PHYSICS 151 – Notes for Online Lecture #16

Potential Energy

We've talked about kinetic energy, which is energy of motion - if something isn't moving, it has no kinetic energy. We defined energy as the ability to do work and saw that we could change the kinetic energy of an object by doing work on it.

Potential energy can be thought of as energy of position, and the link between work and energy also applies in potential energy. Let's say that I want to lift a dumbbell from the floor to over my head. When the dumbbell is on the floor, it has no kinetic energy. When it is over my head, it also has no kinetic energy; however, I did some work in lifting it. I have to work against gravity. While I do change its kinetic energy while I move it, in the two positions that we're looking it, we don't have to consider kinetic energy.

If I did work in lifting the dumbbell, where did the work go? The answer is that the work went to giving the object potential energy. The dumbbell can do work if I drop it.

> **The potential energy that an object in a certain position acquires is equal to the amount of work it takes to put the object in that position.**

Gravitational Potential Energy

If I drop a book from the top of a building, it will be able to do more work when it hits something on the ground than if I drop it from the first story. This suggests that the potential energy of an object increases the further it is from the ground.

We haven't yet talked about one of the alternate definitions of work, which I'll introduce here and we'll talk about more in another lecture.

$$\text{Work} = \text{force times displacement in the direction of the force} = Fx\cos(\theta)$$

If I picture the book being dropped straight down a distance h, the force and the displacement are in the same direction, so $\cos(\theta) = 1$ and the work done by gravity will be:

$$W_{gravity} = Fx = (-mg)(0-h) = mgh$$

This is the amount of work necessary to raise the object through a height h. In doing so, the hammer acquires the ability to do work = mgh. The work done by gravity is of concern to us in many situations, so we give this quantity a special name and call it the **gravitational potential energy**.

$$PE_{grav} = mgy$$

where y is the height of the object above some arbitrary reference level. If we calculate the work done when the hammer falls from height y_1 to y_2, we find that

$$\begin{aligned} W_{grav} &= mgh \\ &= mg\,(y_1 - y_2) \\ &= mgy_1 - mgy_2 \end{aligned}$$

h ——————— y_1

0 ——————— y_2

$$= PE_1 - PE_2$$

Arbitrary Zero for gravitational potential energy. You'll notice a slight difference here between the two types of energy we've discussed so far: the gravitational potential energy depends only on the distance between the two points – you can set zero to be anywhere. Kinetic energy is referenced to an object at rest having kinetic energy = 0. If an object falls a distance of three feet, it doesn't really matter whether I measure it starting at 6 feet and ending at 3 feet, or starting at 3 feet and ending at 0. From either perspective, the net distance traversed is the same. The physically important quantity in this type of energy is the *change* in potential energy, because this is the quantity that can be measured via the work done. The only caveat is that we have to be consistent throughout the calculation with where we choose our zero reference point.

Conservative and Non-Conservative Forces

I mentioned earlier that the gravitational potential energy is path independent – that is, it depends only on the starting and ending points of the motion and not on how you get there. The forces for which the amount of work depends only on the starting and ending points are called *conservative* forces. Conservative forces include gravity, elastic or spring, and others.

The nonconservative force that you will encounter most often is friction. If you push something around, you must work against friction over the entire path. If the distance of one path is 2 times the distance of another path, you must do twice the work in overcoming friction.

The reason we make this differentiation is that only conservative forces can give objects potential energy. Although potential energy is always associated with a force, we can't write a potential energy for just any force – the force must be conservative.

# Conservation of Energy and the Work-Energy Principle

We have discussed the work-energy principle, which states that the net work done on an object is equal to the difference between the final and initial kinetic energy. We can extend this principle to include the effects of potential energy. This is a powerful tool that will allow us to solve a wide variety of problems

Let's start with a general situation, in which we have an object that is initially moving with velocity v_1 and has potential energy PE_1. At some later time, the object now has velocity v_2 and potential energy PE_2. In order to make the change from the first situation to the second, work must be done on the object. That work can be divided into two parts: a part due to conservative forces, W_C and a part due to non-conservative forces W_{NC}.
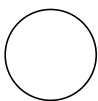
$$W = W_C + W_{NC}$$

PE = PE$_1$
v = v$_1$
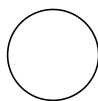
PE = PE$_2$
v = v$_2$

where W is the total work (or net work) done on the object. The work-energy theorem tells

us

$$W = \Delta KE$$
$$W = W_{NC} + W_C = \Delta KE$$
$$W_{NC} + W_C = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2$$
$$W_{NC} = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2 - W_C$$

We know that the work done by conservative forces can be expressed in terms of the difference in potential energy: (Note that this is a tricky definition. This quantity is the work that would have to be done in order to return the system to its initial configuration.)

$$W_C = PE_1\text{-}PE_2 = \Delta PE$$

$$W_{NC} = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2 - W_C$$
so
$$W_{NC} = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2 - (PE_1 - PE_2)$$
$$W_{NC} = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2 - PE_1 + PE_2$$
$$W_{NC} = \Delta KE + \Delta PE$$

The work W_{NC} done by non-conservative forces acting on an object is equal to the total change in kinetic and potential energies.

In many cases – notably when we ignore friction – there are no non-conservative forces acting on the system, which means $W_{NC} = 0$. This equation reduces to a particularly simple form:

$$\Delta KE + \Delta PE = 0$$
$$\tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2 + PE_2 - PE_1 = 0$$
$$\tfrac{1}{2}mv_2^2 + PE_2 = \tfrac{1}{2}mv_1^2 + PE_1$$

If we define the **total mechanical energy** to be the sum of the kinetic and potential energies:

$$E = KE + PE = \tfrac{1}{2}\,mv^2 + PE,$$

this means that

$$E_2 = E_1$$

that is, the total mechanical energy of the system remains the same. We call any quantity that does not change a 'conserved' quantity. We call this law '**conservation of energy**'.

> **Warning: Conservation of energy holds only when there are no non-conservative forces acting on the system. If friction must be considered, you can't use this version of the law.**

In words, if only conservative forces are acting, the total mechanical energy of a system neither increases nor decreases. You can think of this like financial assets. You have liquid assets, like cash, that can be used immediately, and frozen assets, like CDs or bonds. Regardless of how these things are distributed, you have the same amount of money at any given time. At one time, all your assets may be liquid and at another, they may be frozen.

You now see why we called these forces 'conservative' - when they are acting on a system, the total energy is conserved.

Ex. 16-1: A 4.00 kg rock falls from a height of 5.00 m. Find its kinetic, potential and total energy when:

    a) the rock is 5.00 m above the ground and has not yet started to fall

    b) the rock is 2.50 m above the ground

    c) the rock is just about to hit the ground (i.e. h = 0, but the rock hasn't stopped yet)

Make a table of the three different situations to fill in PE, KE and total E

Solution: <u>At any point along the motion, the total energy will be constant.</u> Let's take y = 0 to be the ground, and measure distances from there. The potential energy is zero at y = 0.

The total energy is $E = KE + PE$

a) The energy at position a is E_a, where $E_a = (KE)_a + (PE)_a$

At the top of the motion, the rock is sitting still, so the kinetic energy is 0.

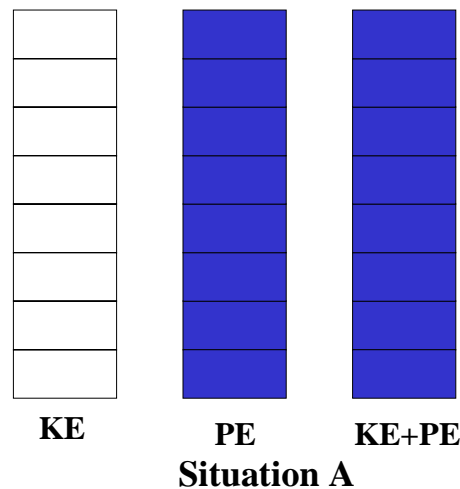$$(KE)_a = 0 \text{ because } v_a = 0$$

This means that the total energy must be equal to the potential energy, which we can calculate. The potential energy is entirely due to the gravitational potential energy, and is

$$PE_a = mgh_a$$
$$PE_a = (4.00\,\text{kg})\left(9.80\,\tfrac{\text{m}}{\text{s}^2}\right)(5.00\,\text{m})$$
$$PE_a = 196\,\text{J}$$

One way of illustrating how the energy is distributed is to use a bar chart

    KE   **PE**   **KE+PE**

Situation A

b) When the rock has fallen halfway, the potential energy decreases because the height above the ground is not as large:

$$PE_b = mgh_b$$
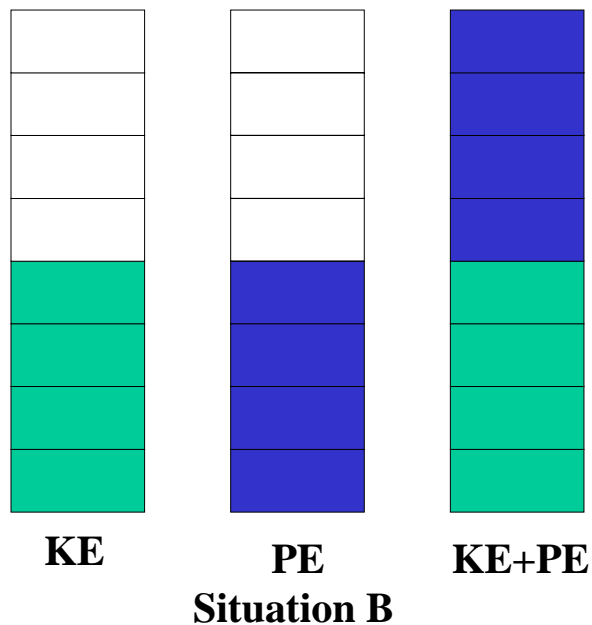$$PE_b = (4.00\,\text{kg})\left(9.80\,\tfrac{\text{m}}{\text{s}^2}\right)(2.50\,\text{m})$$
$$PE_b = 98.0\,\text{J}$$

The kinetic energy is given by $\frac{1}{2} mv_b^2$ but we don't know v_b. However, we do know that the total mechanical energy must remain constant, so

$$KE_b = \text{Total Energy} - PE_b = 196 \text{ J} - 98.0 \text{ J} = 98.0 \text{ J}$$

KE   **PE**   **KE+PE**

Situation B

c)   Finally, when the rock is just at the bottom of its motion, the potential energy is zero, because we've taken $y = 0$ there. The kinetic energy must be all of the energy and must therefore be 196 J.


KE   **PE**   **KE+PE**

Situation C
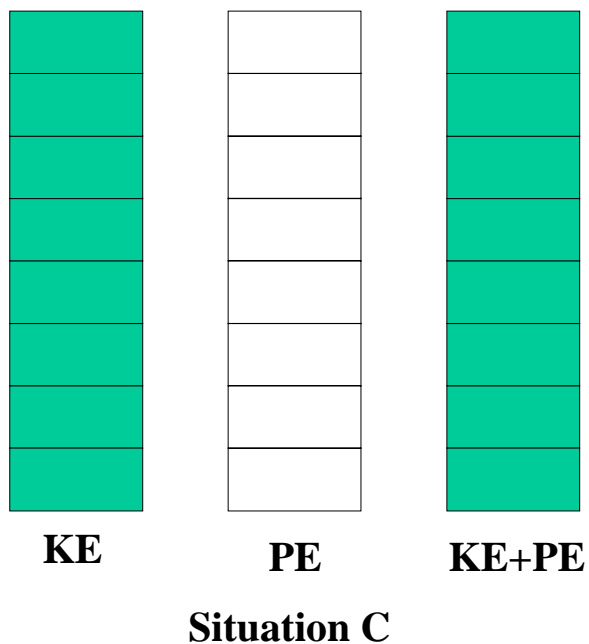
Or, using a chart:

	KE	PE	Total
A	0	196 J	196 J
B	98.0 J	98.0 J	196 J
C	196 J	0 J	196 J

Ex. 16-2: When a 5.00 kg rock is 8.00 m from the ground, its potential energy is 390.0 J. When the same rock is 2.00 m from the ground, the potential energy is 98.0 J. How fast is the rock moving when it is 2.00 m from the ground?

Solution: Since the PE is 98.0 J and the total energy is 390.0 J, everything else must be KE
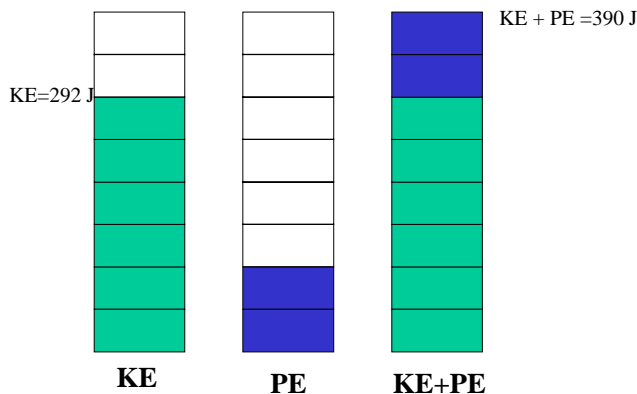
$$KE = E - PE = 390.0 \text{ J} - 98.0 \text{ J} = 292 \text{ J}$$

$$KE = \tfrac{1}{2} mv^2$$

$$\pm\sqrt{\frac{2(KE)}{m}} = v^2$$

$$\pm\sqrt{\frac{2(292\,J)}{5.00 \text{kg}}} = v^2$$

$$-10.8 \tfrac{m}{s} = v_c$$

We choose the minus sign because the rock is heading downward.

Some of you may be wondering "Why didn't we just use kinematic equations?"

$$v^2 = v_o^2 + 2a(y - y_o)$$
$$v^2 = 2a(0 - y_o)$$
$$v^2 = -2ay_o$$
$$v^2 = -2\left(-9.8 \tfrac{m}{s^2}\right)(6.00\,\text{m})$$
$$v = -10.8 \tfrac{m}{s}$$

Let's look at a different type of example to see where the energy equations are much easier to use.

Ex. 16-3: A roller coaster designer designs a hill of 2.00×10^2 m, followed by a hill of 1.50×10^2 m, as shown in the figure. If the car starts with zero velocity at the top of the first hill, how fast will it be going when it reaches the top of the second hill? Ignore friction for the moment.

If the velocity of the car at the top of the first hill is zero, what is the speed when it reaches the top of the second hill?

200 m

150 m
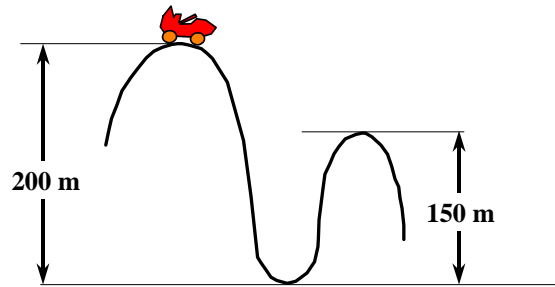
Picking a coordinate system so that you can use kinematic equations is tricky. Here's where energy conservation comes in handy

Call the top of the first hill position 1. At this point:

$v_1 = 0$, so $KE_1 = 0$. This means that all the energy is in the form of potential energy. Calling the ground h=0 zero,

$$E_1 = PE_1 + KE_1$$
$$E_1 = mgh_1 + 0$$
, where $h_1 = 200$ m.

When the car reaches position 2 – the top of the 1.50×10^2 m hill, it will have both potential and kinetic energy, according to

$$E_2 = PE_2 + KE_2$$
$$E_2 = mgh_2 + \tfrac{1}{2}mv_2^2$$

There are no nonconservative forces at work here, so the two energies must be equal.

$$E_1 = E_2$$
$$mgh_1 = mgh_2 + \tfrac{1}{2}mv_2^2$$

We can then solve for v_2, which is the quantity we want

$$mgh_1 = mgh_2 + \tfrac{1}{2}mv_2^2$$
$$gh_1 - gh_2 = \tfrac{1}{2}v_2^2$$
$$\sqrt{2g(h_1 - h_2)} = v_2$$
$$\sqrt{2(9.80\tfrac{m}{s^2})(2.00 \times 10^2 \text{ m} - 1.50 \times 10^2 \text{ m})} = v_2$$
$$31.3\tfrac{m}{s} = v_2$$

Note that we didn't need the mass of the car to figure this out. There's no reasonable way to do this problem with kinematics. Using conservation of energy makes things much simpler.

Elastic Potential Energy

As the spring is stretched or compressed, energy is converted from the motion of the mass and spring to energy stored in the coils and back again. The elastic potential energy due to a spring (and other stretchy things like rubber bands) is:

$$PE_{el} = \frac{1}{2}kx^2$$
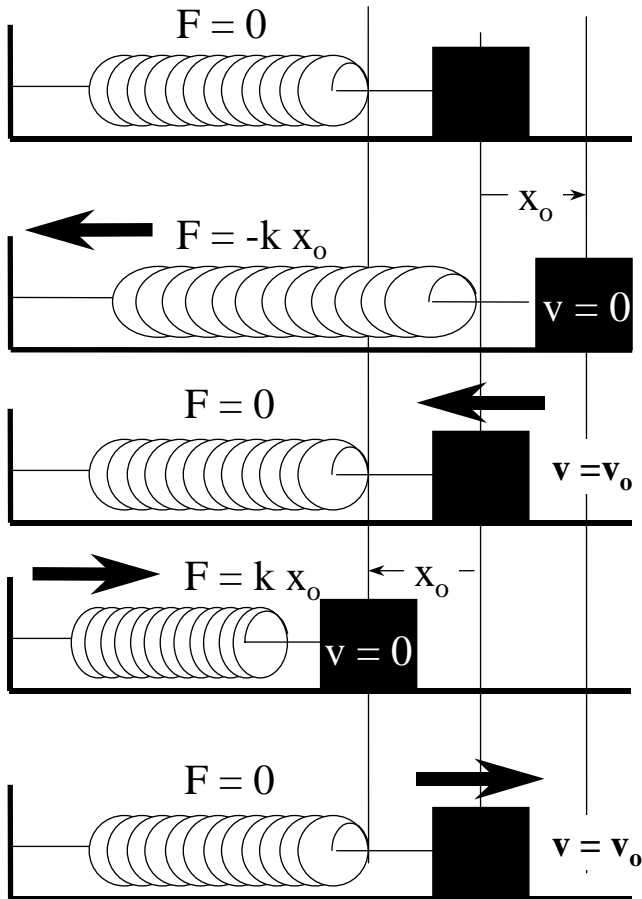
where, unlike gravitational potential energy, we take the zero to be the equilibrium position of the spring (i.e. $x = 0$ corresponds to the point at which there is zero potential energy).

We can write the total mechanical energy for a spring as:

$$E = KE + PE$$

$$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

At the maximum displacement ($x = x_o$), the mass is momentarily standing still. The total energy is then:

$$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

$$E = \frac{1}{2}kx_0^2$$

All potential energy!

When $x = 0$, the mass has a velocity v_o, which is the maximum velocity that the mass can have. The total energy is then:

All kinetic energy

$$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

$$E = \frac{1}{2}mv_o^2$$

Because the total energy is constant at every place along the motion,

$$\frac{1}{2}mv_o{}^2 = \frac{1}{2}kx_o^2$$

$$v_o{}^2 = \frac{k}{m}x_o^2$$

$$v_o = \sqrt{\frac{k}{m}}x_o$$

Ex. 16-4: A spring of spring constant k = 25 N/m has a mass of 0.5 kg attached to it. If the mass is displaced 0.7 m from its equilibrium position, what are:

a) the total energy
b) the maximum velocity
c) the potential and kinetic energies when the mass is 0.4 m from the equilibrium position and
d) the velocity when the mass is at 0.4 m from the equilibrium position.

a) The energy will be entirely potential when the spring is stretched to 0.7 m. We'll take the equilibrium point as the where the mass fully extends the spring but doesn't stretch it. Then	$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$ $E = \frac{1}{2}kx_o^2$ $E = \frac{1}{2}\left(25\tfrac{N}{m}\right)(0.7m)^2$ $E = 6.1J$
b) The maximum velocity will occur when all of the energy is kinetic energy (i.e. at x = 0)	$E = \frac{1}{2}mv_o^2$ $\frac{2E}{m} = v_o^2$ $\sqrt{\frac{2E}{m}} = v_o$ $\sqrt{\frac{2(6.13J)}{0.5kg}} = v_o$ $4.95\tfrac{m}{s} = 5.0\tfrac{m}{s}$
c) when x = 0.4 m from equilibrium: Since the total energy is constant, E = KE + PE, so KE = E-PE	$PE = \frac{1}{2}kx^2$ $PE = \frac{1}{2}\left(25\tfrac{N}{m}\right)(0.4m)^2$ $PE = 2.0J$ KE = 6.1 J-2.0 J = 4.1 J

d) We can use the results of part c to find the velocity of the mass at this point:	$KE = \dfrac{1}{2}mv^2$
	$\sqrt{\dfrac{2KE}{m}} = v$
	$\sqrt{\dfrac{2(4.1J)}{0.5kg}} = v$
	$4.0\tfrac{m}{s} = v$