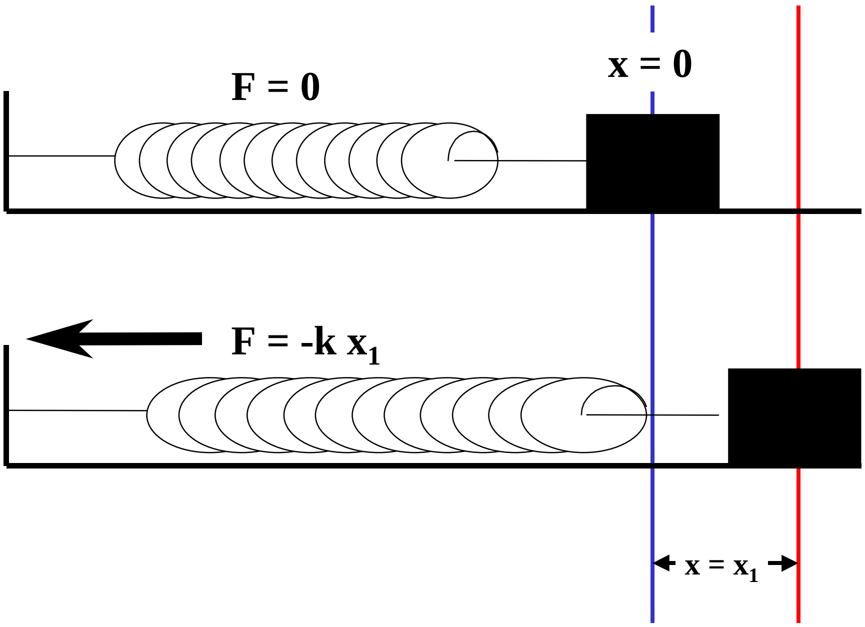
PHYSICS 151 – Notes for Online Lecture #13

Hook's Law

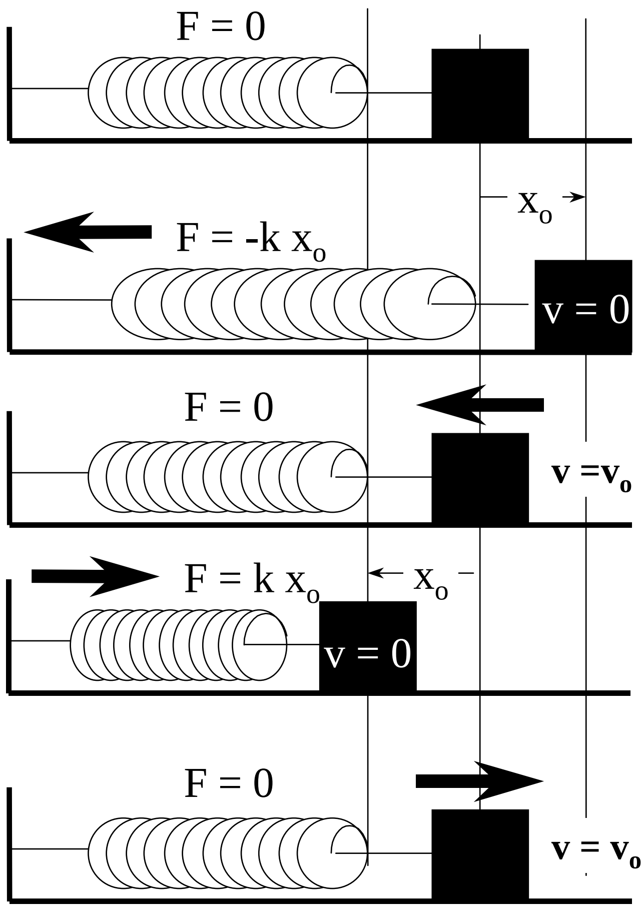


Let's start with a horizontal spring, resting on a frictionless table.

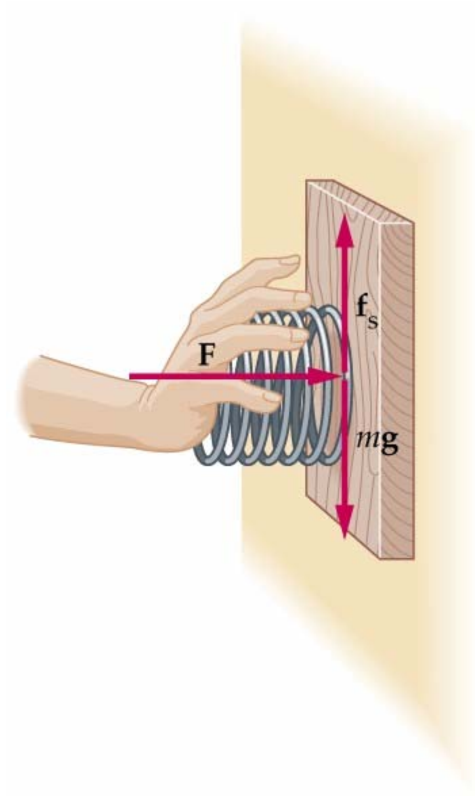
We pick a reference point on the mass –for example, the center of the mass. The position of the center of the mass when the spring is unstretched is called the '**equilibrium point'** (x = 0). Now I pull the spring an arbitrary distance **x** to the right. The spring exerts a force in the direction opposite the displacement (to the left in this case). The force is given by Hooke's Law:

$$F = -kx$$

where k is the spring constant and has units of N/m. If I pull the spring to the right, the spring exerts a force to the left. Alternately, I can push the spring in a distance x. Now the spring exerts a force toward the *right*. Remember that Hooke's law only works when the displacements are small. If you make a very large displacement, Hooke's law doesn't apply anymore and none of what I'm about to tell you will apply either.



Ex. 13-1 – A spring with a force constant of 120 N/m is used to push a 0.27 kg block of wood against a wall, as shown. (a) Find the minimum compression of the spring needed to keep the block from falling given that the coefficient of static friction between the block and the wall is 0.46. (b) Does your answer to part (a) change if the mass of the block of wood is doubled?



(a) $\sum F_y = 0$

$$f_s - mg = 0$$
$$\mu_s F_s - mg = 0$$
$$\mu_s(-kx) - mg = 0$$
$$x = \frac{-mg}{\mu_s k}$$
$$x = \frac{-(0.27\text{ kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)}{(0.46)\left(120\ \frac{\text{N}}{\text{m}}\right)} = \boxed{-4.8\text{ cm}}$$

(b) Yes; the spring displacement is proportional to the block's mass.

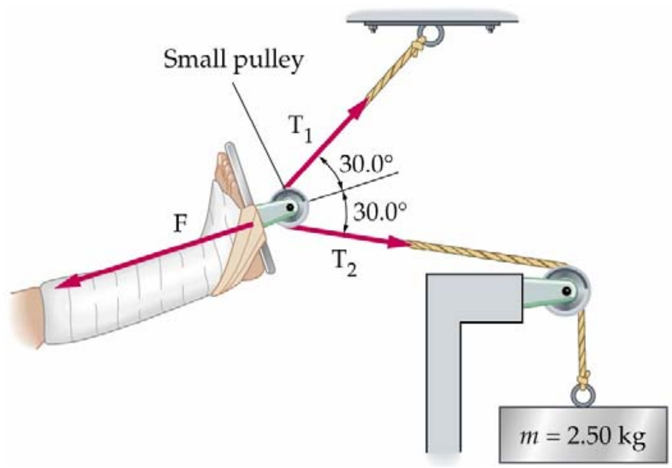
Strings - in a taut string tension pulls equally to the right and left. If the string were cut, tension is the force that would be required to hold the string together.

Pulleys - strings are often used in conjunction with pulleys. A string has the same tension throughout its length and the pulley simply serves to redirect the tension.

Ex. 13-2 – What is the stretching force applied to the broken limb?

The tension in the rope is equal to the weight of the mass = (2.50 kg) (9.8 m/s2) = 24.5 N.

Thus, summing forces about the pulley at the foot yields:



$$\Sigma F_x = T_1 \cos 30° + T_2 \cos 30° - F = 0$$
$$F = 2T\cos 30° = 2(2.50kg)\left(9.81\frac{m}{s^2}\right)\cos 30° = 42.5N$$

Statics is the analysis of situations in which there is no acceleration. There are many applications, including buildings, people's bones when they are in traction, and more.

Statics is a fancy way of saying:

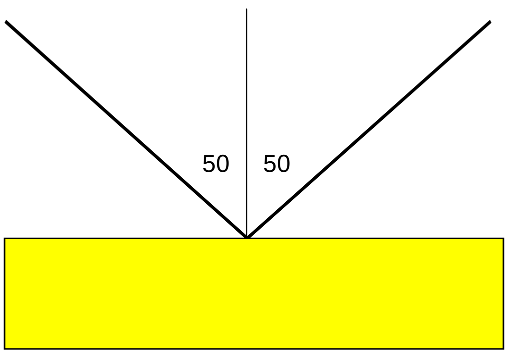
$$\sum \vec{F} = 0$$

Remember though, that this is actually two equations, because it is a vector equation:

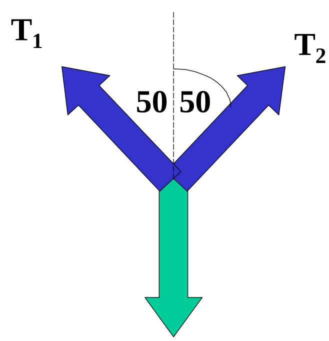
$$\sum F_x = 0$$
$$\sum F_y = 0$$

BOTH of these conditions must be true to have static equilibrium. Note also that we are talking only about translational equilibrium – we haven't discussed rotation yet.

Ex. 13-3: A 20.0 kg sign hangs as shown. What are the tensions in the two wires?



The first thing to do is to identify a point that is in static equilibrium. This point must have the forces that are unknown acting on it. We will choose the point at which the strings meet the sign. The free-body diagram will look like:



We know we will have to work with both components (x and y), so the first thing to do is to break each of the tensions into components.

Remember that for strings, beams, etc., the tension will <u>always</u> be along the string.

T_{1x} = $-T_1$ sin(50)	T_{1y} = $-T_1$ cos(50)
T_{2x} = T_2 sin(50)	T_{2y} = $-T_2$ cos(50)

Now apply Newton's second law in both the x and the y directions. Remember that the acceleration in all directions must be zero.

$\sum F_x = 0$	$\sum F_y = 0$
$-T_1 \sin(50) + T_2 \sin(50) = 0$	$T_1 \cos(50) + T_2 \cos(50) - mg = 0$
$T_1 \sin(50) = T_2 \sin(50)$	

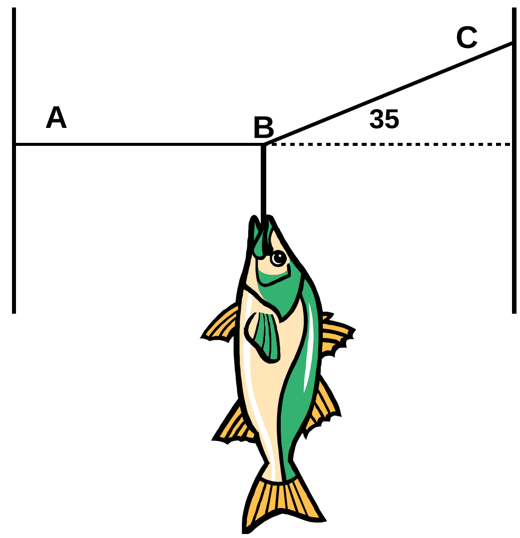
The equation in the x direction tells us that

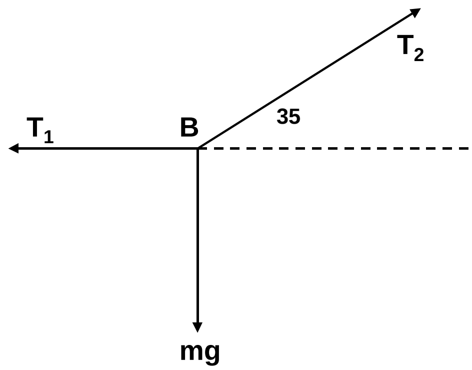
$T_1 = T_2$

We might have already guessed this, because the two angles are equal. Use this in the y-equation. Call $T_1 = T_2 = T$

	$2T\cos(50) = mg$ $T = \dfrac{mg}{2\cos(50)}$ $T = \dfrac{(20.0\,\text{kg})\left(9.80\,\frac{\text{m}}{\text{s}^2}\right)}{2\cos(50)} = 152\,\text{N}$

Ex. 15-2: A fish of mass m is suspended as shown. The string AB will break when the tension exceeds 10.0 N. What is the maximum mass fish that can be supported?





Start by drawing a free body diagram. Since we want to learn something about the tension in AB, we need to draw the FBD for a point on which that force acts. In this case, point B is the best point. First, find all of the components of the forces.

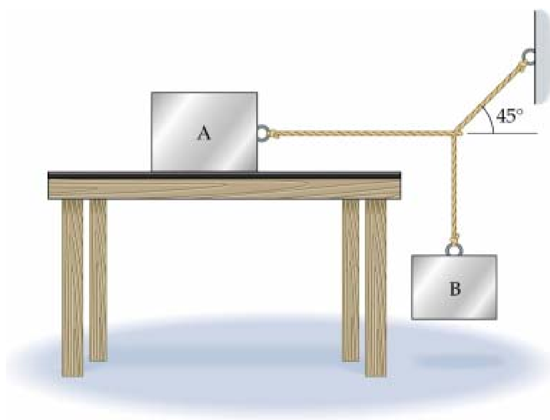
$T_{1x} = -T_1$	$T_{1y} = 0$
$T_{2x} = T_2 \cos(35)$	$T_{2y} = T_2 \sin(35)$

Then apply Newton's second law for the case of a = 0.

$\sum F_x = 0$	$\sum F_y = 0$
$-T_1 + T_2\cos(35) = 0$	$T_2\sin(35) - mg = 0$
$T_1 = T_2\cos(35)$	
The maximum that T_1 can be is 10.0 N, so	
$\dfrac{T_1}{\cos(35)} = T_2$ $\dfrac{10.0\,\text{N}}{\cos(35)} = T_2$ $12.2\,\text{N} = T_2$	
Now use this in the y-equations	

	$T_2 \sin(35) = mg$ $\frac{T_2 \sin(35)}{g} = m$ $\frac{(12.2\,N)\sin(35)}{9.8\,\frac{m}{s^2}} = m$ $0.714\,kg = m$

The system shown is in equilibrium. (a) Find the frictional force exerted on block A given that the mass of block A is 8.50 kg, the mass of block B is 2.25 kg, and the coefficient of static friction between block A and the surface on which it rests if 0.320. (b) If the mass of block A is doubled, does the frictional force exerted on it increase, decrease, or stat the same?



(a) Let T be the tension in the slanting stretch of rope. Then $T \sin 45°$ is the tension in the rope supporting mass B, and $T \cos 45°$ is the tension in the rope pulling on mass A. But $\sin 45° = \cos 45°$, and so

$$f_{\text{s on A}} = T \cos 45° = T \sin 45° = m_B g = (2.25\,\text{kg})\left(9.81\,\frac{\text{m}}{\text{s}^2}\right) = \boxed{22.1\,\text{N}}$$

which is below

$$f_{\text{s, max}} = \mu_s m_A g = 0.320(8.50\,\text{kg})\left(9.81\,\frac{\text{m}}{\text{s}^2}\right) = 26.7\,\text{N}.$$

(b) So long as mass A is heavy enough for $f_{\text{s, max}} \geq 22.1\,\text{N}$, f_s is not affected by changes in mass A; f_s stays the same.