

PHYSICS 151 – Notes for Online Lecture #15

WORK: In physics, we define

Work = displacement * force responsible for the displacement.

Distinguishing between displacement and distance. Remember that we make a distinction between distance and displacement! Displacement is how far you got from where you started (a vector). Distance is the total amount you traveled (a scalar).

Ex. 15-1: A 50 N weight is moved a distance of 20 m and back again, with the force always in the direction of the displacement. What is the total work done?

- a) 100 J
- b) 50 J
- c) 200 J
- d) 400 J
- e) 0 J

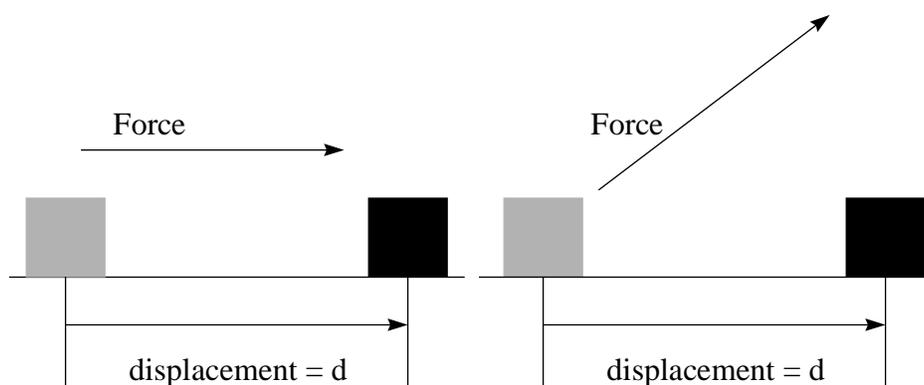
This is another one of those tricky physics questions that forces you to remember that displacement is a vector. Since

work = (force parallel to the displacement) times (displacement),

if the displacement is zero, there is no work done.

So, for example, if you go to lift weights and do a complete repetition of lifting the weight up and back to its starting point, according to a physicist, you have done no work!

This is a very special case, but it reminds you that you have to consider **only** the displacement in calculating the work done. The second part of the definition tells us that only that part of the force that is along the direction of the displacement will contribute to the work done.



Only the part of the force in the direction of the displacement counts!

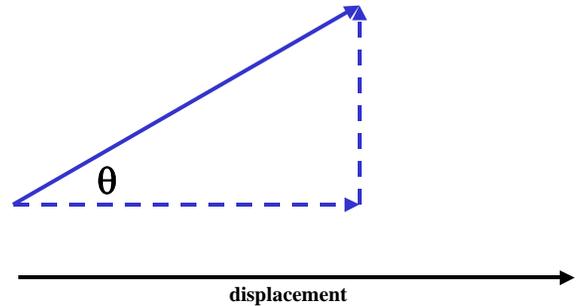
Let's compare two situations: In the first, I pull the box over a displacement d with a force that is entirely horizontal. In the second case, I am pulling with a force that has the same magnitude at the first case, but is applied at an angle to the displacement.

In both cases, the displacement is the same;

however, in the second case, not all of the force goes into moving the object. Some of the force – the part in the y-direction – affects the normal force, but doesn't cause any movement.

We can break the force into a part parallel to the displacement and a part perpendicular to the displacement.

The part of the force perpendicular to the displacement doesn't contribute to motion in the direction of the displacement. Because we define work to be the product of the displacement times the force responsible for that displacement, the part of F perpendicular to the displacement does no work because there is no displacement in the y-direction.



So a more accurate definition of work is this:

Work = Force parallel to the displacement * displacement.

so we often write $W = F \cos \theta d$

where θ is the (smallest) angle between the force and the displacement

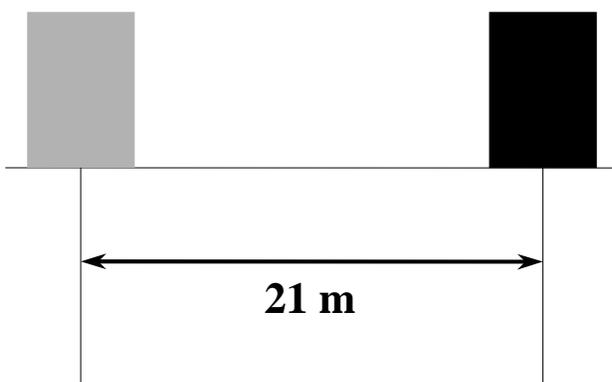
Note that **work is a scalar**. It depends on two vectors (force and displacement), but the way in which they are multiplied together results in a scalar.

The Units of Work

$$[W] = [F][d]$$

$$[W] = [N][m]$$

The units of work are N-m, which we call a Joule. We saw from before that work and energy have the same units, and this definition is consistent with our earlier discoveries.



Ex. 15-2: A 50 N weight is moved a distance of 21 m. Find the work needed to move the box if a) a 125 N force is applied horizontally and b) if the same force is applied at a 60° angle to the horizontal.

In part a, all of the force is applied parallel to the displacement, so the amount of work done is

$$W = F \cos(\theta) x$$

$$W = (125 \text{ N})(\cos (0))(21\text{m})$$

$$W = 2625 \text{ J} = 2600 \text{ J (2 sig. fig.)}$$

In part b, the same force is applied, but at a

60° angle, so the work is now given by

$$\begin{aligned}
 W &= F \cos(\theta) x \\
 W &= (125 \text{ N})(\cos(60))(21 \text{ m}) \\
 &= 1312.5 \text{ J} \\
 &= 1300 \text{ J}
 \end{aligned}$$

So although the person applies the same amount of force each time, less work is done in the case where the force is applied at an angle.

There are other forces acting on this box, including the normal force and gravity. Do they do any work on the box as it moves across the floor?

NO - the normal force and gravity are perpendicular to the displacement. $\cos(90) = 0$, so there is no work done by these forces.

Note that work is done **on** objects or **by** forces and that the sign is important in telling you which object is doing the work and which object is being worked on. Any force can do work on any object.

Ex. 15-3: Let's revisit the box being moved 21 m with an applied force of 125 N applied strictly horizontally, but now let's consider a frictional force of magnitude 43 N. Find a) the work done by the frictional force on the object b) the net force acting in the direction of the displacement and c) the net work done by forces acting in the direction of the displacement.

a) The frictional force will act in the direction opposite the motion. If we write

$$\begin{aligned}
 W &= F \cos(\theta) x \\
 W_{\text{fr}} &= F_{\text{fr}} (\cos\theta) x \\
 W_{\text{fr}} &= (43 \text{ N})(\cos 180)(21 \text{ m}) = -900 \text{ J}
 \end{aligned}$$

b) The net force will be the sum of all the forces acting in the x direction

$$\begin{aligned}
 \Sigma F_x &= F_{\text{fr}} + F_A \\
 \Sigma F_x &= -40 \text{ N} + 125 \text{ N} \\
 \Sigma F_x &= +85 \text{ N}
 \end{aligned}$$

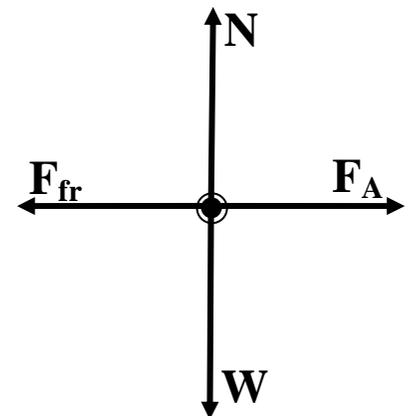
c) We learned from the last example that a 125 N force does a positive 2600 J of work in moving the box across the floor.

The total amount of work done ON the box is:

$$\begin{aligned}
 &2600 \text{ J (from the applied force)} \\
 &-900 \text{ J (from friction)}
 \end{aligned}$$

Total = 1700 J of work

Note that since this is positive, it means that there is net work being done **on** the box. Note that the net work done on the box is the work done due to all of the forces acting on the box – in chapter 4, we called this the **NET FORCE**. Be very careful in working the problems to make sure you know which forces are acting on which objects.



Energy is the ability to do work

Energy is yet another one of those words that has a different meaning in physics than the way we normally use it.

There are many different kinds of energy:

Type of energy	example
Mechanical energy	falling mass on a piledriver
Chemical energy	a candle, gunpowder, a battery
Thermal energy	steam
Electrical Energy	From a lightning bolt
Nuclear energy	Energy stored in the nucleus of an atom

Conservation of Energy I.

Energy cannot be created or destroyed – it can only be changed from one form to another. This is called conservation of energy and it is a very important principle. If you think, for example, about an archer releasing an arrow, there is energy stored in the bow when it is pulled back. That energy is used to give kinetic energy to the arrow. We will discuss using the principle of conservation of energy in a later lecture. For now, we're going to talk about different types of energy.

Mechanical Energy

For the moment, we're going to concentrate on **mechanical** energy, as opposed to electrical, thermal or other types of energy. There are two primary types of mechanical energy: kinetic and potential.

Kinetic Energy

Kinetic energy is **energy of motion**. Translational kinetic energy is the kinetic energy something has because it is changing its position. Objects can have other types of kinetic energy. For example, something can be standing in one place, but rotating. We say that this object has rotational kinetic energy. We're only going to be dealing with translational kinetic energy for the moment, so we will just refer to it as 'kinetic energy'. We'll make a distinction later if it is necessary. The amount of kinetic energy a moving object has is given by:

$$KE = \frac{1}{2}mv^2$$

Let's examine the units for energy:

$$\begin{aligned}
\text{KE} &= \frac{1}{2}mv^2 \\
&= (\text{kg})\left(\frac{\text{m}}{\text{s}}\right)^2 \\
&= \frac{\text{kg m}^2}{\text{s}^2} \\
&= \frac{\text{kg m}}{\text{s}^2} \text{ m} \\
&= \text{N m}
\end{aligned}$$

So the units of kinetic energy – and all energy for that matter – are Newtons times meters. We have a special name for these units:

$$1\text{J} = 1\text{N} \cdot \text{m}$$

where J stands for Joule. (pronounced “jewel”)

There are other units used for energy, and they vary depending on the application. So, for example, if you are talking about energy, the unit “kW-h” (kiloWatt-hour) is used. The Calorie is also a unit of energy. To give you an idea of magnitudes:

Total amount of energy used in one year in the US in 1990	$8.6 \times 10^{16} \text{ J}$
Energy generated by Hoover Dam in one year	$2 \times 10^{16} \text{ J}$
Energy from burning 1 gallon of gasoline	$2 \times 10^8 \text{ J}$
Energy of one calorie	4 J
Energy of one molecule of air	$6 \times 10^{-21} \text{ J}$

The Work-Energy Theorem

EXAMPLE 15-4: You start running at a pace of 1.5 m/s. As you warm up, you decide to run faster – say 3.0 m/s. By how much does your kinetic energy change and what causes it to change? Assume that your mass is $7.0 \times 10^1 \text{ kg}$.

When you were running 1.5 m/s, your kinetic energy was:

$$\begin{aligned}
\text{KE}_1 &= \frac{1}{2}mv_1^2 \\
&= \frac{1}{2}(70\text{kg})\left(1.5\frac{\text{m}}{\text{s}}\right)^2 \\
&= 79\text{J}
\end{aligned}$$

After you have increased your velocity, your new kinetic energy is:

$$\begin{aligned}
\text{KE}_2 &= \frac{1}{2}mv_2^2 \\
&= \frac{1}{2}(70\text{kg})\left(3.0\frac{\text{m}}{\text{s}}\right)^2 \\
&= 320\text{J}
\end{aligned}$$

So your kinetic energy increased by $320 \text{ J} - 79 \text{ J} = 240 \text{ J}$ (to 2 significant figures)

What caused this increase in kinetic energy? Your body had to do some **work**. Your body converted the energy stored in your muscles and fat into kinetic energy. The amount of work done is equal to the amount of the energy change. If you want to increase your kinetic energy by 320 J, you have to do 320 J of work. This leads us to our first definition of work

Work is the process of transferring energy

Mathematically, we would say:

$$W = \Delta KE$$

$$W = KE_2 - KE_1$$

$$W = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

Or that the work done is equal to the change in kinetic energy. This is called the **work-energy theorem**.

In this example your body did work. It is also possible for another object to do work on your body.

Ex. 15-5: You are not watching where you are going and you run into a building (bringing your velocity to 0 rather quickly). What work is done and is it done by or on you?

In the beginning, you have a velocity of 3.0 m/s and a kinetic energy of 320 J, as we calculated in Example 17-1. When you hit the building, you have a velocity of zero and a kinetic energy of zero.

$$W = \Delta KE$$

$$W = KE_2 - KE_1$$

$$W = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

$$W = 0\text{J} - 320\text{J}$$

$$W = -320\text{J}$$

In this case, your velocity decreased, so your kinetic energy decreased. The minus sign indicates that work was done on you (or, alternately, work was done by the wall).

Ex. 15-6: How much work was done by the brakes of a 1500 kg car in slowing it from 15 m/s to 5 m/s. What was the force exerted by the brakes on the car? We used the work-energy theorem to find:

$$W = KE_2 - KE_1$$

$$W = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

$$W = \frac{1}{2}(1500\text{ kg})(5\frac{\text{m}}{\text{s}})^2 - \frac{1}{2}(1500\text{ kg})(15\frac{\text{m}}{\text{s}})^2$$

$$W = -1.5 \times 10^5 \text{ J}$$

We can use our definition of work to determine the force of friction was if we know how far the car traveled (its displacement) while its velocity was changing from 15 m/s to 5 m/s. Let's say that the change occurs in 37 m.

$$W = F_{fr} \cdot x$$

$$\frac{W}{x} = F_{fr}$$

$$\frac{-1.5 \times 10^5 \text{ J}}{37 \text{ m}} = F_{fr}$$

$$-4.1 \times 10^3 \text{ N} = F_{fr}$$