{mg}{L}s$$

so that the 'spring constant' for this problem is

$$F = -\frac{mg}{L}s$$

$$F = -\left(\frac{mg}{L}\right)s$$

$$F = -"k"s$$

where the effective spring constant is given by

$$k = \frac{mg}{L}$$

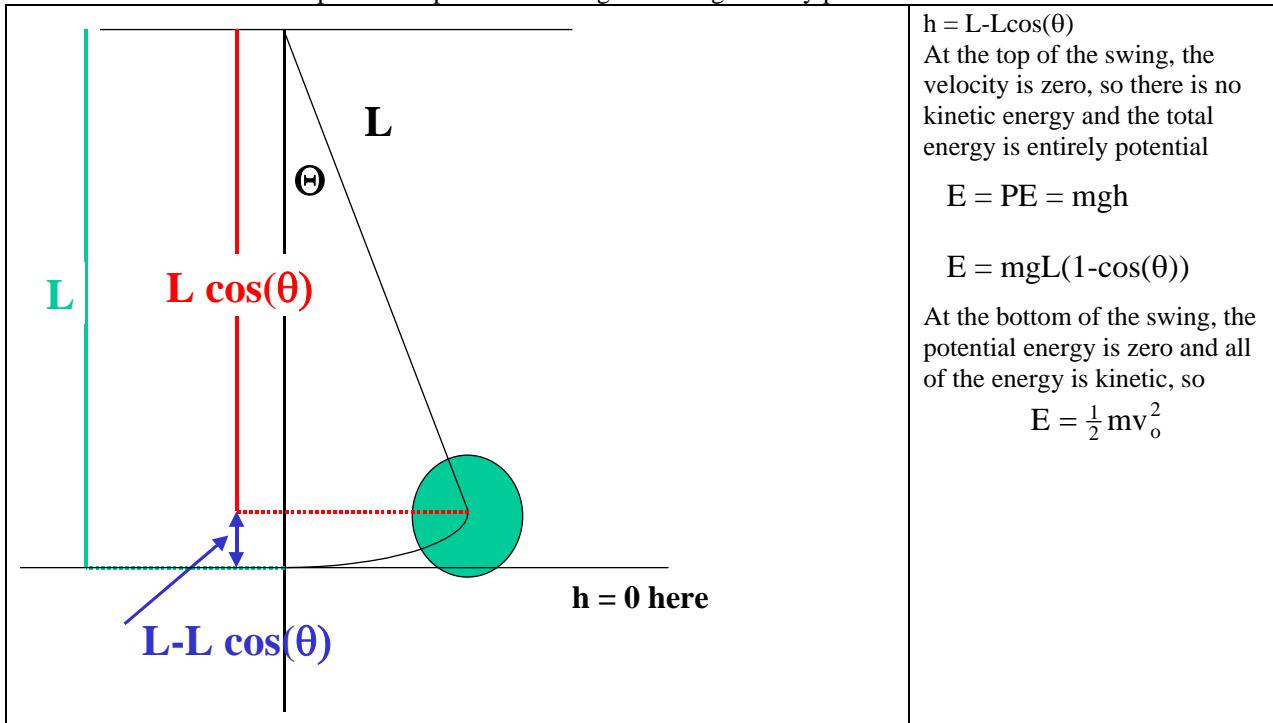
We can use the same relationships we derived for the mass on a spring to find the similar quantities for the pendulum

Spring	Pendulum
$f = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$	$f = \frac{1}{2\pi} \sqrt{\frac{g}{L}}$
$T = \frac{1}{f} = 2\pi \sqrt{\frac{m}{k}}$	$T = \frac{1}{f} = 2\pi \sqrt{\frac{L}{g}}$

Note that, for the pendulum, all of these results are independent of the mass of the pendulum.

Energy Concerns for the Pendulum

Take $h = 0$ to be the lowest part of the pendulum swing. The height at any point is then



$$h = L - L \cos(\theta)$$

At the top of the swing, the velocity is zero, so there is no kinetic energy and the total energy is entirely potential

$$E = PE = mgh$$

$$E = mgL(1 - \cos(\theta))$$

At the bottom of the swing, the potential energy is zero and all of the energy is kinetic, so

$$E = \frac{1}{2}mv_o^2$$

where v_o is the maximum velocity.

From conservation of energy, the total energy at these two points must be equal.

$$E = \frac{1}{2}mv_o^2 = mgL(1 - \cos(\theta))$$

$$v_o^2 = 2gL(1 - \cos(\theta))$$

$$v_o = \sqrt{2gL(1 - \cos(\theta))}$$

Ex. 24-1: A pendulum of length 0.50 m makes 21.0 oscillations in 30.0 seconds. What is the value of g ?

$$\text{frequency} = \frac{\text{number of oscillations}}{\text{time}} = \frac{21.0}{30.0} = 0.7 \text{ Hz}$$

$$f = \frac{1}{2\pi} \sqrt{\frac{g}{L}}$$

$$(2\pi)^2 f^2 = \frac{g}{L}$$

$$(2\pi)^2 f^2 = \frac{g}{L}$$

$$(2\pi)^2 f^2 L = g$$

$$(2\pi)^2 (0.7 \frac{1}{s})^2 (0.5 \text{ m}) = g$$

$$9.67 \frac{\text{m}}{\text{s}^2} = g$$

Ex. 24-2: A pendulum with a frequency of 6 oscillations per second is taken to the moon, where gravity is $1/6^{\text{th}}$ the gravity of earth. What will the pendulum's frequency be on the moon?

Solution: The pendulum length is constant.

$$f = \frac{1}{2\pi} \sqrt{\frac{g}{L}}$$

$$f(2\pi) = \sqrt{\frac{g}{L}}$$

$$(2\pi f)^2 = \frac{g}{L}$$

$$L = \frac{g}{(2\pi f)^2}$$

$$L = \frac{1}{4\pi^2} \frac{g}{f^2}$$

Earth	Moon
$L = \frac{1}{4\pi^2} \frac{g_{\text{Earth}}}{(f_{\text{Earth}})^2}$	$L = \frac{1}{4\pi^2} \frac{g_{\text{Moon}}}{(f_{\text{Moon}})^2}$

These two must be equal, so

$$L = \frac{1}{4\pi^2} \frac{g_{\text{Earth}}}{(f_{\text{Earth}})^2} = \frac{1}{4\pi^2} \frac{g_{\text{Moon}}}{(f_{\text{Moon}})^2}$$

$$\frac{g_{\text{Earth}}}{(f_{\text{Earth}})^2} = \frac{g_{\text{Moon}}}{(f_{\text{Moon}})^2}$$

$$g_{\text{Earth}} (f_{\text{Moon}})^2 = (f_{\text{Earth}})^2 g_{\text{Moon}}$$

$$(f_{\text{Moon}})^2 = (f_{\text{Earth}})^2 \frac{g_{\text{Moon}}}{g_{\text{Earth}}}$$

$$f_{\text{Moon}} = f_{\text{Earth}} \sqrt{\frac{g_{\text{Moon}}}{g_{\text{Earth}}}}$$

$$f_{\text{Moon}} = f_{\text{Earth}} \sqrt{\frac{\frac{1}{6} g_{\text{Earth}}}{g_{\text{Earth}}}}$$

$$f_{\text{Moon}} = f_{\text{Earth}} \sqrt{\frac{1}{6}}$$

$$f_{\text{Moon}} = 6 \text{ Hz} \sqrt{\frac{1}{6}} = 2.5 \text{ Hz}$$