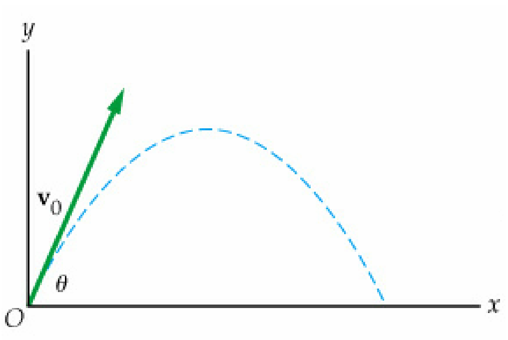
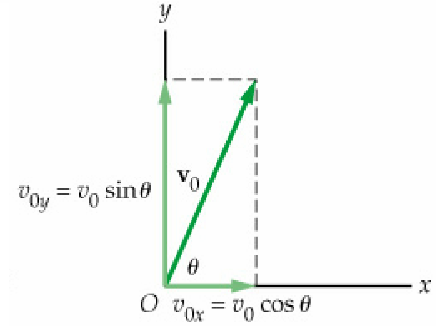
PHYSICS 151 – Notes for Online Lecture #9

Projectile Motion

In this lecture we will look at projectile problems with a general launch angle – projectile launched with velocity v_0 at angle θ.

We will one again make use of the fact that the horizontal and vertical motions are entirely independent. If we break the initial velocity into horizontal and vertical components we can treat the two motions separately with different initial velocities.





The x component of velocity is $v_{0x} = v_0\cos\theta$

The y component of velocity is $v_{0y} = v_0\sin\theta$

Ex. 9-1 A cork shoots out of a champagne bottle at an angle of 40.0° above the horizontal. If the cork travels a horizontal distance of 1.50 m in 1.25 s, what was its initial speed?

We can ignore the vertical motion and just consider the horizontal motion as due to the horizontal component of the initial velocity.

$$v_x = \frac{x}{t} = \frac{1.50\text{ m}}{1.25\text{ s}} = 1.20\ \frac{\text{m}}{\text{s}} = v_{0x}$$
$$v_0 = \frac{v_{0x}}{\cos\theta} = \frac{1.20\ \frac{\text{m}}{\text{s}}}{\cos 40.0°} = 1.57\ \text{m/s}$$

Ex. 9-2 The "hang time" of a punt is measured to be 4.50s. If the ball was kicked at an angle of 63.0° above the horizontal and was caught at the same level from which it was kicked, what was its initial speed?

The maximum height is achieved at time $t = \frac{1}{2}(4.50\text{ s}) = 2.25\text{ s}$,

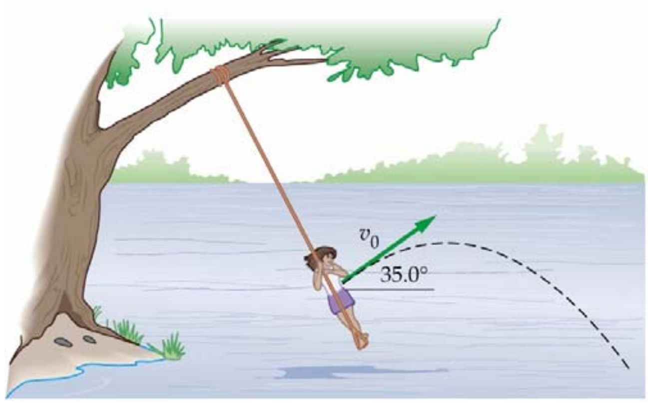
and at that time $v_y = 0$.

So considering the motion from that time on and since $v_y = v_{0y} - gt$,

$$v_{0y} = \left(9.81\frac{\text{m}}{\text{s}^2}\right)(2.25\text{s}) = 22.07\ \frac{\text{m}}{\text{s}}$$

So, $v_0 = \dfrac{22.07\ \frac{\text{m}}{\text{s}}}{\sin 63.0°} = 24.8\ \text{m/s}$.

Ex 9-3: On a hot summer day, a young girl swings on a rope above the local swimming hole. When she lets go of the rope her initial velocity is 2.25 m/s at an angle of 35.0° above the horizontal. If she is in flight for 1.60 s, how high above the water was she when she let go of the rope?



$$v_{oy} = v_0 \sin\theta$$
$$= \left(2.25\frac{m}{s}\right)\sin(35°)$$
$$= 1.29\frac{m}{s}$$

Known: v_{0y} = 1.29 m/s   Solve: y   NI: v_y

a = -9.81 m/s^2

t = 1.60 s

$$y = v_{oy}t + \frac{1}{2}at^2$$
$$= \left(1.29\frac{m}{s}\right)(1.60s) + \frac{1}{2}\left(-9.81\frac{m}{s^2}\right)(1.60s)^2$$
$$= -10.5m$$

What was the girl's greatest height above the water?

$$v_y^2 = v_{0y}^2 + 2ay$$
$$y = \frac{-v_{0y}^2}{2a} = \frac{-\left(1.29\frac{m}{s}\right)^2}{2\left(-9.81\frac{m}{s^2}\right)} = 0.08m$$

So adding this to 10.5 m gives only 10.6 m.

Ex. 9-4 : A football is kicked at an angle of 30 degrees to the horizontal with an initial velocity of 24.0 m/s. How far does it go?

At t_o		At t	
$x_0 = 0$	$y_0 = 0$	$x = R$	$y = h$
$v_{x0} = v_o \cos\theta$	$v_{y0} = v_o \sin\theta$	$v_x = ?$	$v_y = ?$
$a_x = 0$	$a_y = -9.8\ m/s^2$		

$v_{x0} = v_o \cos\theta = (24.0\ m/s \cos(30)) = 20.8\ m/s$
$v_{y0} = v_o \sin\theta = (24.0\ m/s \sin(30)) = 12.0\ m/s$
At the top of the motion, $v_y = 0$ (note that v_x is not 0)

$$v_y = v_{y0} + a_y t$$

$$\frac{v_y - v_{y0}}{a_y} = t = \frac{0 - 12.0\frac{m}{s}}{-9.8\frac{m}{s^2}} = 1.22s$$

From this, we can find out how far it goes in the x direction:

$$x = x_0 + v_{x0}t + \frac{1}{2}a_x t^2$$

$$x = v_{x0}t = 20.8\tfrac{m}{s}(1.22s) = 25.4m$$

This is half as far as it goes, so the total length is 50.8 m

Note that the time in the air T = 2.44 s is $T = \dfrac{2v_0 \sin\theta}{g}$

And the total distance traveled R (the horizontal range) is $R = v_0(\cos\theta)T$

Substituting the first equation into the second yields the **horizontal range formula**:

$$R = \frac{2v_0^2}{g}\sin\theta\cos\theta = \frac{v_0^2}{g}\sin 2\theta$$

Which in this example yields

$$R = \frac{v_0^2}{g}\sin 2\theta = \frac{(24m/s)^2}{(9.8m/s^2)}\sin 60^\circ = 50.9m$$

Now we can ask the question: At what angle should one kick a ball to get the maximum horizontal range? We want the sin function to have its maximum value of 1 which occurs when the argument is 90°. If 2θ = 90°, then θ = 45°.

The maximum height can also be defined. $v_y^2 = v_{0y}^2 + 2ay$

$$y_{\max} = \frac{v_{0y}^2}{2a} = \frac{v_0^2 \sin^2\theta}{2g}$$