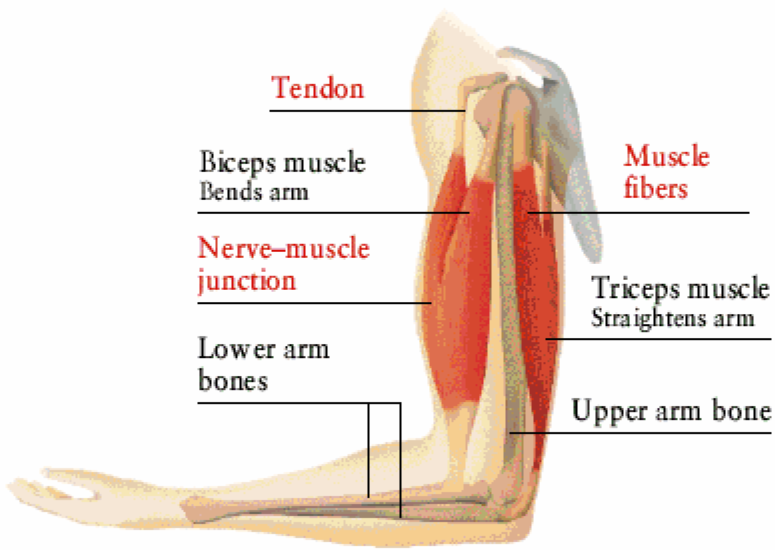
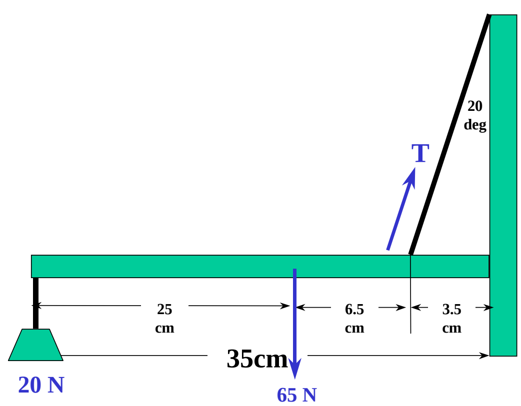
PHYSICS 151 – Notes for Online Lecture #22

Ex. 22-1: To bend your arm, you have to move your biceps muscle. The biceps is attached to the shoulder and to the elbow by tendons. When a person holds a weight, the muscle will have some tension in it, and there will be a force at the elbow.

If the lower arm weighs 65.0 N and the person holds a 20.0 N weight (roughly 5 pounds), find the tension in the muscle and the forces on the elbow.



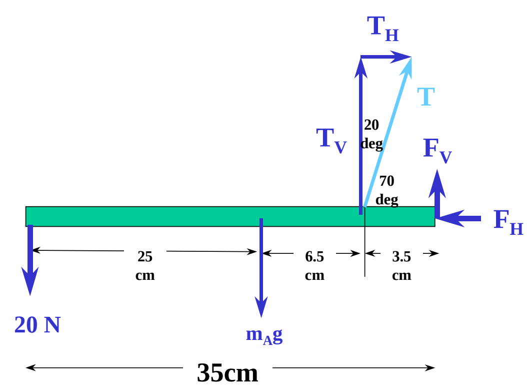


We can model the arm as shown. Note that the CM (center of mass) is NOT at the geometric center of the object: this is because the mass is not uniformly distributed along the arm. The location of the CM would have to be given to you for you to work the problem.

First, draw the free-body diagram for the lower arm

Sign Convention: up is positive and torques that cause a ccw rotation are positive

Note that: $T_H = T\sin(20)$ $T_V = T\cos(20)$



Write the translational equilibrium equations

x-equation	y-equation
$\Sigma F_x = 0$ $T_H - F_H = 0$ $T\sin(20) - F_H = 0$ $T\sin(20) = F_H$	$\Sigma F_y = 0$ $T_V + F_V - 20.0\,N - 65.0\,N =$ $T\cos(20) + F_V = 85.0\,N$

Unfortunately, there are still three unknowns here: T, F_V and F_H, so we must write the torque equation. We can eliminate two of the forces - F_V and F_H - by taking the axis of rotation to be at the hinge.

$$\Sigma \tau = 0$$
$$(20.0\,N)(0.35\,m) + (65.0\,N)(0.10\,m) - T_y(0.035m) = 0$$
$$(20.0\,N)(0.35\,m) + (65.0\,N)(0.10\,m) = T_y(0.035m)$$
$$\frac{7.0\,Nm + 6.5\,Nm}{(0.035m)} = T_y$$
$$390\,N = T_y$$

Using $T_y = T\cos(20)$:

$$T = \frac{T_y}{\cos(20)}$$
$$T = \frac{390\,N}{\cos(20)}$$
$$T = 410\,N$$

Now using the x-equation from summing the forces:	$\Sigma F_x = 0$ $T_H - F_H = 0$ $T\sin(20) - F_H = 0$ $T\sin(20) = F_H$ $410\,N\sin(20) = F_H$ $140\,N = F_H$

So the horizontal force is 141 N. Use the y-equations from summing the forces to get the last piece of information	$\Sigma F_y = 0$ $T_V + F_V - 20N - 65N = 0$ $T\cos(20) + F_V = 85N$ $F_V = 85N - T\cos(20)$ $F_V = 85N - 411N\cos(20)$ $F_V = -301N$

Note that the value for F_V turned out to be negative. This means that I assumed it to act in the wrong direction. Instead of acting up, it must act down.

Using:

$$F = \sqrt{F_H^2 + F_V^2}$$

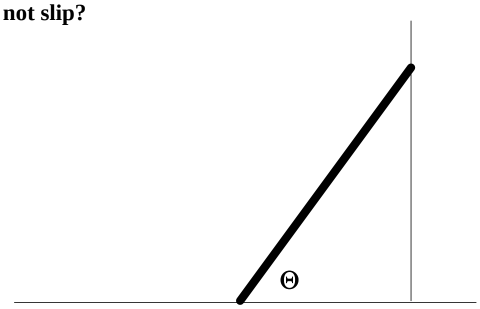
The total magnitude of the force on your elbow is $F = \sqrt{(141N)^2 + (-301N)^2}$.

$$F = 332\,N$$

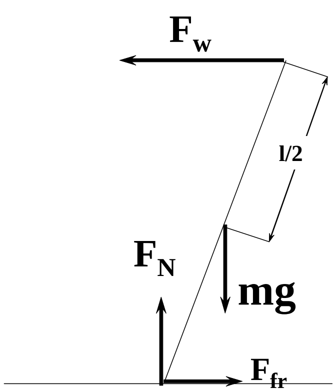
That's like a 74lb weight acting directly on your elbow

Ex 22-2: The Ladder Problem

A ladder is leaning up against a smooth wall. If the coefficient of friction between the ladder and the ground is 0.25, what is the maximum angle at which the ladder will not slip?



Usual coordinate system. Free body diagram for the ladder:



$$\Sigma F_x = 0$$
$$F_{fr} - F_W = 0$$
$$F_{fr} = F_W$$

$$\Sigma F_y = 0$$
$$F_N - mg = 0$$
$$F_N = mg$$
$$F_N = (20kg)\left(9.8\,\tfrac{m}{s^2}\right)$$
$$F_N = 196N$$

Note that the resultant of F is NOT along the ladder!

Take torques about the ground; we need to find the lever arms. We don't know any heights so let's call the length of the ladder l. When the ladder is at an angle θ with the ground, the height against the wall is sinθ=h/l or h = l sin (θ)

The distance from the wall, d is

cosθ = d/l or d = lcos(θ)

the lever arm for the weight of the ladder is going to be at half that distance (l/2) cos (θ), so the torque due to the weight is

τ_{Weight} = -(l/2)cos(θ)(mg)

the lever arm for the torque the wall is creating is the height

τ_{Wall} = +lsin(θ)F_W

$$\Sigma \tau = 0$$
$$-\left(\frac{l}{2}\right)\cos(\theta)(mg) + l\sin(\theta)(F_W) = 0$$

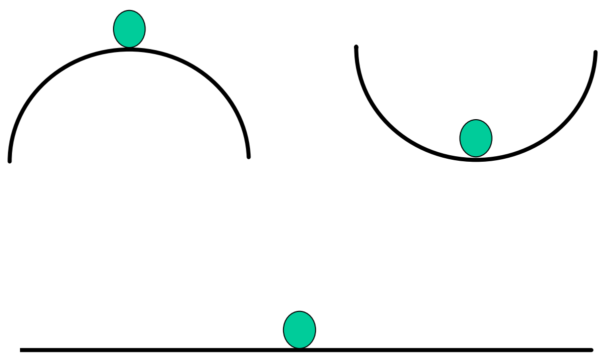
From above, we know that $F_W = F_{fr} = \mu mg$:

$$-\left(\frac{l}{2}\right)\cos(\theta)(mg) + l\sin(\theta)(F_W) = 0$$
$$-\left(\frac{l}{2}\right)\cos(\theta)(mg) + l\sin(\theta)(\mu mg) = 0$$
$$\left(\frac{l}{2}\right)\cos(\theta)(mg) = l\sin(\theta)(\mu mg)$$
$$\left(\frac{1}{2}\right)\cos(\theta) = \sin(\theta)(\mu)$$
$$\left(\frac{1}{2\mu}\right) = \frac{\sin(\theta)}{\cos(\theta)}$$
$$\left(\frac{1}{2\mu}\right) = \tan(\theta)$$
$$\left(\frac{1}{2\mu}\right) = \tan(\theta)$$
$$\left(\frac{1}{2(0.25)}\right) = \tan(\theta)$$
$$63.4° = \theta$$

We've been talking about equilibrium and taken it to mean the following:

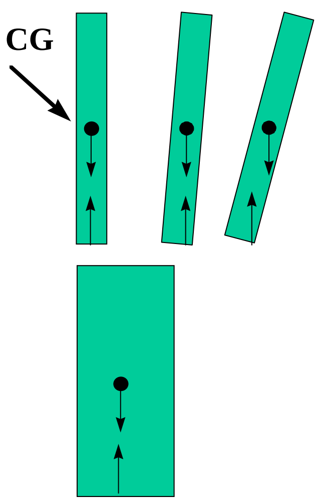
$\Sigma F_x = 0 \qquad \Sigma F_y = 0 \qquad \Sigma \tau = 0$

So any situation where these equations are satisfied is called equilibrium. But there is a difference as to the types of equilibrium. Consider three different situations: In each one, the system is in equilibrium. Now let's consider what would happen if I were to push the ball slightly. In the first situation, the ball would move away from it's equilibrium point and not come back to it. In the second, the ball would move back toward it's equilibrium and eventually settle back down there. In the third, if I pushed the ball a little, it would stay where I pushed it. The first case is an example of an *unstable* equilibrium, the second a case of *stable* equilibrium and the third a case of *neutral* equilibrium.



You may have noticed another simplification we've been making in dealing with these equilibrium cases. We've always taken the pivot point to be just that – a point. If I try to stand a meter stick up on its end, it doesn't stay up very well. That's an example of an unstable equilibrium – if I set it just right, it stays standing, but if I make just a little perturbation, it will fall down. Now suppose that I look at something with a broader base, like an eraser. Turns out that its equilibrium is a whole lot more stable. There is much more of the base on the table.

We can predict whether something is going to be stable by looking at the position of its center of gravity. When CG is over the base, the object will be stable. So let's compare the ruler and the eraser.

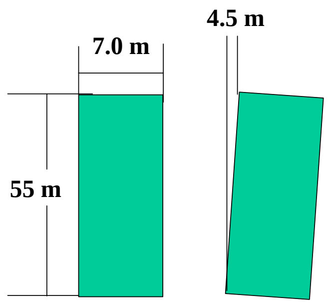


As soon as the ruler's CG is no longer over the base, the force of gravity causes an unbalanced torque and the ruler will fall. The eraser has a much broader base, and therefore can be tipped further before the cg extends over the base.

I mentioned last week that people have a center of gravity. If you stand and try to lean, you'll find that, at some point, you fall over. That's because your CG is outside of your base, which is your feet. When you bend over to pick something up, your CG shifts. There's now a torque on the top half of your body. To compensate, your hips move outward.

Ex. 28-3: The leaning tower of Pisa is 55 m tall and 7 m in diameter. Presently, the top is 4.5 m from where it would be if it were straight. How far can the tower move before it would become unstable?

The leaning tower of Pisa is 55 m tall and 7.0 m in diameter. Presently, the top is 4.5 m from where it would be if it were it straight. How far can the tower move before it would become unstable?



The center of gravity will be located at the middle of the tower – we're treating it as if it were uniform.

So the CG is 55/2 m up and 3.5m in drawing.

Where is it now? it's tilted already. The angle of tilt is:

$$\sin(\theta) = \frac{4.5\text{m}}{55\text{m}}$$

$$\theta = 4.7°$$

Call α the critical angle of tilt – when the CM is directly over the corner which we can consider to be a pivot point.

Then, the position of the CM is: $\sin\alpha = \dfrac{x}{27.5m}$ If x = 0 is the central point, we want to find out when the value of x is just equal to 3.5 m

$$\sin\theta = \frac{3.5\text{m}}{27.5\text{m}}$$

$$\sin\theta = 7.3°$$