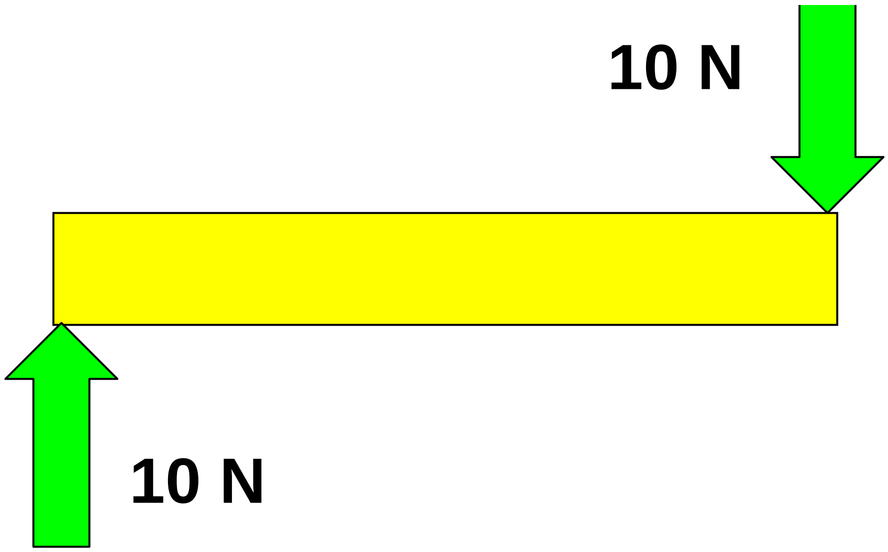
PHYSICS 151 – Notes for Online Lecture #20

Torque: The whole reason that we want to worry about centers of mass is that we are limited to looking at point masses unless we know how to deal with rotations. Let's revisit the meterstick. Say I apply a force of 10 N on one end and 10 N on the other end as shown



We can first attack this problem using what we learned when we were studying statics.

$$\Sigma\vec{F} = 0$$

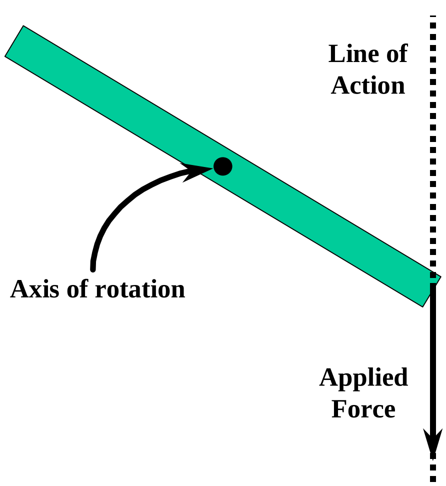
Since there are only forces in the y direction, we can add them up directly. Taking up as positive:

$$-10\text{ N} + 10\text{ N} = 0$$

So according to our old definition of equilibrium, this system is in equilibrium. However, we know that the meterstick is going to move, even though the net force is zero. This tells us that there is something missing from our definition of equilibrium.

TRANSLATIONAL EQUILIBRIUM means that the vector sum of the forces is zero.

$$\Sigma\vec{F} = 0$$



The reason our definition has failed us is that we now are allowing objects to have spatial extent and so they can rotate. There are two things that determine the characteristics of the rotation: the magnitude of the force and how far from the pivot the force acts.

Important Torque Definitions

AXIS OF ROTATION: The axis about which the object moves (or would move if the object is in equilibrium.)

LINE OF FORCE or **LINE OF ACTION**: A straight line running directly through the applied force.

LEVER ARM: The lever arm is the perpendicular distance from the axis of rotation to the line of action.

Finding the lever arm is usually one of the hardest parts of torque problems.

Start by finding ‘r’, which is the distance from the axis of rotation to the point where the force is applied. Note that r is always going to lie along the object that the force is acting upon.

We can then draw the lever arm by drawing a line that is perpendicular to the line of force and passes through the axis of rotation.

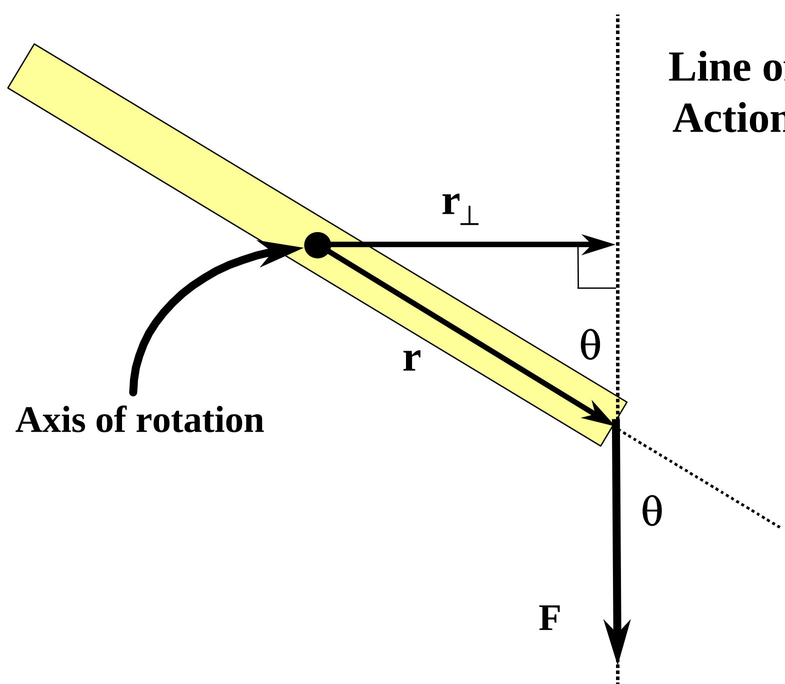
We can call the lever arm $r_\perp$. The angle between r and F is called θ. Note, then that

$$r_\perp = r\sin(\theta)$$

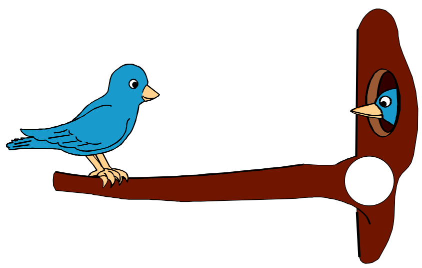
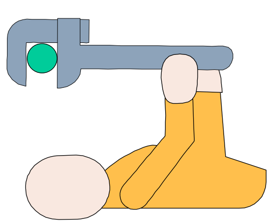
I said that the details of the rotation were determined by the magnitude of the force and the distance from the CM that the force acts. The product of these two quantities is called the **torque**. Torque describes rotation.

$$\tau = r_\perp F$$
$$\tau = rF\sin(\theta)$$

Note: The units of torque are N-m – even though we called that a Joule when we dealt with energy and work, we don't call it a Joule when we're using it to describe torque.

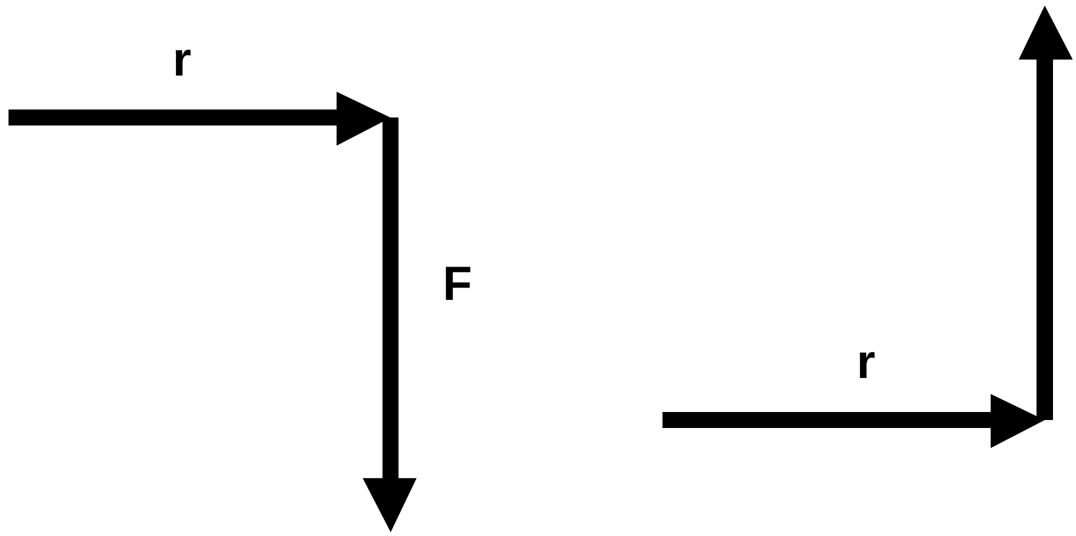


Exercise: Identify the force, show the line of force and the lever arm for each of the following situations in the box next to the drawing. The axis of rotation is denoted by a dot.

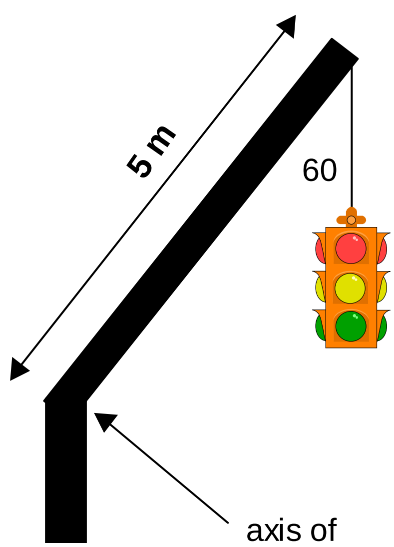
Direction: We use the right hand rule to determine the direction of the torque. If you curl your fingers from r to F, your thumb will point in the direction of the torque.

Counterclockwise = positive Clockwise =negative



In the left picture, the torque will point into the page. In the right-hand picture, the torque will point out of the page.

Ex 20-1: A civil engineer needs to hang a traffic light over an intersection from a pole, as shown. The light weighs 70 N.



a) Draw the lever arm.

b) What is the magnitude of the lever arm assuming the base of the support as the axis of rotation? (sin 60 = $\sqrt{3}/2$, cos 60 = 1/2)

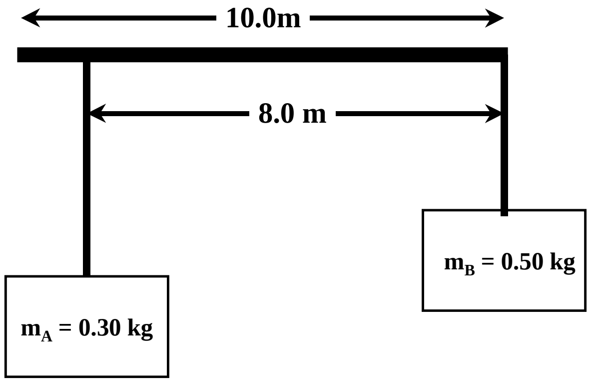
c) What is the direction of the torque?

d) What is the magnitude of the torque?

e) Describe different ways that the engineer could decrease the amount of torque on the support.

Ex. 20-2: An artist is designing a mobile, as shown. Where must the mobile be hung to keep the rod horizontal and what is the tension in the hanging string? Assume that the rod is massless.

An artist is designing a mobile as shown. Find where along the rod the mobile must be hung to keep the rod level. a) Assume the rod is massless and b) Assume the rod has a mass of 1.0 kg



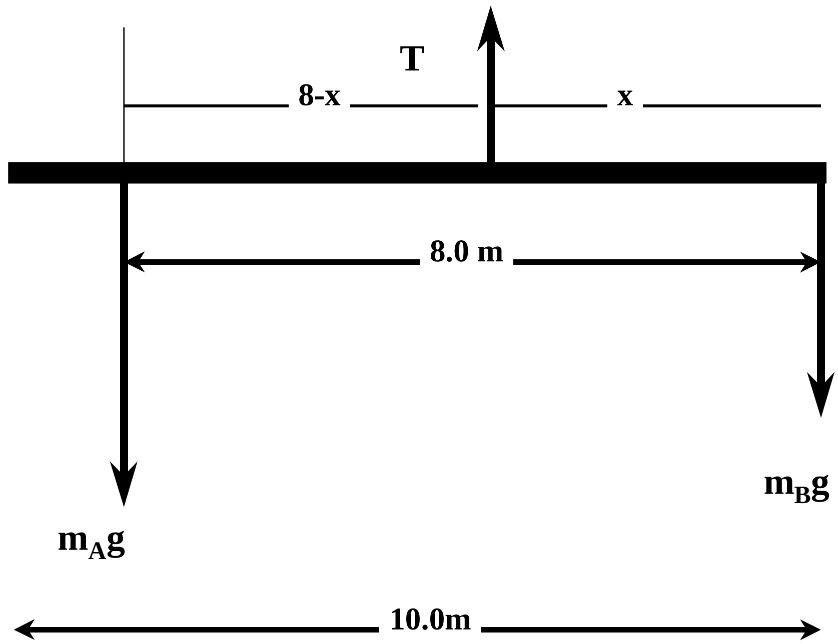
a) Assuming first that the rod has no mass. Let's assume that the point at which the mobile is to be hung is a distance x from the right end of the rod.

First, draw a free-body diagram to show all of the forces acting on the body.

We can then start by applying the condition for translational equilibrium - the math for translational equilibrium is often easier, so that's the first one we'll try applying.

$$\Sigma F_y = 0$$
$$T - m_A g - m_B g = 0$$
$$T = m_A g + m_B g$$
$$T = (m_A + m_B)g$$
$$T = (0.3\text{kg} + 0.5\text{kg})\left(9.8\tfrac{\text{m}}{\text{s}^2}\right)$$
$$T = 7.84\text{N} = 7.9\text{N}$$

This tells us the tension, but not where the support must be placed. For that, we have to apply the torque equation. We can take torques about any point. Let's take them about the string from which the mobile balances. This has the advantage that, if we couldn't figure out the tension in the string, it won't enter the problem.

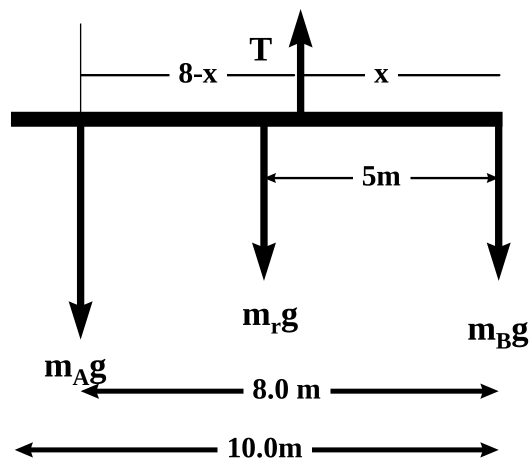


$$\Sigma\tau = 0$$
$$-m_B g x + m_A g(8 - x) = 0$$
$$m_B x = m_A(8 - x)$$
$$m_B x = 8m_A - x m_A$$
$$(m_A + m_B)x = 8m_A$$
$$x = \frac{8m_A}{(m_A + m_B)}$$
$$x = \frac{8(0.3\text{kg})}{(0.3\text{kg} + 0.5\text{kg})} = 3\text{m}$$

Note that we could also take the torques about a different point. Let's calculate them about the right end of the rod. The sum of the forces remains the same.

Ex. 20-3: How does the distance change if the rod has a mass of 1.0 kg?

We now have to include the weight of the rod. The weight of the rod acts at the CM. Since the rod is uniform, this will be the geometric center of the rod.



$$\Sigma F_y = 0$$

$$T - m_A g - m_B g - m_r g = 0$$

$$T = (m_A + m_B + m_r)g$$

$$T = (0.3\text{kg} + 0.5\text{kg} + 1.0\text{kg})\left(9.8\,\tfrac{\text{m}}{\text{s}^2}\right)$$

$$T = 17.6\text{N} = 18\text{N}$$

So the tension increases, as we would expect.

I'm going to take the axis of rotation to be at the right end again.

$$\Sigma\tau_{rightend} = 0$$

$$m_A g(8) + m_r g(5) - Tx = 0$$

$$m_A g(8) + m_r g(5) = Tx$$

$$\frac{(8m_A + 5m_r)g}{T} x$$

$$x = \frac{[8(0.3kg) + 5(1.0kg)]\left(9.8\,\tfrac{m}{s^2}\right)}{18\,N} = 4.0m$$