PHYSICS 151 – Notes for Online Lecture #8

In the real world, we don't exist in just one dimension ⇒ We need to describe how things move in two and three dimensions! We will use subscripts for the velocities and accelerations to make it clear when we're referring to the x direction and when we're referring to the y-direction.

x	y
$a_x = 0$	$v_y = v_{y0} + a_y t$
$x = v_x t$	$y = v_{y0}t + \frac{1}{2}a_y t^2$
	$v_y{}^2 = v_{y0}^2 + 2a_y(y - y_0)$
	$y = \bar{v}_y t = \frac{(v_{oy} + v_y)}{2}t$

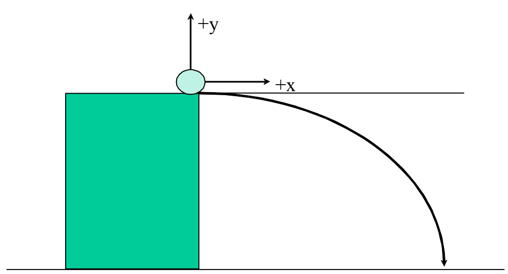
Ex. The motion of a ball in flight

- The path is parabolic in nature. You may remember studying things called conic sections in geometry. You need special equations to describe complicated shapes like this. Luckily for us, it turns out that motion in the x and y directions are independent of each other. That is, if I were to drop one ball straight down and shoot the other off the table, the y motion would be identical (Pop and drop). Any motion– any displacement or velocity or acceleration –can be broken down into its x and y components. **The important thing is that the x and y motions can be analyzed separately.** What you will notice, however, is that time is the variable linking the two sets of equations.

Ex. 9-1 A ball is shot from a toy gun with an initial velocity of 6.7 m/s horizontally. If the gun is 1.16 m off the floor

a) how long does the ball take to hit the floor and
b) how far from the starting point does the ball travel?

For our two times, let's take the naughts to be the point where the ball has just been fired from the gun and the final time to be just before the ball hits the floor.



Remember that we are working with constant acceleration. This means that $a_{xo} = a_x$ and $a_{yo} = a_y$. We need to establish a coordinate system. Let's take x = 0 and y = 0 to be at the point where the ball is at t_o. Then $x_o = 0$ and $y_o = 0$. $a_x = 0$ and $a_y = -9.8\ m/s^2$.

Remember that the motions are totally independent of each other: the time it takes the ball to reach the bottom is the same regardless of the horizontal motion. The t that we calculate will be the time it takes the ball to reach the ground. So realize that we need to solve for time from the y-motion. (How long does it take a ball to fall from this height? The x-motion doesn't affect this! We will then move time to the x-motion side of the problem and use it to solve for how far the ball has traveled in the x-direction.

Known: $v_{yo} = 0$ m/s Solve: t Not Involved: v_y

$a_y = -9.8$ m/s^2

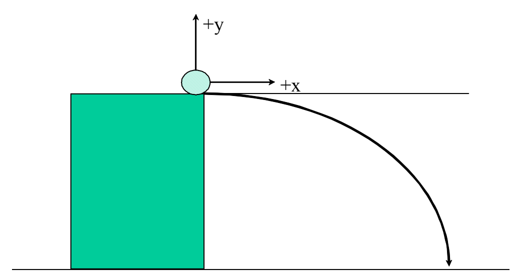
$y = -1.16$ m

$$y = v_{y0}t + \frac{1}{2}a_y t^2$$
$$y = \frac{1}{2}a_y t^2$$
$$t = \sqrt{\frac{2y}{a_y}} = t = \sqrt{\frac{2(-1.16m)}{-9.8\frac{m}{s^2}}} = 0.49s$$

Now that we have the time, realize that this time can be used in the x equations to figure out how far the ball travels.

$$x = v_{x0}t$$
$$x = v_{x0}t$$
$$x = \left(6.7\tfrac{m}{s}\right)\left(0.49s\right) = 3.3\ m$$

Ex. 9-2 A ball thrown from a building horizontally with a velocity of 4.2 m/s strikes the ground 16.0 m from the building. How tall is the building?



We will again need to use the fact that the vertical and horizontal components of motion are entirely independent. Let's place the origin of our coordinate system right where the ball leaves the edge of the building. We can use the x-motion to solve for the time in the air and then use this time in the y-motion and ask "from what height does it take an object this long to fall to the ground?".

<u>x-motion</u>

Since a_x=0, we can simply use $x = v_x t$

$$t = \frac{x}{v_x} = \frac{16.0\,m}{4.2\frac{m}{s}} = 3.8\,s$$

<u>y-motion</u>

Known: $v_{yo} = 0$ m/s Solve: y Not Involved: v_y

$a_y = -9.8$ m/s^2

$t = 3.8$ s

$$y = v_{0y}t + \frac{1}{2}a_y t^2$$
$$y = \frac{1}{2}a_y t^2 = \frac{1}{2}\left(-9.8\frac{m}{s^2}\right)(3.8s)^2 = -71\,m$$

An archer shoots an arrow horizontally at a target 15 m away. The arrow is aimed directly at the center of the target, but it hits 52 cm lower. What was the initial speed of the arrow?

Ex. 9-3 A cougar leaps horizontally from the top of a cliff with an initial velocity of 8.25 m/s. The cliff is 6.43 m tall. What are location and velocity when the cougar impacts the ground?

We know the path is parabolic. We can calculate the time it takes for an object to fall from this height.

$t = \sqrt{\frac{2y}{a}} = \sqrt{\frac{2(-6.43m)}{-9.8\frac{m}{s^2}}} = 1.15\,s$ and from this the horizontal distance. $x = v_x t = (8.25m/s)(1.15s) = 9.5m$

To calculate the velocity at the impact point we will need to know the x and y components of velocity at that point. The x velocity is a constant 8.25 m/s in this problem since $a_x = 0$. The y velocity it the speed an object has after falling 6.43 m.

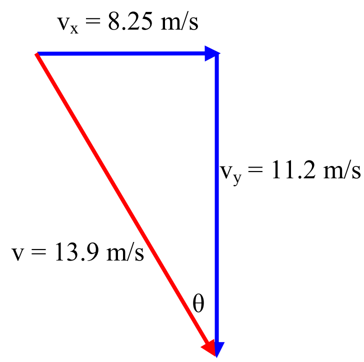
$$v_y^2 = v_{0y}^2 + 2a_y y$$

$$v_y = \sqrt{2a_y y} = \sqrt{2\left(-9.81\frac{m}{s^2}\right)(-6.43\,m)} = -11.2\frac{m}{s}$$

Thus we need to vectorially combine v_x=8.25 m/s and v_y=-11.2 m/s. Using the Pythagorean Theorem:

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{\left(8.25\frac{m}{s}\right)^2 + \left(-11.2\frac{m}{s}\right)^2} = 13.9\frac{m}{s}$$

$$\theta = \tan^{-1}\frac{opp}{adj} = \tan^{-1}\frac{8.25\frac{m}{s}}{11.2\frac{m}{s}} = 36.3^\circ$$



. A crow is flying horizontally with a constant speed of 2.7 m/s when it releases a clam from its beak. The clam lands on the rocky beach 2.10 s later. Just before the clam lands what is (a) the horizontal component of velocity and (b) its vertical component of velocity? (c) How would your answers to parts (a) and (b) change if the speed of the crow were increased?