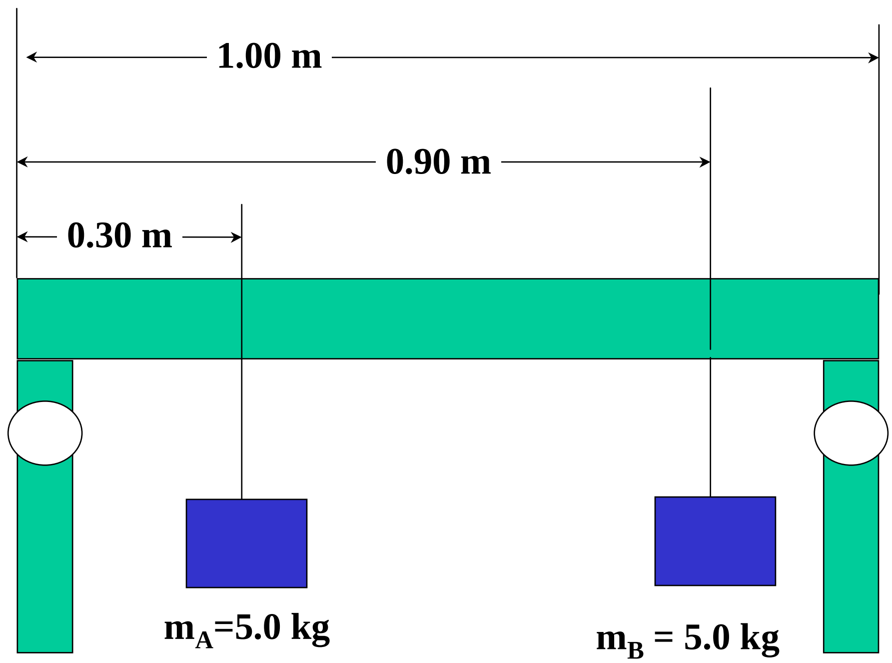
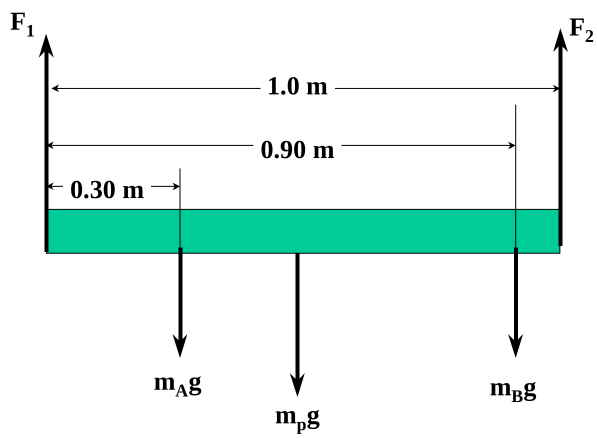
PHYSICS 151 – Notes for Online Lecture #21

More Advanced Torque Problems

Ex. 21-1: A plank of mass m_p = 1.0 kg is placed on two scales. Masses are placed as shown. M_A = 5.0 kg at a distance 0.30 m and M_B = 5.0 kg at a distance 0.90 m. What do the two scales read?



First, draw the free body diagram for the plank, then apply the conditions for translational equilibrium.



$$\Sigma F_y = 0$$

$$F_1 + F_2 - m_A g - m_B g - m_p g = 0$$

$$F_1 + F_2 = (m_A + m_B + m_p)g$$

$$F_1 + F_2 = (5.0\,\text{kg} + 5.0\,\text{kg} + 1.0\,\text{kg})\,g$$

Again, we're stuck here, so we need to do the torque equations. Let's start by taking the axis of rotation about the left scale. This eliminates F_1 from the equations, allowing us to solve for F_2. **Ccw rotations will be considered positive**.

$$\Sigma \tau_{F_1} = 0$$

$$-m_A g\,(0.30\,m) - m_p g\,(0.50\,m) - m_B g\,(0.90\,m) + F_2(1.0m) = 0$$

$$\left[m_A(0.30\,m) + m_p(0.50\,m) + m_B(0.90\,m)\right] g = F_2(1.0\,m)$$

$$\frac{\left[m_A(0.30\,m) + m_p(0.50\,m) + m_B(0.90\,m)\right]}{(1.00\,m)} = \frac{F_2}{g}$$

$$\frac{\left[(5.0\,kg)(0.30) + (1.0\,kg)(0.50) + (5.0\,kg)(0.90)\right]}{(1.0)} = \frac{F_2}{g}$$

$$6.5\,kg = \frac{F_2}{g} = m_2$$

Now, let’s do the same thing, but use F_2 as the pivot point.

$$\Sigma \tau_{F_2} = 0$$

$$+m_A g(0.7m) + m_p g(0.5m) + m_B g(0.1m) - F_1(1.0m) = 0$$

$$m_A g\,(0.70m) + m_p g\,(0.50\,m) + m_B g\,(0.10\,m) = F_1(1.0\,m)$$

$$\frac{m_A(0.70\,m) + m_p(0.50\,m) + m_B(0.10\,m)}{(1.0\,m)} = \frac{F_1}{g}$$

$$\frac{m_A(0.70) + m_p(0.50) + m_B(0.10)}{(1.0)} = \frac{F_1}{g}$$

$$\frac{(5.0\,kg)(0.70) + (1.0\,kg)(0.50) + (5.0\,kg)(0.10)}{(1.0)} = \frac{F_1}{g}$$

$$4.5\,kg = \frac{F_1}{g}$$

These results can be checked by putting the results into the force equation

$$\Sigma F_y = 0$$

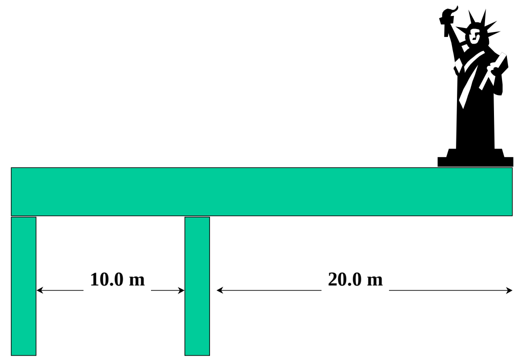
$$F_1 + F_2 - m_A g - m_B g - m_p g = 0$$

$$F_1 + F_2 = \left(m_A + m_B + m_p\right) g$$

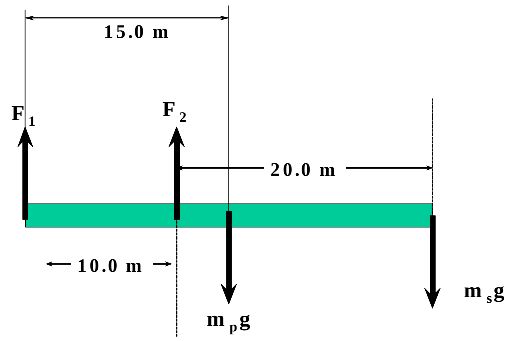
$$4.5\,kg\,(g) + 6.5\,kg\,(g) = \left(5.0\,kg + 5.0\,kg + 1.0\,kg\right) g$$

$$11\text{ kg} = 11\text{ kg}$$

Ex. 21-2: Cantilevers. A statue is to be placed at the end of the plank as shown. If the statue has a mass of 1.25 x 10^3 kg and the plank has mass = 2.00 x 10^3 kg, what forces must the two supports exert?



Draw a free-body diagram:



First, write that the sum of the forces is zero

$$\Sigma F_y = 0$$
$$F_1 + F_2 - m_p g - m_s g = 0$$
$$F_1 + F_2 = (m_s + m_p)g$$

Now, write the torque equations taking the torque about the leftmost pt. Taking torques that cause ccw rotations to be positive,

$$\Sigma \tau_{F_1} = 0$$
$$m_s g(30.0\,m) + m_p g\,(15.0\,m) - F_2(10.0\,m) = 0$$
$$\frac{\left[m_s(30.0\,m) + m_p(15.0\,m)\right]g}{(10.0\,m)} = F_2$$
$$\frac{\left[1.250\,x10^3 kg\,(30.0\,m) + 2.00\,x10^2\,kg(15.0\,m)\right]\left(9.8\,\tfrac{m}{s^2}\right)}{(10.0\,m)} = F_2$$
$$3.97x10^4\,N = F_2$$

To get F_1, we can either use the equation from force, or take torques about another axis. I'm going to use the force equation.

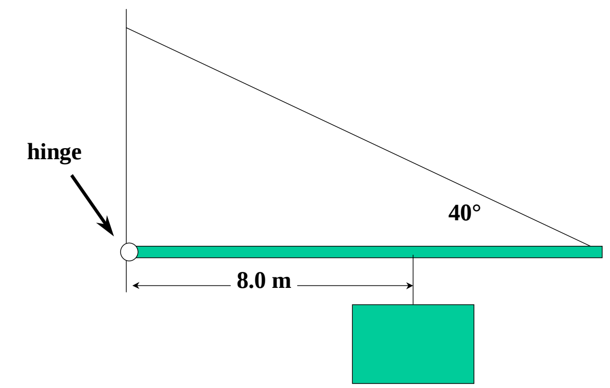
$$F_1 + F_2 = (m_s + m_p)g$$

$$F_1 = (m_s + m_p)g - F_2$$

$$F_1 = (1.25\text{x}10^3\text{ kg} + 2.00\text{x}10^2\text{ kg})(9.8\tfrac{\text{m}}{\text{s}^2}) - 3.97\text{x}10^4\text{ N}$$

$$F_1 = -2.55\text{x}10^4\text{ N}$$

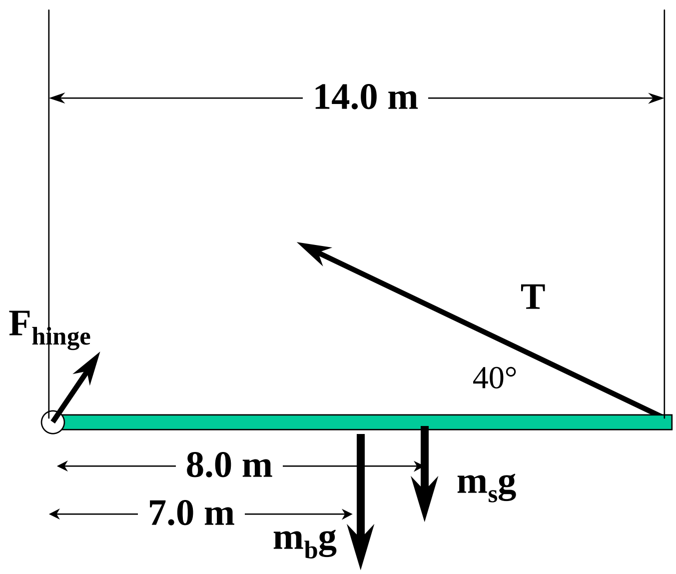
Note that the expression for F_1 came out negative, which shows us that we assumed the wrong direction for the F_1.

Ex. 21-3: A 50.0-kg sign is to be hung from a 20.0-kg beam of length 14.0 m as shown. a) What is the tension in the wire b) What are the horizontal and vertical components of the force exerted by the hinge?	

Choose sign conventions as usual. (torques that cause ccw rotations are positive, up is positive). Pick the beam as the object for the free body diagram.

We know that the tension will have both x and y components, so go ahead and break it down into components right away.

T_x = -T cos(40°) T_y = T sin(40°)

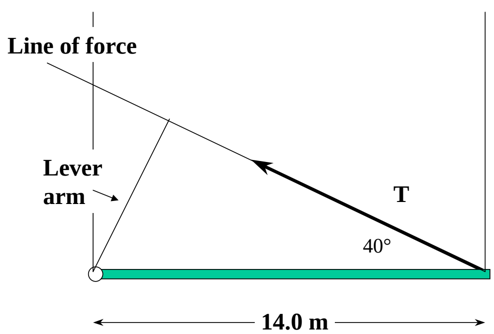
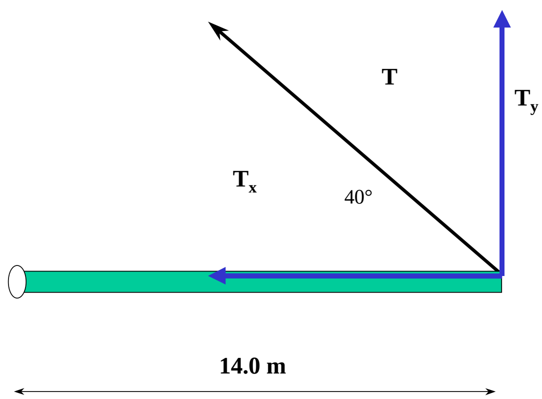


We know that the hinge has x and y components. (Actually, we might not know that, but unless we know that it definitely **doesn't** have both x and y components, we'd better assume that it has both.) Since we don't know the angle at which the hinge force acts, we'll call these components F_{Hx} and F_{Hy}. Write the conditions for translational equilibrium.

$\Sigma F_x = 0$	$\Sigma F_y = 0$
$T_x + F_{Hx} = 0$	$T_y + F_{Hy} - m_s g - m_b g = 0$
$-T\cos(40°) + F_{Hx} = 0$	$T\sin(40°) + F_{Hy} - m_s g - m_b g = 0$
$F_{Hx} = T\cos(40°)$	$T\sin(40°) + F_{Hy} - (m_s + m_b)g = 0$

Take torques about the end where the hinge is so that we can eliminate the hinge force from the equations.

We can calculate the torque due to the tension in one of two ways

Method 1: Find the Lever arm	Method 2: Use the components of the tension. Because the T_x component causes no torque (it passes through the axis of rotation.) only the y component can cause torque.
$r_\perp = r\sin(40)$	$\tau = rF_y$
$r_\perp = (14.0\,\text{m})\sin(40)$	$\tau = (14.0\,\text{m})\,T_y$
$\tau = r_\perp F$	$\tau = (14.0\,\text{m})\,T\sin(40)$
$\tau = (14.0\,\text{m})\sin(40)\,T$	
	

Regardless of which method you use, you get the same answer.

Now write the condition for rotational equilibrium. Watch the signs!

$$\Sigma\tau_{hinge} = 0$$
$$-m_b g(7.0\,m) - m_s g(8.0\,m) + T_y(14.0\,m) = 0$$
$$m_b g(7.0\,m) + m_s g(8.0\,m) - T\sin(40°)(14.0\,m) = 0$$
$$T\sin(40°)(14.0\,m) = \left(m_b(7.0m) + m_s(8.0\,m)\right)g$$
$$T = \frac{\left(m_b(7.0\,m) + m_s(8.0\,m)\right)g}{\sin(40°)(14.0\,m)}$$
$$T = \frac{\left((20.0\,kg)(7.0\,m) + (50.0\,kg)(8.0\,m)\right)\left(9.8\,\frac{m}{s^2}\right)}{\sin(40°)(14.0\,m)}$$
$$T = 590\,N$$

Once we have the magnitude of the tension, we can use this information to determine the forces of the hinge. Insert this result in $\Sigma F_x = 0$

$F_{Hx} = T\cos(40°)$ $F_{Hx} = (590\,N)\cos(40°)$ $F_{Hx} = 450\,N$	$T\sin(40°) + F_{Hy} - (m_s + m_b)g = 0$ $F_{Hy} = (m_s + m_b)g - T\sin(40°)$ $F_{Hy} = (50.0\,kg + 20.0\,kg)\left(9.8\,\frac{m}{s^2}\right) - (590\,N)\sin(40°$ $F_{Hy} = 310\,N$

Note that we could also take torques about the end where the Tension is applied. We have to get the same answers!

$$\Sigma\tau_{RightEnd} = 0$$
$$-m_b g\,(7.0\,m) - m_s g\,(6.0\,m) + F_{Hy}(14.0\,m) = 0$$
$$F_{Hy}(14.0\,m) = m_b g\,(7.0\,m) + m_s g\,(6.0\,m)$$
$$F_{Hy}(14.0\,m) = \left[m_b(7.0\,m) + m_s(6.0\,m)\right]g$$
$$F_{Hy} = \frac{\left[m_b(7.0\,m) + m_s(6.0\,m)\right]g}{(14.0\,m)}$$
$$F_{Hy} = \frac{\left[(20.0\,kg)(7.0\,m) + (50.0\,kg)(6.0\,m)\right]\left(9.8\,\frac{m}{s^2}\right)}{(14.0\,m)}$$
$$F_{Hy} = 310\,N$$