PHYSICS 151 – Notes for Online Lecture #14

In this lecture we will consider objects moving along circular paths at constant speed -- this is referred to as uniform circular motion. From Newton's first law you know that there must be a force acting on this object, otherwise it would move in a straight line instead of a circle. We call this the centripetal (center seeking) force.

We will need a few new equations to handle uniform circular motion. If an object moves a distance s around a circular path of radius r, this arc length is related to the angle through which the particle moves through

$$\theta = \frac{s}{r}$$

Where θ is measured in radians. 360º=2π, so 1 radian = 57.3º

Similarly, if an object moves with velocity v around a circular path of radius r, this velocity is related to the angular velocity ω through which the particle moves through

$$\omega = \frac{v}{r} = \frac{2\pi}{T} = 2\pi f$$

Where T is the period of motion in seconds and f the frequency of motion in s^{-1} or Hertz.

The centripetal acceleration is given by $a_c = \frac{v^2}{r} = \omega^2 r$

The centripetal force always points toward the center of the circle. It is not a new force - it is made up of forces with which you are already familiar such as weight, gravity, tension, and friction.

Thus, whenever an object moves in a circle there is a centripetal force with magnitude $m\frac{v^2}{r}$ pointing toward the center of the circle which causes this motion. Centripetal force is not a new force! It is made up of forces with which we are already familiar – gravity, friction, tension, and the normal force.

Ex. 15-1 - Jupiter's moon Europa has an average orbital radius of 6.67×10^8 m and a period of 85.2 h. Calculate the magnitude of (a) its average orbital speed, (b) the angular velocity and (c) the centripetal acceleration of Europa.

$$v = \frac{2\pi r}{T} = \frac{2\pi\left(6.67 \times 10^8 m\right)}{\left(85.2h\right)\left(\frac{3600s}{1hr}\right)} = 1.37 \times 10^8 m$$

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{\left(85.2h\right)\left(\frac{3600s}{1hr}\right)} = 2.05 \times 10^{-5}\frac{rad}{s}$$

$$a_c = \omega^2 r = \frac{4\pi^2 r}{T^2} = 0.280\frac{m}{s^2}$$

Ex 15-2 - A stunt driver drives a car so fast that it leaves the ground as it tops a hill. If the hill can be approximated by a 165-m-radius vertical circle, what speed must the car exceed if it is to leave the ground?

First draw a free body diagram for the car.

$$\sum F_r = mg - N = m\frac{v^2}{r}$$

Assume that we solve for the situation where the car just stays on the road -- that the normal force is zero.

$$\sum F_r = mg = m\frac{v^2}{r}$$

$$g = \frac{v^2}{r}$$

$$v = \sqrt{gr} = \sqrt{\left(9.81\frac{m}{s^2}\right)(165m)} = 40.2\frac{m}{s}$$

Thus, if velocity is any larger than this, the car will leave the surface.

*** Very similar ideas apply to a ball on a loopty-loop track.

Ex 15-3 - What angle does a plumb bob line make with the vertical in a train rounding a 300-m-radius curve at 27 m/s?

First draw the free body diagram (which must include gravity and tension).

$$\sum F_y = T\cos\theta - mg = 0$$

$$T\cos\theta = mg$$

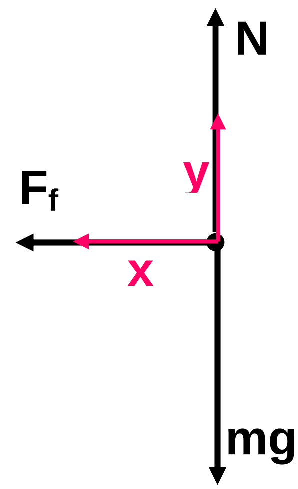
$$\sum F_x = T\sin\theta = m\frac{v^2}{r}$$

$$\tan\theta = \frac{v^2}{gr}$$

$$\theta = \tan^{-1}\frac{v^2}{gr} = \tan^{-1}\frac{\left(27\frac{m}{s}\right)^2}{\left(9.81\frac{m}{s^2}\right)(300m)} = 13.9°$$

Ex 15-4 - A coin placed on a turntable rotating at 33.3 rev/min will stay there if its center is placed no further than 8.5 cm from the axis of rotation. What is the coefficient of friction between the coin and turntable?

The free body diagram shows that the frictional force here must be the centripetal force.



$$\sum F_r = \mu mg = m\frac{v^2}{r}$$

$$\mu = \frac{v^2}{gr}$$

where the velocity is

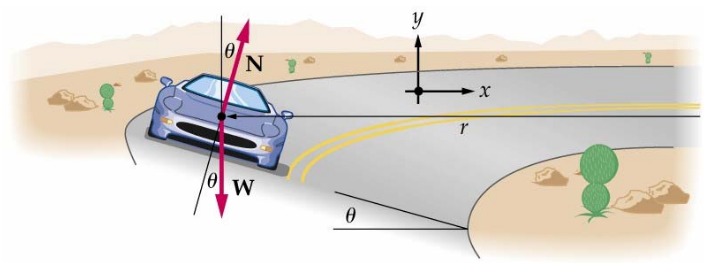
$$v = 2\pi rf = 2\pi(0.085m)(33.3/\min)\left(\frac{1\min}{60s}\right) = 0.296\frac{m}{s}$$

$$\mu = \frac{v^2}{rg} = \frac{\left(0.296\frac{m}{s}\right)^2}{(0.085m)\left(9.81\frac{m}{s^2}\right)} = 0.11$$

*** The ideas above are similar to what goes on in the Rotor – the amusement park ride where people are spun in a cylinder and the floor drops out.

Banked Roads

-- we would like the car to stay on the road without any help from friction since that comes and goes with the weather and tire conditions. At what angle should the road be banked for speed v and radius r?



$$\sum F_y = N\cos\theta - mg = 0$$

$$N = \frac{mg}{\cos\theta}$$

$$\sum F_x = N\sin\theta = m\frac{v^2}{r}$$

$$\frac{mg}{\cos\theta}\sin\theta = m\frac{v^2}{r}$$

$$\tan\theta = \frac{v^2}{gr}$$

$$\theta = \tan^{-1}\frac{v^2}{gr}$$

Thus, banked roads are designed for specific speeds and when drivers exceed those speeds they are depending on friction to help keep them on the road.