PHYSICS 151 – Notes for Online Lecture #5

Freefall - A Special Case of Uniformly Accelerated Motion

The force of gravity causes a uniform acceleration. Galileo proved that all objects – in the absence of air resistance – fall at the same rate (see feather and coin demo). Explain air resistance using sheets of paper (1 crumbled up and 1 flat).

You can measure the acceleration due to gravity fairly easily. The magnitude of the acceleration due to gravity is called g and

$$g = 9.8\ \text{m/s}^2.$$

Note that g is a vector! This means that you have to worry about the direction!

$$F_{gravity} = \frac{GMm}{r^2} = mg$$

$$g = \frac{GM}{r^2} = \frac{\left(6.67\times10^{-11}\,\frac{N\cdot m^2}{kg^2}\right)\left(5.97\times10^{24}\,kg\right)}{\left(6.37\times10^{6}\,m\right)^2} = 9.81\frac{m}{s^2}$$

This value is smaller near the equator and at higher elevations, but this value gives a good representation for our purposes.

When dealing with falling objects, we are going to use the y-axis and not the x-axis. Remember that there are a number of coordinate systems in which you can work. You have to be very careful about choosing your directions because they can change the sign of g.

Review our four equations: rewrite them for the y-axis.

$$v^2 = v_0^2 + 2a(y - y_0) \quad \text{(constant acceleration only!)}$$

$$y = y_0 + v_0 t + \frac{1}{2}at^2 \quad \text{(constant acceleration only!)}$$

$$y = y_0 + \bar{v}t = y_0 + \frac{v_0 + v}{2}t \quad \text{(constant acceleration only!)}$$

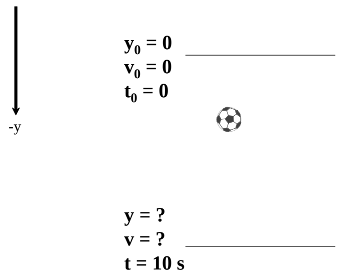
$$\vec{v} = \vec{v}_0 + \vec{a}t \quad \text{(constant acceleration only!)}$$

Note that the velocities are now the velocities in the y direction, and the acceleration is also an acceleration in the y-direction.

If I chose to take y upward being positive and y downward being negative, what is the sign of g?

Answer: g will be negative in this case, because gravity acts to move things in an increasingly negative direction.

Example 6-1: A person drops a ball from rest down a stairwell. How far does it fall in 10 seconds?



KNOWN:	UNKNOWN
$v_0 = 0$	y
a = g = -9.8 m/s^2	
t = 10s	

$$y = v_0 t + \frac{1}{2} a t^2$$

$$y = 0(10s) + \frac{1}{2}(-9.8 \tfrac{m}{s^2})(10s)^2$$

$$y = \frac{-9.8 \times 100}{2} (\tfrac{m}{s^2}) s^2$$

$$y = -490\, m$$

Note that the minus sign is correct: it indicates that the ball is going toward negative y

Example 6-2: Compare the answer above with the case in which the person throws the ball with an initial velocity of 3 m/s downward - now how far does the ball fall in 10 s?

$$y = v_0 t + \frac{1}{2} a t^2$$

$$y = \left(-3 \tfrac{m}{s}\right)(10s) + \frac{1}{2}(-9.8 \tfrac{m}{s^2})(10s)^2$$

$$y = \left(-3\right)(10)\left[\tfrac{m}{s} s\right] + \frac{1\left(-9.8\right)(100)}{2}\left[\frac{m}{s^2} s^2\right]$$

$$y = -30m + -490m = -520m$$

What if you're not sure of the sign of g?

Check it out:

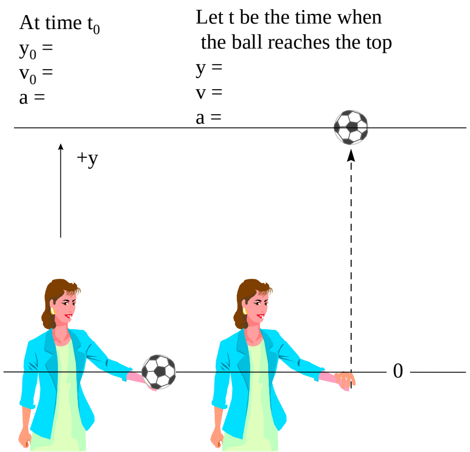
$v = v_0 + at$

Assume at $v_0 = 0$
$v = at$

If you drop the ball downward, the sign of the velocity will be the same sign as the axis pointing downward. Time is always positive, so the sign of g must be the same as the sign of the downward axis.

Example 6-3: A person throws a ball upward with an initial velocity of 3.0 m/s. a) What is the velocity of the ball at the highest point it reaches? b) How high does the ball go?

KNOWN	UNKNOWN
v_0 = 3 m/s	y
v = 0 m/s	
a = -9.8 m/s^2	



As you throw the ball upward, the velocity is in the positive direction; however, gravity will slow the ball down. At the top of the path, the velocity will be zero

What is the acceleration of the ball at the highest point it reaches? (-9.8 m/s^2) Don't forget that the acceleration due to gravity is **always** 9.8 m/s downward!

b) How high does the ball go?

What's important here is the first part of the motion: from the throw to the peak of the motion.

$$v^2 = v_0^2 + 2ay$$

$$v_0^2 = -2ay$$

$$\frac{v_0^2}{-2a} = y$$

$$\frac{\left(3.0\,\frac{m}{s}\right)^2}{-2\left(-9.8\,\frac{m}{s^2}\right)} = y$$

0.46m=y

Can also calculate: How much time does it take to go to the top of the motion?
Simplest way is probably:

$$v = v_0 + at$$

$$t = \frac{v - v_o}{a}$$

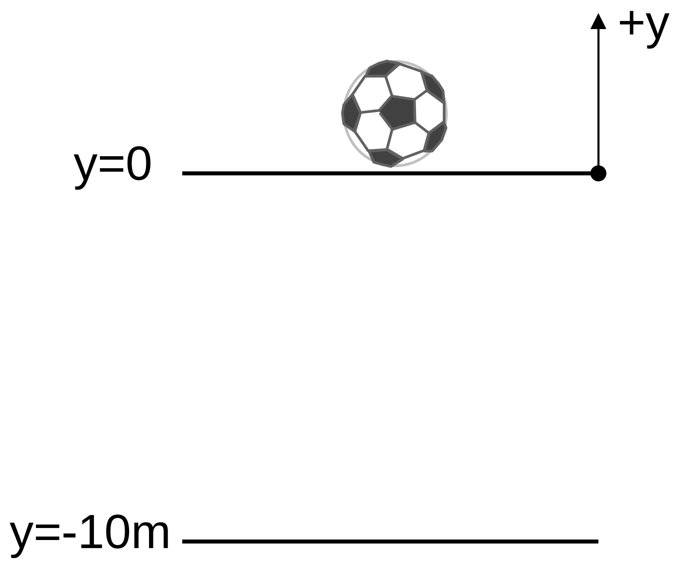
$$t = \frac{0 - 3.0\,\frac{m}{s}}{-9.8\,\frac{m}{s^2}} = 0.31s$$

KNOWN	UNKNOWN
v_0 = 3.0 m/s	t
v = 0 m/s	
a = -9.8 m/s^2	
y = 0.46 m	

Note that there are a great many quantities that may be obtained from symmetry arguments when objects are thrown in the air. Since g is a constant 9.8 m/s^2 downward throughout the path it increases the speed on the way down in exactly the same way it increased the speed on the way up. Thus the two y velocities (on the way up and on the way down) for any horizontal level must have the same magnitude but different signs. The time the object takes to go up to the peak must also be the same as the time it takes to fall from the peak to the horizontal level it began at.

We can demonstrate a potential sign problem by returning to the first example.

A person drops a ball from rest down a stairwell. What is the velocity after 10 m?



$v_0 = 0$
$y = -10$ m
$\vec{g} = -9.8$ m/s^2

$$v^2 = v_0^2 + 2ay$$
$$v^2 = 0 + 2\left(-9.8\tfrac{m}{s^2}\right)(-10m)$$
$$v^2 = 196\tfrac{m^2}{s^2}$$
$$v = \sqrt{196\tfrac{m^2}{s^2}} = \pm 14\tfrac{m}{s}$$

Note, however, that this is a speed! The object is headed downward, so the velocity is negative.