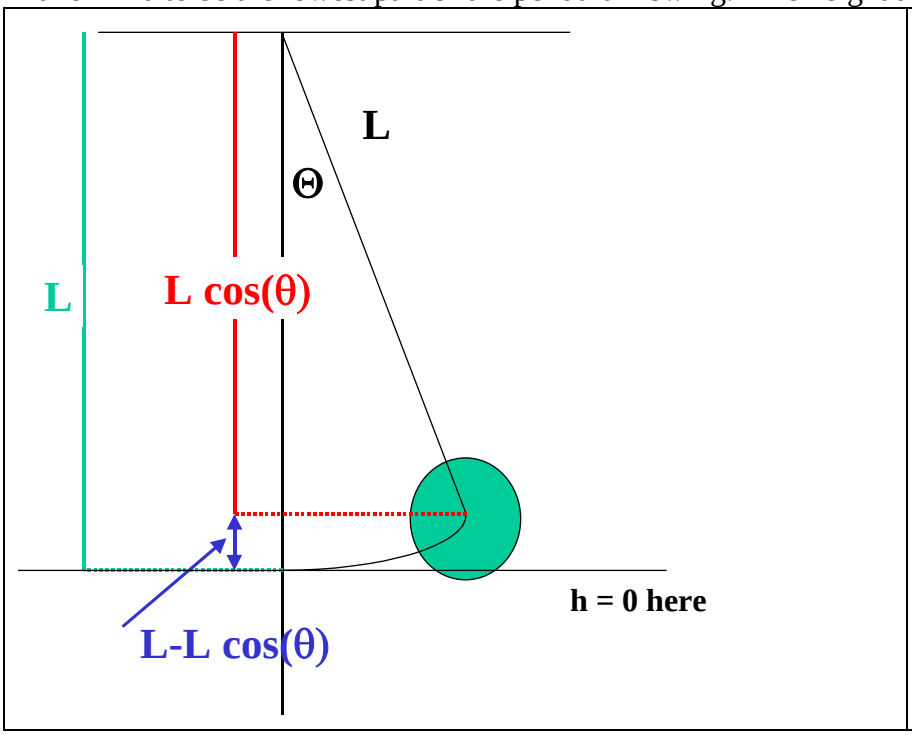
PHYSICS 151 – Notes for Online Lecture #17

Energy Concerns for the Pendulum

Take h = 0 to be the lowest part of the pendulum swing. The height at any point is then



h = L-Lcos(θ)

At the top of the swing, the velocity is zero, so there is no kinetic energy and the total energy is entirely potential

E = PE = mgh

E = mgL(1-cos(θ))

At the bottom of the swing, the potential energy is zero and all of the energy is kinetic, so

$E = \frac{1}{2}mv_o^2$

where v_o is the maximum velocity.

From conservation of energy, the total energy at these two points must be equal.

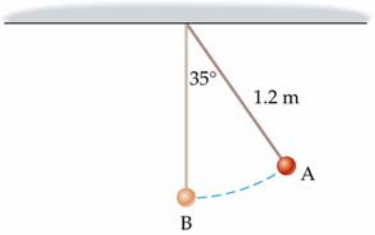
$$E = \tfrac{1}{2}mv_o^2 = mgL(1-\cos(\theta))$$

$$v_o^2 = 2gL(1-\cos(\theta))$$

$$v_o = \sqrt{2gL(1-\cos(\theta))}$$

Example 17-1: Suppose that the pendulum bob has a mass of 0.4 kg. If it is released from position A find (a) the velocity at position B and (b) the velocity when the pendulum is at an angle of 10°.

(a) We want to set $E_A = E_B$ and have position B define the zero level potential.



$$E_A = E_B$$

$$mgl(1-\cos 35°) = \frac{1}{2}mv^2$$

$$v = \sqrt{2gl(1-\cos 35°)} = \sqrt{2\left(9.8\frac{m}{s^2}\right)(1.2m)(1-\cos 35°)} = 1.2\frac{m}{s}$$

$$E_{10°} = E_B$$

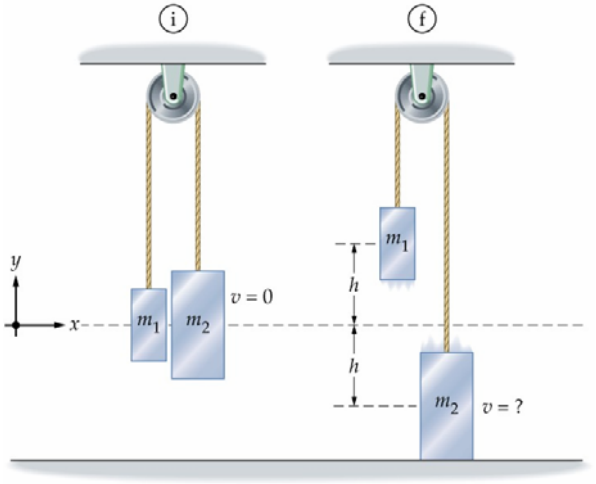
$$\frac{1}{2}mv_{10°}^2 + mgl(1-\cos 10°) = \frac{1}{2}mv^2$$

$$v_{10°}^2 + 2gl(1-\cos 10°) = v^2$$

$$v_{10°} = \sqrt{v^2 - 2gl(1-\cos 10°)} = \sqrt{\left(1.2\frac{m}{s}\right)^2 + 2\left(9.8\frac{m}{s^2}\right)(1.2m)(1-\cos 10°)} = 1.04\frac{m}{s}$$

Example 17-2: The two masses in the Atwood's machine shown here are initially at rest at the same height. After they are released, the large mass m_2 falls through a height *h* and hits the floor, and the small mass m_1 rises through a height *h*.
(a) Find the speed of the masses just before m_2 lands, giving your answer in terms of m_1, m_2, g and *h*. Assume the ropes and pulley have negligible mass and that friction can be ignored.
(b) Evaluate your answer to part (a) for the case h = 1.2 m, m_1 = 3.7 kg, and m_2 = 4.1 kg.

The kinetic energy of the two mass system is given by:



$$E_i = E_f$$

$$0 = \frac{1}{2}(m_1 + m_2)v_f{}^2 - m_2 gh + m_1 gh$$

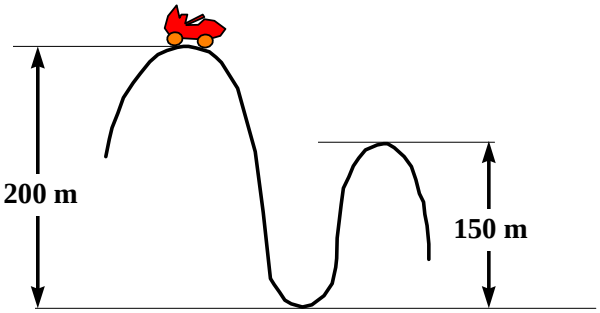
$$v_f = \boxed{\sqrt{2gh\left(\frac{m_2 - m_1}{m_1 + m_2}\right)}}$$

$$v_f = \sqrt{2\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(1.2\ \text{m})\left(\frac{4.1\ \text{kg} - 3.7\ \text{kg}}{3.7\ \text{kg} + 4.1\ \text{kg}}\right)} = \boxed{1.1\ \text{m/s}}$$

Non-Conservative Forces

So what about the energy that goes into those non-conservative forces that we excluded last lecture? According to conservation of energy, the work done by friction must be included if we are to truly keep an accurate count of the energy. Note that friction does work. This work is manifested as energy – in this case heat energy. If you brake quickly, you smell your brake linings burning – friction generates heat. You can see this yourself if you rub your hands together very quickly – the friction between your hands makes them warm. Let's investigate what happens in the roller coaster problem if we now consider friction.

If the velocity of the car at the top of the first hill is zero, what is the velocity when it reaches the top of the second hill?



Ex. 17-3: A roller coaster designer has a hill of 2.00×10^2 m, followed by a hill of 1.50×10^2 m. The car starts with zero velocity at the top of the first hill and is measured to have a speed of 25.0 m/s at the top of the second hill? How much work did friction do and where did the energy 'go'? The mass of the car is 1.50×10^3 kg.

We solved this problem with no friction using the fact that, if there are no non-conservative forces acting, the total energy must be equal:

$$E_2 = E_1$$

And we found that the speed of the car at the top of the next hill is 31.3 m/s. But this is theory. An engineer measures the actual speed of the car and it's only 25.0 m/s. – what happened to the rest of the energy?

If you recall when we derived that the energies were equal, at one point we said that we would assume that there are no non-conservative forces. Before we made that assumption, we had:

$$W_{NC} = (KE_2 + PE_2) - (KE_1 + PE_1)$$

$$W_{NC} = (KE_2 + PE_2) - (KE_1 + PE_1)$$

$$W_{NC} = \tfrac{1}{2}mv_2^2 + mgh_2 - \left(\tfrac{1}{2}mv_1^2 + mgh_1\right)$$

$$W_{NC} = \tfrac{1}{2}mv_2^2 + mg(h_2 - h_1)$$

$$W_{NC} = \tfrac{1}{2}(1.50 \times 10^3 \text{ kg})\left(25.0 \tfrac{\text{m}}{\text{s}}\right)^2 + (1.50 \times 10^3 \text{ kg})\left(9.8 \tfrac{\text{m}}{\text{s}^2}\right)(1.50 \times 10^2 \text{ m} - 2.00 \times 10^2 \text{ m})$$

$$W_{NC} = -2.66 \times 10^5 \text{ J}$$

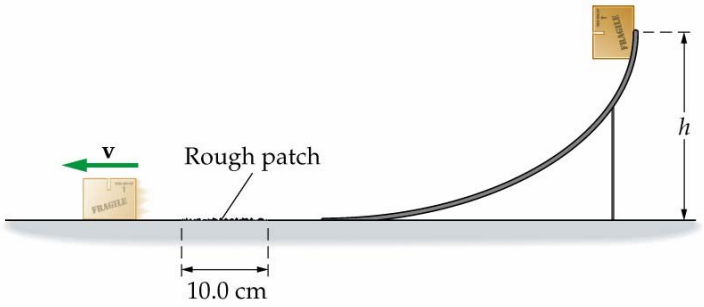
This tells us that the work done by non-conservative forces is of magnitude 2.66×10^5 J. Why the minus sign?

The culprit in this case is friction: We know that the force of friction always acts in a direction opposite that of the displacement, so the work done on the car is negative. The energy went to heat in the brakes, noise, etc.

Ex. 21-3 A 1.80 kg block slides on a rough, horizontal surface. The block hits a spring with a speed of 2.00 m/s and compresses it a distance of 11.0 cm before coming to rest. If the coefficient of kinetic friction between the block and the surface is $\mu_k = 0.560$, what is the force constant of the spring?

$$W_{nc} = \Delta PE + \Delta KE$$
$$W_{nc} = PE_f - PE_i + KE_f - KE_i$$
$$W_{nc} = PE_f - KE_i$$
$$-\mu_k mgx = \frac{1}{2}kx^2 - \frac{1}{2}mv_i^2$$
$$k = \frac{m(-2\mu_k gx + v_i^2)}{x^2}$$
$$= \frac{(1.80\text{ kg})\left[(-2)(0.560)\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(0.110\text{ m}) + \left(2.00\ \frac{\text{m}}{\text{s}}\right)^2\right]}{(0.110\text{ m})^2}$$
$$= 415\ \text{N/m}$$

Ex 21-4 The figure below shows a 1.50-kg block at rest on a ramp of height h. When the block is released, it reaches the bottom of the ramp and moves across a surface that is frictionless except for one section of width 10.0 cm that has a coefficient of kinetic friction $\mu_k = 0.64$. Find h such that the block's speed after crossing the rough patch is 3.50 m/s.



$$W_{nc} = \Delta PE + \Delta KE$$
$$W_{nc} = PE_f - KE_i$$
$$-\mu mgd = -mgh + \frac{1}{2}mv^2$$
$$h = \frac{\frac{v^2}{2} + ugd}{g}$$
$$= \frac{v^2}{2g} + ud$$
$$= \frac{\left(3.50\ \frac{\text{m}}{\text{s}}\right)^2}{2\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)} + 0.640(0.100\text{ m})$$
$$= \boxed{68.8\text{ cm}}$$