PHYSICS 151 – Notes for Online Lecture #4

Acceleration

The gas pedal in a car is also called an accelerator because pressing it allows you to change your velocity.

Acceleration is how fast the velocity changes. So if you start from rest and began accelerating at 3 m/s^2, at the end of 1s your velocity is 3 m/s, at the end of 2s your velocity is 6 m/s, at the end of 3s your velocity is 9 m/s, etc.

We can define an instantaneous acceleration as

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t}$$

and we can define an average acceleration.

$$\bar{\vec{a}} = \frac{\Delta \vec{v}}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1}$$

Example 5-1: When you enter the expressway, you have to change your velocity. Let's assume that the ramp is 30 m long. At the start of the ramp, you're going about 35 mph which is 16 m/s. It takes you 25 seconds to get to the end of the ramp, at which time you're going 30 m/s (about 65 mph). What is the average acceleration?

$$\bar{\vec{a}} = \frac{\Delta \vec{v}}{\Delta t} = \frac{v_2 - v_1}{\Delta t} = \frac{30\frac{m}{s} - 16\frac{m}{s}}{25s} = 0.56\left[\frac{\frac{m}{s}}{s}\right]$$

Let's take a look at the units here

$$\left[\frac{\frac{m}{s}}{s}\right] = \frac{m}{s^2}$$

So the answer is +0.56 m/s^2. Ask yourself whether this is a reasonable answer. Acceleration measures how fast the velocity is changing. Here, the velocity is increasing, so it makes sense that the acceleration is positive.

Example 5-2: You're approaching a school zone and have to slow down from 16 m/s to 11 m/s. You didn't notice the police car and have to step on the brakes quickly, making the change in 2.5 seconds. What is the acceleration?

$$\bar{\vec{a}} = \frac{\Delta \vec{v}}{\Delta t} = \frac{v_2 - v_1}{\Delta t} = \frac{11\frac{m}{s} - 16\frac{m}{s}}{2.5s} = -2.0\frac{m}{s^2}$$

Note that the negative sign means that the velocity is decreasing. We sometimes call a negative acceleration a deceleration.

The instantaneous acceleration is defined similarly to the instantaneous velocity

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t}$$

Uniformly Accelerated Motion

There are many cases of interest in which the acceleration is a constant. In most cases, we know the position or velocity at one time and want to find either position or velocity at a second time.

We're going to subscript all of our starting parameters with a zero. So at the start of time (which we'll take to be zero), the object is at position x_0 and has velocity v_0. We want to find parameters at a general time, t

Start at time $t_0 = 0$ displacement at $t_0 = x_0$ velocity at $t_0 = v_0$	At some later time, t displacement at t = x velocity at t = v

The average velocity at any time t will be (from our definition of average velocity)

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{x - x_0}{t}$$

$$\bar{v} = \frac{x - x_0}{t}$$

Note that for constant acceleration, the average acceleration is equal to the instantaneous acceleration. From the definition of acceleration

$$a = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{v - v_0}{t}$$

$$a = \frac{v - v_0}{t}$$

Note that this equation is **only** good for the case when the acceleration is constant in time!

We can rearrange this equation by multiplying both sides by t to get

$$at = v - v_0$$

and, rearranging, we find

$$v = v_0 + at \quad \text{(constant acceleration only!)}$$

To figure out the position, things are a little trickier.

$$\bar{v} = \frac{x - x_0}{t}$$

or

$$x = x_0 + \bar{v}t$$

Note that our first equation has the instantaneous velocity and not the average velocity. When the acceleration is constant, the average velocity is the midpoint between the initial and the final velocities.

$$x = \bar{v}t = \frac{v_0 + v}{2}t \quad \text{(constant acceleration only)}$$

Again, note that this is true only for constant acceleration!

$x = x_0 + \bar{v}t$ use the above eq.

$$x = x_0 + \left(\frac{v_0 + v}{2}\right)t$$

now plug in our expression for v

$$x = x_0 + \left(\frac{v_0 + (v_0 + at)}{2}\right)t$$

A little algebra

$$x = x_0 + v_0 t + \frac{1}{2}at^2 \quad \text{constant acceleration only!}$$

Your book derives a fourth equation, which I'm not going to spend time deriving, but which is very useful. You'll notice that all three of the equations we've derived contain time. There are going to be some cases in which time is not known.

$$v^2 = v_0^2 + 2a(x - x_0) \quad \text{constant acceleration only!}$$

For completeness t_0 and x_0 are left in the above discussion as variables. However, 99% of the time both of them are zero. A typical kinematics problem will involve 4 of the five variables (x, v, v_0, a, & t). You will know 3 of these and will be asked to solve for a fourth. The 5th variable is "not involved" and identifying this variable is your best clue to help you identify the proper equation.

Equation	Variable not involved
$v = v_0 + at$	x
$x = \bar{v}t = \frac{v_0 + v}{2}t$	a
$x = v_0 t + \frac{1}{2}at^2$	v
$v^2 = v_0^2 + 2ax$	t

We can handle problems where x_0 is not zero, by letting substituting $x - x_0$ for x in the above formulas.

Example 5-3: A car moves from rest with a constant acceleration of 3 m/s^2.

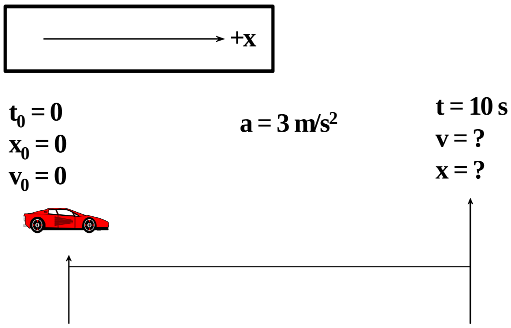
a) What is its velocity after 10 s

b) How far has the car moved after 10 s?

Solution:

KNOWN:	UNKNOWN
a = 3 m/s^2	v
v_0 = 0	
t = 10s	

Picture:



$$v = v_0 + at$$

$$= 0 + \left(3\frac{\text{m}}{\text{s}^2}\right)(10\text{s})$$

$$= \mathbf{30\ m/s}$$

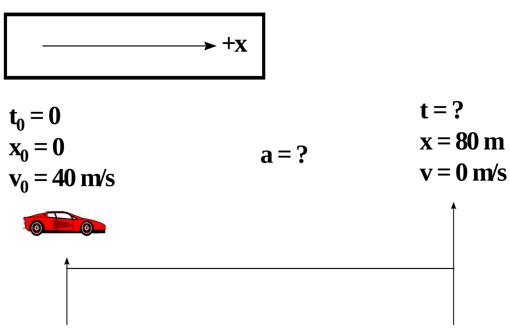
$$x = v_0 t + \frac{1}{2}at^2$$

$$x = 0 + \frac{1}{2}\left(3\tfrac{m}{s^2}\right)(10s)^2$$

$$x = \left[\frac{1}{2}(3)(100)\right]\left(\tfrac{m}{s^2}\right)(s)^2$$

$$x = \frac{300}{2}m = 150m$$

Example 5-4: The police find skid marks of length 80 m at an accident scene. If the person was traveling 40 m/s (about 90 mph), what was the acceleration?



KNOWN:	UNKNOWN
x=80 m	a
v_0 = 40 m/s	
v = 0 m/s	

Note that we don't know t in this problem, nor do we **need** to know it. There's only one equation that doesn't use t:

$$v^2 = v_0^2 + 2a(x - x_0)$$
$$0 = v_0^2 + 2ax$$
$$v_0^2 = -2ax$$
$$a = -\frac{v_0^2}{2x}$$
$$a = \frac{-\left(40\,\frac{m}{s}\right)^2}{2(80m)}$$
$$a = -\frac{1600\left(\frac{m^2}{s^2}\right)}{160m}$$
$$a = -10\,\frac{m}{s^2}$$

Does the minus sign make sense? Yes! The car is slowing down, so the acceleration **should** be negative

You Try It!

A rocket blasts off and moves straight upward from the launch pad with constant acceleration. After 3.0 s the rocket is at a height of 80.0 m.

(a) What are the magnitude and direction of the rocket's acceleration?

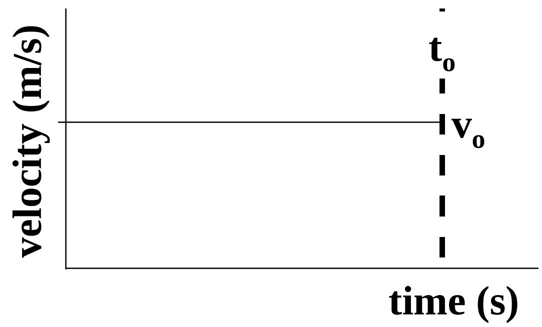
(b) What is its speed at this time?

Graphs and Acceleration

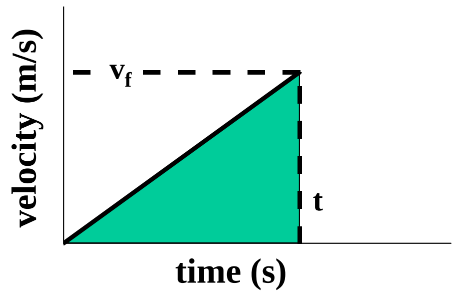
The instantaneous acceleration is defined similarly to the instantaneous velocity

$$\vec{a} = \lim_{\Delta t \longrightarrow 0} \frac{\Delta \vec{v}}{\Delta t}$$

This means that they have the same graphical interpretation also: the instantaneous acceleration is the slope of the velocity-time graph. If an object is moving with constant velocity, the velocity-time graph will look like this:



The acceleration in this case is zero, because $\Delta\vec{v} = v_2 - v_1$ = 0 everywhere. So when there is constant velocity, there is zero acceleration.



In the case of constant non- graph will look velocity v_f. equals the triangle, so the

zero acceleration, the velocity vs. time like the graph at right. Call the final Recall that the area underneath the curve displacement. This is just a right area is given by ½ (base) (height) or

$$A = ½\, t\, v_f$$

This will come in handy next time.

Determine the acceleration and distance traveled during each of the four segments below.