PHYSICS 151 – Notes for Online Lecture #12

Friction: The basic facts of macroscopic (everyday) friction are:

1) Frictional forces depend on the two materials that are sliding past each other. A box sliding over a waxed floor experiences less friction than a box sliding over an unwaxed floor.

2) There are two types of friction for most surfaces: static friction and kinetic (or sliding) friction.

3) The frictional force, F_{fr} always acts in the direction opposite the direction of motion.

4) The frictional force does not depend on the surface area of the objects in contact.

Static friction is the resistance an object has to starting to move. For many interfaces, we must pull on an object with some moderate force before the object begins to move.

For the kinetic case

$$F_{fr(kinetic)} = \mu_k N \quad \text{with direction opposite to the direction of motion}$$

Kinetic friction is the friction experienced by a moving object. Once motion begins, the frictional force decreases and we can exert a smaller force on the object (to balance the kinetic frictional force) to keep the object moving at constant velocity. For many interfaces the dry sliding frictional force is roughly independent of the relative speed of the object over the surface. For surfaces lubricated with liquids or for viscous friction, (as for a boat speeding across a lake) there is some speed dependence.

For the static case

$$F_{fr(static)} \leq \mu_s N$$

Everything we know about friction is summarized in a parameter called the coefficient of friction. The coefficient of friction is different for the static and kinetic cases -- μ_s is the coefficient of static friction. μ_k is the coefficient of kinetic friction. Both of these numbers are always less than 1. For example, steel moving along steel has a kinetic frictional coefficient around 0.6. If you lubricate the surfaces, you can reduce the coefficient of kinetic friction to around 0.2.

Also, you might notice that it is often harder to get the object moving than to keep it moving at a constant velocity. This means that the maximum value of the static coefficient of friction is always greater than the kinetic coefficient of friction.

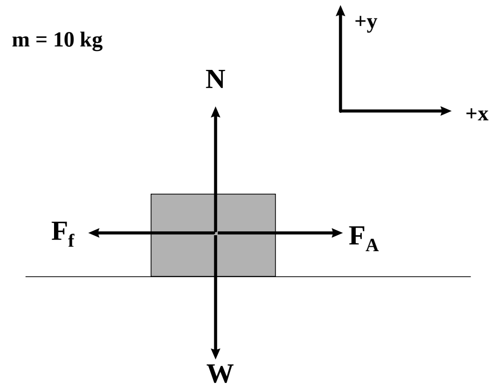
Notice that in the equation for the static frictional force, there is a less than or equal to sign.

$$F_{fr(static)} \leq \mu_s N$$

The static frictional force is a strangely adaptable force. If I push lightly on a heavy box sitting on the floor, the box does not move. I can push a little harder and still there is no motion. Since there is no acceleration in either of these cases, the static frictional force must adapt itself to just balance whatever force I apply to the box so that the net force on the box is zero.

An investigation of friction naturally leads to wondering what happens at the interface at the atomic level. We know that friction is different for different materials; so the particular structure of each material must be significant. However, we will entirely focus on the macroscopic, where blocks of materials can be characterized by a coefficient of friction.

Ex. 12-1: A box of mass 10.0 kg is pulled at an acceleration of 2.0 m/s^2 with the force applied strictly horizontally. If the coefficient of friction is 0.20, what force is now needed to maintain this acceleration?



x-direction	y-direction
$\sum F_x = ma_x$ $F_A - F_{fr(kinetic)} = ma_x$ $F_A - \mu_k N = ma_x$	$\sum F_y = ma_y$ $0 = N - W$
We now need to find the magnitude of the Normal Force. We do this from the y-equations	
	$0 = N - W$ $N = W = mg$
Plus this result into the x-equations	
$F_A = \mu N + ma_x$ $F_A = \mu mg + ma_x$ $F_A = m(\mu g + a_x)$ $F_A = 10.0\,kg\left(0.20(9.8\tfrac{m}{s^2}) + 2.0\tfrac{m}{s^2}\right)$ $F_A = 39.6\,N = 40\,N$	

In the frictionless case we would only need to apply a force of 20 N (let μ = 0 to see this), so we have to apply an additional 20 N to overcome friction.

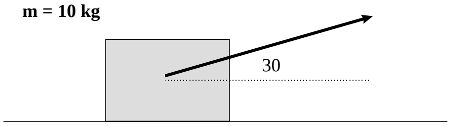
Ex. 13-2: Let's return to the box of mass 10.0 kg pulled at an acceleration of 2.0 m/s^2 but now with the force applied at an angle of 30 degrees with the horizontal.

a) If the coefficient of friction is 0.20, what force is now needed to maintain this acceleration?

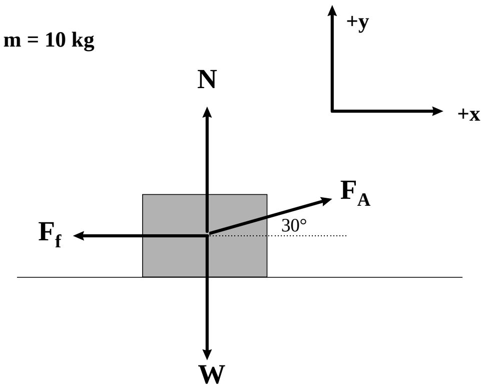
b) What is the magnitude of the normal force?

A box of mass 10 kg is pulled at an acceleration of 2 m/s^2. If the force is applied at an angle of 30° with respect to the horizontal and the coefficient of friction is 0.2, what force is necessary to maintain this acceleration?

m = 10 kg



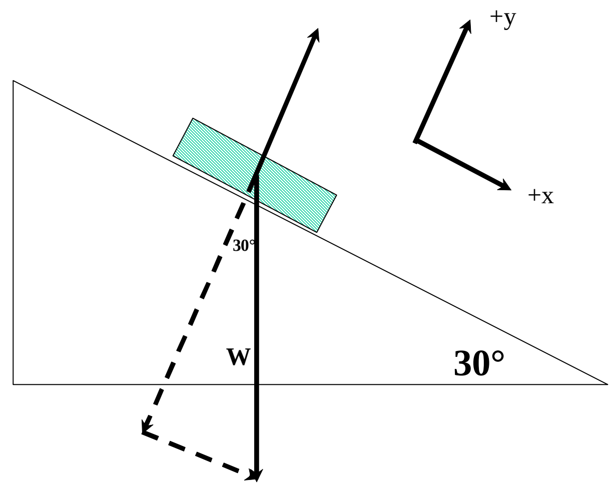
m = 10 kg



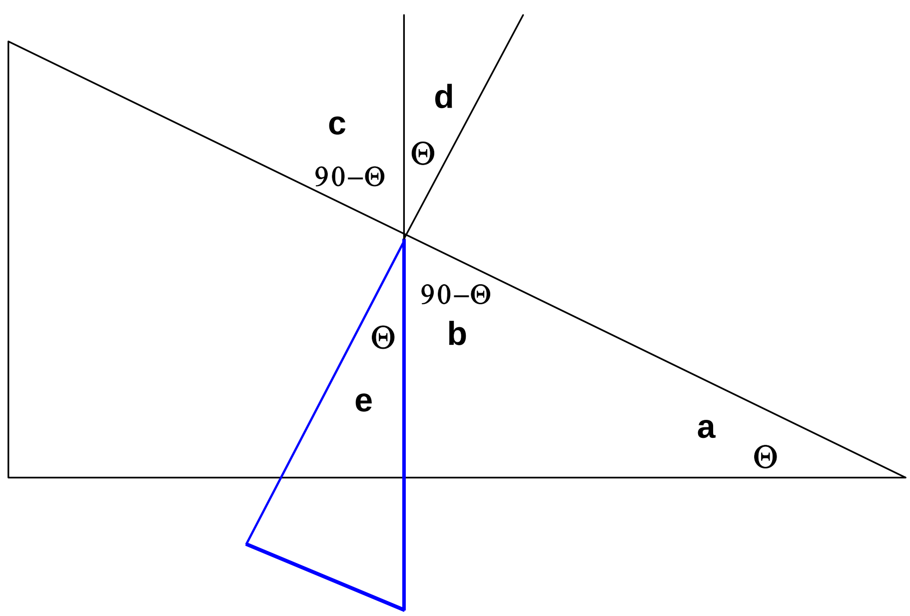
x-components	**y-components**
Break the applied force into components	
$F_{Ax} = F_A \cos(30)$	$F_{Ay} = F_A \sin(30)$
Write Newton's Second Law for each direction	
$\sum F_x = ma_x$ $F_{Ax} - F_f = ma_x$ $F_A \cos(30) - \mu N = ma_x$	$\sum F_y = ma_y$ $N - W + F_{Ay} = 0$
To get the applied force, we need the normal force	
	$N = W - F_{Ay}$ $N = mg - F_A \sin(30)$
Plug this result back into the x-equation	
$F_A \cos(30) - \mu N = ma_x$ $F_A \cos(30) - \mu(mg - F_A \sin(30)) = ma_x$ $F_A(\cos(30) + \mu \sin(30)) = ma_x + \mu mg$ $F_A(\cos(30) + \mu \sin(30)) = m(a_x + \mu g)$ $F_A = \frac{m(a_x + \mu g)}{(\cos(30) + \mu \sin(30))}$	
$F_A = \frac{(10\,\text{kg})\left(2.0\,\frac{\text{m}}{\text{s}^2} + (0.20)\left(9.8\,\frac{\text{m}}{\text{s}^2}\right)\right)}{(\cos(30) + 0.20 \sin(30))}$ $F_A = \frac{39.6\,\text{N}}{(0.966)} = 41\,\text{N}$	
Compare this to the result if there is no friction (23 N). Now use this to find the magnitude of the normal force	
	$N = W - F_{Ay}$ $N = +mg - F_A \sin(30)$ $N = (10\text{kg})\left(9.8\,\frac{\text{m}}{\text{s}^2}\right) - 41\,\text{N} \sin(30)$ $N = 78\,\text{N}$
Note that the normal force is reduced due to the applied force being upward.	

Ex. 12-3: A book is on an inclined plane. Is the normal force:

a) greater than mg
b) less than mg
c) equal to mg

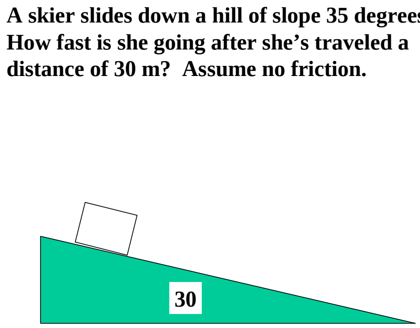


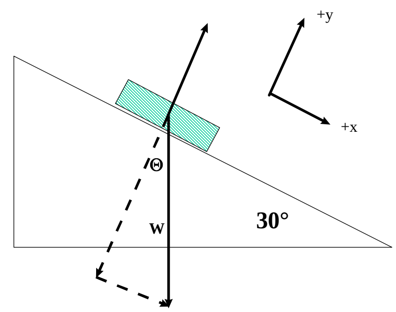
As shown in the free body diagram below, the normal force will be balanced by the component of the weight parallel to the normal direction. For inclined plane problems, it's often easier to change the coordinate axes, as shown. The component of the weight in the y direction will be proportional to the cosine of the angle marked θ. Regardless of what θ is, the sin or cos will be less than one, so the resulting component will be less than the value of the weight. So the normal force will be less than mg. In our last example, there were components in both directions, x and y. Remember from our analysis of motion in 2D that the x and the y components act independently. In the free body diagram I've shown above, what must be happening to the book? Answer is: it must be accelerating, as we have $\sum \vec{F}_x = F\sin\theta = ma$ Now you know why understanding vectors is so important!



We are given that the plane is at an angle of θ (angle a). Looking at the right triangle formed by the inclined plane and the weight vector, angle b must be 90-θ. By the alternate interior angle theorem (I think that's the theorem), the angle directly opposite this angle (c) must also be 90-θ. Then angle d must be 90 – (90-θ) = θ. Using the same theorem, the angle that we're interested in – angle e –must be θ.

Ex. 12-4: A skier slides down a hill of slope 30 degrees. She starts at rest. How fast is she going by the time she's traveled 30 m. Assume no friction.





With this choice of coordinate axes, all of the motion is taking place in the x-direction.

$$v_x{}^2 = v_{x0}^2 + 2a_x x$$
$$v^2 = 2a_x x$$

But we have to know a_x!

$$\sum F_x = ma_x$$

The Normal force is entirely along the y-axis; however, the weight is not entirely along either axis, so we have to decompose the weight.

From the picture, $W_x = +W \sin\theta$
$W_y = -W \cos\theta$

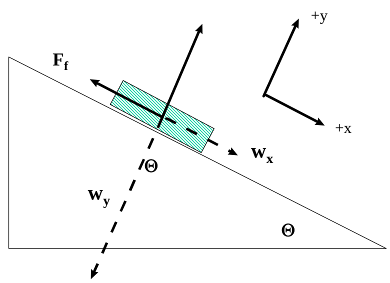
$$\sum F_x = ma_x$$
$$W_x = ma_x$$
$$W \sin\theta = ma_x$$
$$\frac{W \sin\theta}{m} = a_x$$
$$\frac{mg \sin\theta}{m} = a_x$$
$$g \sin\theta = a_x$$
$$\left(9.8\tfrac{m}{s^2}\right)\sin 30 = a_x$$
$$4.9\tfrac{m}{s^2} = a_x$$

Now put this back in the equation for velocity:

$$v^2 = 2a_x x$$
$$v^2 = 2(4.9\tfrac{m}{s^2})(30m)$$
$$v^2 = 294\tfrac{m^2}{s^2}$$
$$v = 17.1\tfrac{m}{s}$$

Now let's include friction. Assume the coefficient of friction is 0.15 and find the velocity after 30 m.

What changes? All we have to do is to add the frictional force to the free body diagram.



$$\sum F_x = ma_x$$
$$W_x - F_f = ma_x$$
$$mg\sin\theta - \mu N = ma_x$$

$$\sum F_y = ma_y$$
$$-W_y + N = 0$$
$$-W\cos\theta + N = 0$$
$$W\cos\theta = N$$

But now we have to find the normal force.

$$mg\sin\theta - \mu mg\cos\theta = ma_x$$
$$g\left(\sin\theta - \mu\cos\theta\right) = a_x$$
$$\left(9.8\tfrac{m}{s^2}\right)\left(\sin 30 - 0.15\cos 30\right) = a_x$$
$$3.63\tfrac{m}{s^2} = a_x$$

$$v^2 = 2a_x x$$
$$v^2 = 2(3.63\tfrac{m}{s^2})(30m)$$
$$v^2 = 218\tfrac{m^2}{s^2}$$
$$v = 14.8\tfrac{m}{s}$$