PHYSICS 151 – Notes for Online Lecture #19

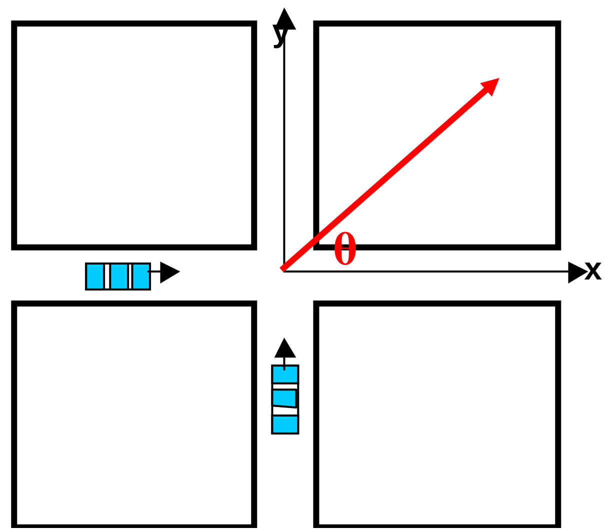
Conservation of Linear Momentum in 2-D

In this lecture we will tackle the vector nature of momentum. As we did with forces we will look at momentum in both the x and y direction. Thus our equations will become:

$$p_{x-initial} = p_{x-final}$$

$$p_{y-initial} = p_{y-final}$$

We will need to indicate an x-y coordinate system on our drawing and break any momentum not in one of these two directions into its x and y components.



Ex. 19-1: Two cars approach each other along streets that meet at a right angle. They collide at the intersection. After the collision they stick together. If the car traveling north has a mass of m_1 = 1300 kg and an initial speed of v_1 = 2.25 m/s and the eastbound car has a mass of m_2 = 1800 kg and an initial speed of v_2 = 4.50 m/s, what will be their speed and direction immediately after impact?

$$p_{x-initial} = p_{x-final}$$

$$m_2 v_2 = (m_1 + m_2) v \cos\theta$$

$$p_{y-initial} = p_{y-final}$$

$$m_1 v_1 = (m_1 + m_2) v \sin\theta$$

Let's go after the angle θ first by eliminating v. We need to solve the x-equation for v and then plug this expression into the y equation.

$$v = \frac{m_2 v_2}{(m_1 + m_2)\cos\theta}$$

$$m_1 v_1 = (m_1 + m_2)\frac{m_2 v_2}{(m_1 + m_2)\cos\theta}\sin\theta$$

$$m_1 v_1 = m_2 v_2 \tan\theta$$

$$\theta = \tan^{-1}\left(\frac{m_1 v_1}{m_2 v_2}\right) = \tan^{-1}\frac{(1300kg)(2.25m/s)}{(1800kg)(4.50m/s)} = 20°$$

Now we can easily solve for the velocity after the collision from either the x or y momentum equation.

$$m_2 v_2 = (m_1 + m_2) v \cos\theta$$

$$v = \frac{m_2 v_2}{(m_1 + m_2)\cos\theta}$$

$$= \frac{(1800\,kg)\left(4.50\frac{m}{s}\right)}{(1300\,kg + 1800\,kg)(\cos 20°)} = 2.78\frac{m}{s}$$

Inelastic Collisions

Collisions in which kinetic energy is conserved are known as elastic collisions. Although only collisions between subatomic particles are truly elastic, many collisions in the real world (such as pool balls) are close enough to be considered elastic. This allows us to have a momentum equation and a kinetic energy equation.

$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'$$

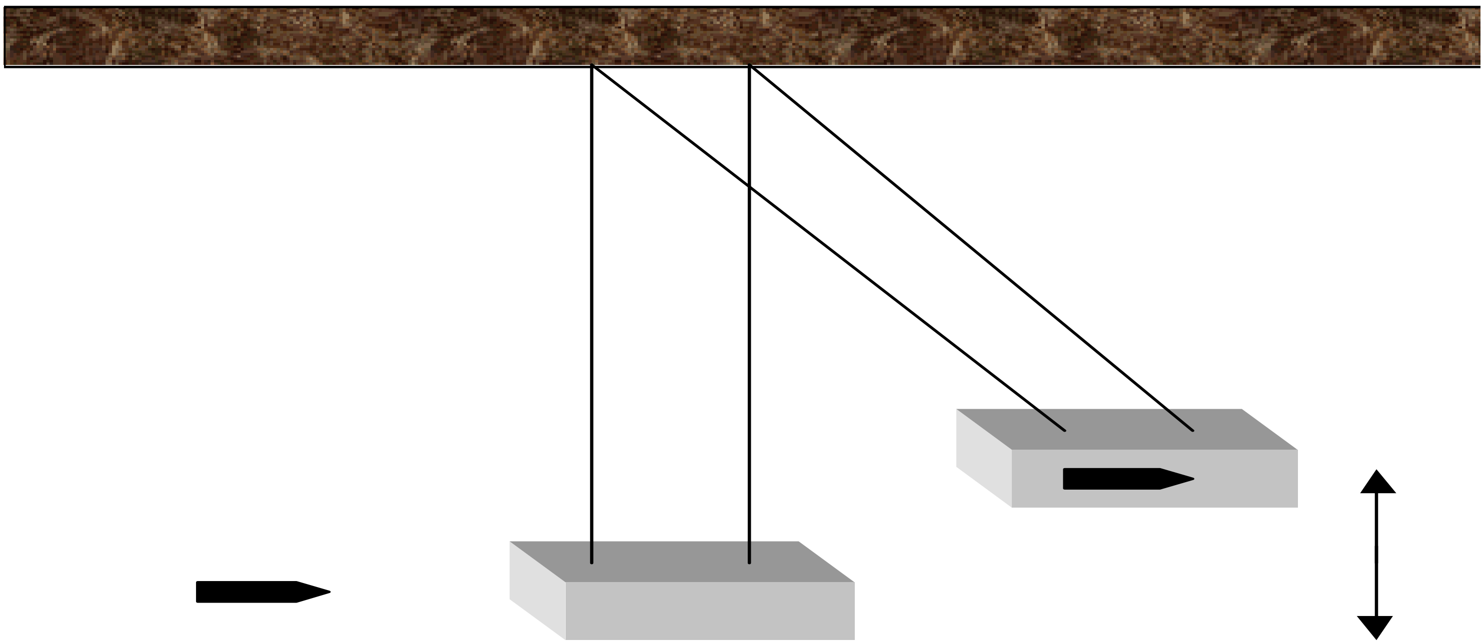
$$\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_1 v_1'^2 + \frac{1}{2} m_2 v_2'^2$$

Which allows you to solve for two unknowns in these problems. This also leads to considerable algebraic complexity. For this reason inelastic collisions will not be covered in this course.

In inelastic collisions, kinetic energy is not conserved. Realize that energy is always conserved, but some of it is transformed into heat which is difficult to account for in problems. Energy is always transformed into heat when the colliding objects are deformed. Almost all real world collisions are inelastic.

Realize that although energy is not conserved in a "useful manner" during the collision, immediately after the collision energy methods are often useful again. A good example of this is the ballistic pendulum – an apparatus for measuring the muzzle velocity of bullets.

Ex. 19-2: The 3.56-g bullet from a 22-250 rifle is fired into the 1.174-kg block of a ballistic pendulum. The bullet sticks within the block, which swings back rising 0.595 m. What was the speed of the bullet just before impact?



We can apply conservation of momentum during the collision (bullet to block). The momentum before the bullet hits must be equal to the momentum after the bullet hits.

$$m_{bul}v_{bul} + m_{block}v_{block} = (m_{bul} + m_{block})v_{final}$$

This simplifies since the initial velocity of the block is zero. Solving for the initial velocity of the bullet yields:

$$v_{bul} = \frac{(m_{bul} + m_{block})v_{final}}{m_{bul}}$$

We cannot solve this equation since we do not know the velocity of the block-bullet pair after the collision.

Now during the collision, kinetic energy is not conserved since the objects undergo deformation. Considerable energy goes into heating up the block. However, immediately after the collision energy conservation is once again useful. So we can get the velocity of the block-bullet pair by noting how high the ballistic pendulum swings up.

Applying conservation of energy:

$$E_i = E_f$$

$$KE_{right-after-impact} = PE_{high-point-of-swing}$$

$$\frac{1}{2}mv^2 = mgh$$

$$v = \sqrt{2gh} = \sqrt{2(9.8m/s^2)(0.595m)} = \sqrt{11.662m^2/s^2} = 3.41m/s$$

We can now substitute this value for v_{final} into our conservation of momentum equation.

$$v_{bul} = \frac{(m_{bul} + m_{block})v_{final}}{m_{bul}} = \frac{(3.56x10^{-3}\,kg + 1.174kg)}{3.56x10^{-3}\,kg}(3.41m/s) = 1130\,m/s$$

Ex. 19-3: A bullet with a mass of 4.0 g and a speed of 650 m/s is fired at a block of wood with a mass of 0.095 kg. The block rests on a frictionless surface, and is thin enough that the bullet passes completely through it. Immediately after the bullet exits the block, the speed of the block is 23 m/s. (a) What is the speed of the bullet when it exits the block? (b) Is the final kinetic energy of this system equal to, less than, or greater than the initial kinetic energy? (c) Verify your answer to part (b) by calculating the initial and final kinetic energies of the system.

(a) Use momentum conservation. Let the subscripts b and B denote the bullet and the block, respectively.

$$m_b v_{bi} + m_B v_{Bi} = m_b v_{bf} + m_B v_{Bf}$$

$$m_b v_{bi} + 0 = m_b v_{bf} + m_B v_{Bf}$$

$$v_{bf} = \frac{m_b v_{bi} - m_B v_{Bf}}{m_b}$$

$$= \frac{(0.0040\text{ kg})\left(650\ \frac{\text{m}}{\text{s}}\right) - (0.095\text{ kg})\left(23\ \frac{\text{m}}{\text{s}}\right)}{0.004\,\text{kg}}$$

$$= \boxed{1.0\times10^2\ \text{m/s}}$$

(b) The final kinetic energy is less than the initial kinetic energy because energy is lost to the heating and deformation of the bullet and block.

(c)

$$K_i = \frac{1}{2}m_b v_{bi}^2 = \frac{1}{2}(0.0040\text{ kg})\left(650\ \frac{\text{m}}{\text{s}}\right)^2 = \boxed{850\text{ J}}$$

$$K_f = \frac{1}{2}m_b v_{bf}^2 + \frac{1}{2}m_B v_{BF}^2 = \frac{1}{2}(0.0040\text{ kg})\left(103.75\ \frac{\text{m}}{\text{s}}\right)^2 + \frac{1}{2}(0.095\text{ kg})\left(23\ \frac{\text{m}}{\text{s}}\right)^2 = \boxed{47\text{ J}}$$