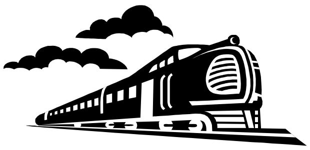
PHYSICS 151 – Notes for Online Lecture #7

Vector Addition of Velocity

One common application of adding and subtracting vectors is in calculating relative velocities. We will begin in 1-D and then work our way to more complicated 2-D problems.

As an example of these types of problems, suppose that an Amtrak train is traveling at 25 m/s. A passenger on board the train walks toward the front of the train at 5 m/s. What is the velocity of the passenger as seen by an observer on the side of the tracks?



When the two objects are moving along the same line, the relative velocity of one to the other is obtained simply by ordinary subtraction. v_{AB} denotes the velocity of object A with respect to object B. Thus if we use v_{AB} to denote the velocity of object A with respect to object B, and v_{AC} to denote the velocity of object A with respect to object C, then the velocity of a relative to C is found by vector addition:

$$v_{AC} = v_{AB} + v_{BC}$$

To apply this to our earlier problem note that the velocity of the train wrt the ground is 25 m/s and that the velocity of the passenger wrt the train is 5 m/s. Thus the velocity of the passenger with respect to the ground is 30 m/s.

Example 7-1: A vendor on a train that is moving in the forward direction at 2.00 m/s pushes her cart toward the rear of the train at 0.47 m/s while an ant on a sandwich on her cart crawls toward the front of the train at 0.01 m/s. What is the velocity of the ground with respect to the ant?

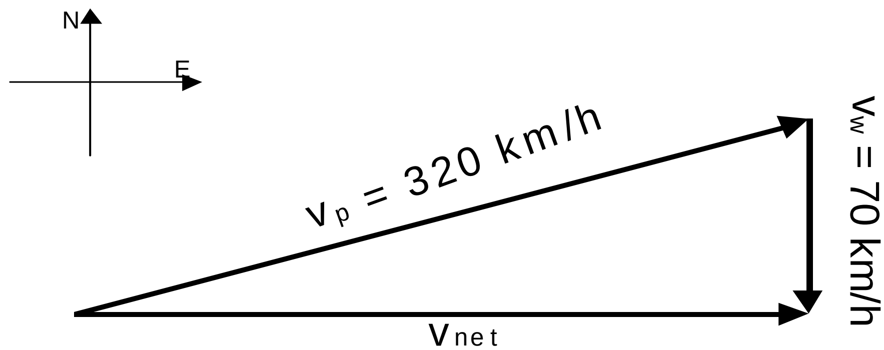
Let's start by considering the motion of the train relative to the ground with the forward direction as positive. The velocity of the train wrt the ground is $v_{TG} = +2.00$ m/s. The cart is moving relative to the train at $v_{CT} = -0.47$ m/s. Thus we can combine to express the velocity of the cart relative to the ground as $v_{CG} = v_{CT} + v_{TG} = -0.47$ m/s + 2.00 m/s = 1.53 m/s.

We still need to include the motion of the ant. The velocity of the ant wrt the cart is $v_{AC} = 0.01$ m/s. Thus combining yields the velocity of the ant wrt the ground: $v_{AG} = v_{AC} + v_{CG} = 0.01$ m/s + 1.53 m/s = 1.54 m/s.

However, the original question we were asked is "What is the velocity of the ground wrt the ant?". The ground's motion relative to the ant is $v_{GA} = -1.54$ m/s.

As an airplane taxies on the runway with a speed of 16.5 m/s, a flight attendant walks toward the tail of the plane with a speed of 1.22 m/s. What is the flight attendant's speed relative to the ground?

Example 7-2: An airplane with a maximum air speed of 320 km/h takes off and heads to its destination, which is due east of its starting point, when a 70 km/h crosswind starts blowing from the north. The schedule calls for the plane to travel a distance of 1590 km between airports in a time of 5.0 h. Will the plane arrive on schedule?



V_p is the maximum airspeed of the plane. When the cross wind starts, the pilot must turn to head somewhat into the wind in order to stay on the due east course. We can compute the angle θ from:

$$\sin\theta = \frac{v_w}{v_p} = \frac{70}{320} = 0.219$$
$$\theta = 12.6°$$

The net speed to the east is $v_{net} = v_p \cos\theta = (320 km/h)\cos 12.6° = 312 km/h$.

Will the plane be on time? The time required to travel 1590 km at a speed of 312 km/h is

$$t = \frac{distance}{speed} = \frac{1590 km}{312 km/h} = 5.09h$$

No, the plane will be late!

A boat capable of making 9.0 km/h in still water is used to cross a river flowing at a speed of 4.0 km/h. (a) At what angle must the boat be directed so that its motion will be straight across the river? What is its resultant speed relative to the shore?

As in the last problem, the boat must head somewhat upstream so that it has a velocity component upstream that just equals the velocity of the river.

$$v_b \sin\theta = v_r$$
$$\theta = \sin^{-1}\frac{v_r}{v_b} = \sin^{-1}\frac{4.0 km/h}{9.0 km/h} = 26°$$

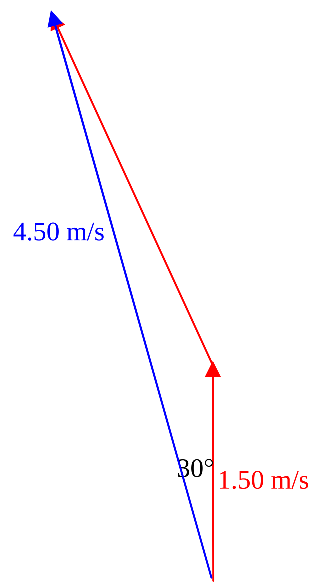
So the boat must be angled 26° in the upstream direction from the direct path across the river.

The resultant velocity of the boat is $v = v_b \cos\theta = 8\, km/h$.

So far all examples have had vectors at right angles.

Example 7-3: A passenger walks from one side of a ferry to the other as it approaches a dock. If the passenger's velocity is 1.50 m/s due north relative to the ferry, and 4.50 m/s at an angle of 30.0° west of north relative to the water, what are the direction and magnitude of the ferry's velocity relative to the water?

	x-component (east)	y-component (north)
v_{pf}	0 m/s	1.50 m/s
v_{fw}	?	?
v_{pw}	-4.50 sin 30°	4.50 cos 30°



Thus, the components of the ferry's velocity relative to the water are

v_x = -2.25 m/s v_y = 2.40 m/s

Applying the Pythagorean theory and the tangent function yields:

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{\left(-2.25\frac{m}{s}\right)^2 + \left(2.40\frac{m}{s}\right)^2} = 3.29\frac{m}{s}$$

$$\theta = tan^{-1}\left(\frac{2.25}{2.40}\right) = 43.2° \quad (W\ of\ N)$$

Note that your textbook would use unit vector notation in specifying the solution. You should convince yourself that the two methods are equivalent.

$\vec{\mathbf{v}}_{\text{pf}}$ = passenger's velocity relative to the ferry

$\vec{\mathbf{v}}_{\text{pw}}$ = passenger's velocity relative to the water

$$\begin{aligned}\vec{\mathbf{v}}_{\text{fw}} &= \text{ ferry's velocity relative to the water} \\ &= \vec{\mathbf{v}}_{\text{fp}} + \vec{\mathbf{v}}_{\text{pw}} \\ &= -\vec{\mathbf{v}}_{\text{pf}} + \vec{\mathbf{v}}_{\text{pw}}\end{aligned}$$

Let north be along the positive *x*-axis, then

$$\begin{aligned}\vec{\mathbf{v}}_{\text{fw}} &= -\left(1.50\ \frac{\text{m}}{\text{s}}\right)\hat{\mathbf{x}} + \left(4.50\ \frac{\text{m}}{\text{s}}\right)\cos(30.0°)\mathbf{x} + \left(4.50\ \frac{\text{m}}{\text{s}}\right)\sin(30.0°)\hat{\mathbf{y}} \\ &= \left(-1.50\ \frac{\text{m}}{\text{s}} + 3.90\ \frac{\text{m}}{\text{s}}\right)\hat{\mathbf{x}} + \left(2.25\ \frac{\text{m}}{\text{s}}\right)\hat{\mathbf{y}} \\ &= \left(2.40\ \frac{\text{m}}{\text{s}}\right)\hat{\mathbf{x}} + \left(2.25\ \frac{\text{m}}{\text{s}}\right)\hat{\mathbf{y}}\end{aligned}$$

$$v_{\text{fw}} = \sqrt{\left(2.40\ \frac{\text{m}}{\text{s}}\right)^2 + \left(2.25\ \frac{\text{m}}{\text{s}}\right)^2} = \boxed{3.29\ \text{m/s}}$$

$$\theta = \tan^{-1}\left(\frac{2.25\ \frac{\text{m}}{\text{s}}}{2.40\ \frac{\text{m}}{\text{s}}}\right) = \boxed{43.2°\ \text{west of north}}$$