PHYSICS 151 – Notes for Online Lecture #17

Linear Momentum

Physicists like conservation laws. We discussed conservation of energy, but there are lots of other conservation laws and physicists are still discovering more. When Newton originally formulated the second law, he actually stated $\sum \vec{F} = m\vec{a}$ in terms of something other than acceleration. He defined something he called the '*quality of motion*', which was simply the product of the mass and the velocity. We call this the momentum and denote it by the symbol $\vec{p}$.

$$\vec{p} = m\vec{v}$$

units: kg(m/s)

Remarkably, no one has come up with another name for these units.
Note that the momentum is a vector because it is proportional to the velocity. The momentum of an object will be in the same direction as its velocity.
We learned the second law as

$$\sum \vec{F} = m\vec{a}$$

however, we can write this in terms of velocities, using

$$\vec{a} = \frac{\Delta v}{\Delta t}$$

$$\sum \vec{F} = m\vec{a}$$

$$\sum \vec{F} = m\frac{\Delta v}{\Delta t}$$

$$\sum \vec{F} = \frac{\Delta(mv)}{\Delta t}$$

$$\sum \vec{F} = \frac{\Delta \bar{p}}{\Delta t}$$

Note that I've assumed here that the mass is constant when I brought the mass into the Δ expression. (When might the mass not be constant? A rocket, where fuel is being expelled and the mass changes significantly.)

Ex. 18-1: A car hits a wall, causing its velocity to change from 15 m/s to zero. If the car has mass 1200 kg, what is the momentum change?

$$\Delta\vec{p} = \vec{p}_{final} - \vec{p}_{initial}$$

$$\Delta\vec{p} = 0 - (1200kg)\left(15\tfrac{m}{s}\right)$$

$$\Delta\vec{p} = -1.8x10^4 \frac{kg - m}{s}$$

Ex. 18-2: A ball hits a wall and bounces directly backward. The mass of the ball is 5 kg. It starts with an initial velocity of 15 m/s and has the same speed when it bounces back. What is the change in momentum?

Solution: The final velocity of the ball is -15 m/s and the initial velocity is + 15 m/s. Remember that "momentum is a vector quantity".

$$\Delta\vec{p} = \vec{p}_{final} - \vec{p}_{initial}$$

$$\Delta\vec{p} = m\vec{v}_{final} - m\vec{v}_{initial}$$

$$\Delta\vec{p} = m\left(\vec{v}_{final} - \vec{v}_{initial}\right)$$

$$\Delta\vec{p} = (5kg)\left(-15\tfrac{m}{s} - 15\tfrac{m}{s}\right)$$

$$\Delta\vec{p} = (5kg)\left(-30\tfrac{m}{s}\right)$$

$$\Delta\vec{p} = -150\frac{kg-m}{s}$$

Conservation of Momentum

Momentum can be tricky because it's a vector, but one of the reasons we like momentum is because the momentum of a system – *under some conditions* – is conserved.

The law of conservation of momentum says that

As ***long as there are no external forces acting on the system***,
the momentum before = momentum after.

That first part is the fine print: you have to make sure that you have satisfied that condition before you apply the second part.

This law was actually discovered experimentally before the mathematics to accompany it was understood. People measured the velocity of objects after colliding, calculated the momenta and compared them.

The most general statement of the law of conservation of momentum is:

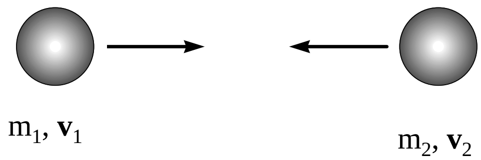
The total momentum of a system of isolated bodies remains constant

Isolated just means here that no external forces act on the system. So if I were to choose a system of three pool balls hitting each other, I would mentally draw a circle around the balls and make sure that there are no outside forces acting (like a cue striking one of the balls). There are forces - the interactions between the balls – but those occur only *within* the system.

If we chose a system, such as a meteor falling to the earth, momentum is not conserved, because there is the external force of gravity acting on the meteor. If we redefine our system to include the Earth, there are no external forces, so momentum between the two is conserved.

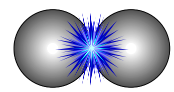
Let's examine the behavior of two balls of equal mass heading toward each other, bouncing together and then rebounding. We can calculate the total momentum in each state.

Let's say that initially the ball on the left has mass m_1 and velocity $\mathbf{v}_1$ to the right. The ball on the left has mass m_2 and velocity $\mathbf{v}_2$ to the left. Taking to the right as positive, the momenta are



$$\mathbf{p_1} = mv_1$$

$$\mathbf{p_2} = -mv_2$$



The balls collide, and then afterwards, the first mass is moving to the left with velocity $\mathbf{v}_1'$ and the second mass is moving to the right with velocity $\mathbf{v}_2'$. The momenta are:

$$\mathbf{p}_1' = -m\mathbf{v}_1'$$

$$\mathbf{p}_2' = m\mathbf{v}_2'$$

Experimentally, even before Newton, people had measured situations like this and found that, although each ball changed it's own momentum, the total momenta of the system was unchanged, as long as there were no net forces acting on the system.

The law of conservation of momentum says that: *As long as there are no external forces acting on the system, the momentum before = momentum after*. For the system of two masses, the law would be:

$$m_1\vec{v}_1 + m_2\vec{v}_2 = m_1\vec{v}_1{}' + m_2\vec{v}_2{}'$$

Ex. 18-3: Two carts, one of mass 1.5 kg and one of mass 3.0 kg are at rest on an air track. A charge is set off between them and they go in opposite directions. If the 1.5 kg cart travels at a speed of 2.0 m/s to the right after the charge has exploded, what is the velocity of the 3.0 kg cart?

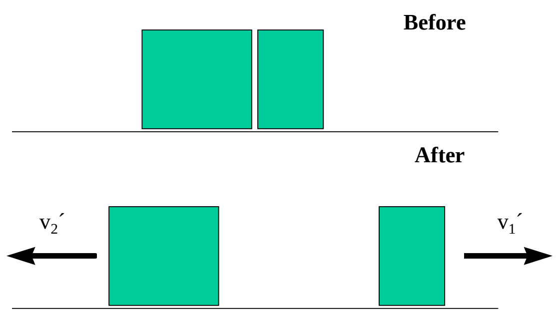
The key to working conservation problems, you'll remember, is that you have to identify two particular cases to analyze. We'll take before to refer to the masses at rest and after to be when the charge has exploded and the masses are moving.

Before: $m_1 = 1.5$ kg $\quad m_2 = 3.0$ kg

$v_1 = 0 \quad v_2 = 0$

After:

$v_1' = +2.0 \quad v_2' = ?$



Apply conservation of momentum:

$$m_1\vec{v}_1 + m_2\vec{v}_2 = m_1\vec{v}_1{}' + m_2\vec{v}_2'$$

$$0 = m_1\vec{v}_1{}' + m_2\vec{v}_2'$$

$$m_1\vec{v}_1{}' = -m_2\vec{v}_2'$$

$$-\frac{m_1}{m_2}\vec{v}_1{}' = \vec{v}_2'$$

$$-\frac{1.5\text{kg}}{3.0\text{kg}}\left(+2.0\,\tfrac{\text{m}}{\text{s}}\right) = \vec{v}_2' = -1.0\,\tfrac{\text{m}}{\text{s}}$$

IMPULSE

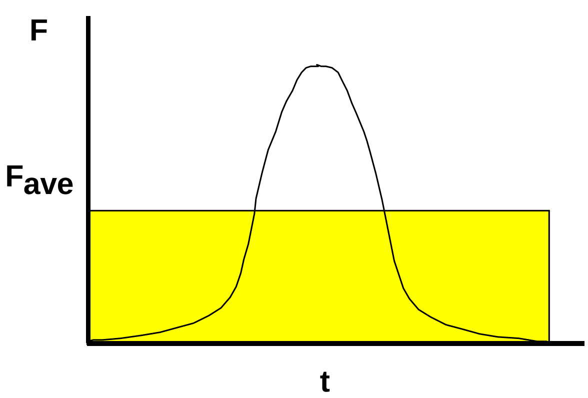
We're working toward being able to analyze collisions – which is important in a wide variety of fields, from traffic accidents, billiard balls and atomic or nuclear collisions.
Collisions are very important in real life, whether you're trying to figure out how a billiard ball travels across a table or calculating what happens when two atomic particles interact.

I showed you earlier that one definition of net force is $\sum \vec{F} = \left(\frac{\Delta \vec{p}}{\Delta t}\right)$

What this tells us is that the net force you feel is not only due to the momentum change - such as the large momentum change that you experience when, say, you hit your head on the dashboard of a car, but also is determined by the amount of time over which that force acts. If the time over which the force acts is short, the force you feel will be large.
This is the motivation behind dashboards. It used to be that dashboards weren't padded. So if your head had a collision with a dashboard, you basically hit a piece of metal. Think about what happens if you hit a something padded, like a pillow. The pillow gives a bit, extending the time of the contact. A larger time means a smaller net force, even though the change in momentum is the same. A padded dashboard extends the time over which the momentum change occurs so that it (theoretically, at least) doesn't hurt as much due to there being less net force. The same argument applies for the difference between aluminum and wood baseball bats. The wood is softer and gives more when the ball hits it, extending the time of contact and decreasing the force.
We have a special name for the momentum change - we call it the impulse.

$$\text{Impulse} = \Delta \mathbf{p}$$

The units for impulse are the same as those for momentum = kg-m/s
In general, the force that we're dealing with varies enormously over the time it acts on the object.



So we approximate the force for all times as an average value. In the figure at left, the rectangle with height equal to the average force covers the same area as the actual curve itself. We usually write the impulse as:

$$\overline{\mathbf{F}}\Delta \mathbf{t} = \Delta \mathbf{p}$$

Ex. 18-4: A ball of mass 3.0 kg hits a wall straight on and bounces backward. It has the same velocity when it leaves the wall as it does when it hits it, 43 m/s. If the time of contact is 1.4 s, what is the average force exerted on the ball?

$$\overline{\mathbf{F}}\Delta \mathbf{t} = \Delta \mathbf{p}$$

$$\overline{\mathbf{F}}\Delta \mathbf{t} = mv_f - mv_i$$

$$\overline{\mathbf{F}} = \frac{m\left(v_f - v_i\right)}{\Delta t}$$

$$\overline{\mathbf{F}} = \frac{3.0\,kg\left(-43\tfrac{m}{s} - \left(43\tfrac{m}{s}\right)\right)}{1.4s}$$

$$\overline{\mathbf{F}} = -180\,N$$