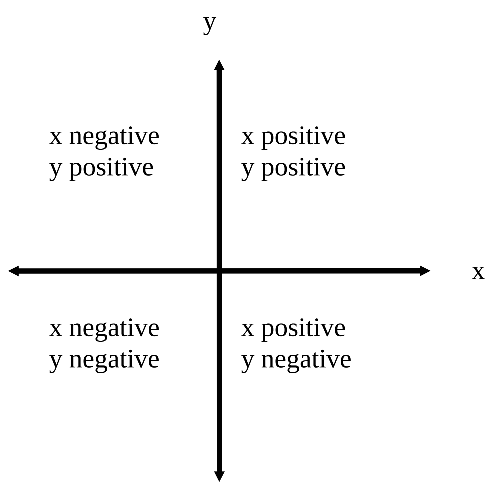
PHYSICS 151 – Notes for Online Lecture #3

Kinematics involves the description of the position and motion of objects as a function of time. In this chapter, we will be limiting that motion to a straight line. A number of quantities in this chapter will be defined (distance, displacement, average velocity, and instantaneous velocity).

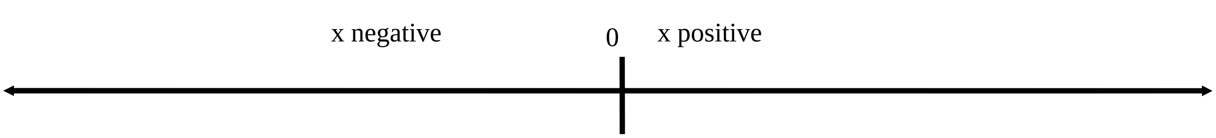
Coordinate Systems and Reference Frames.

To make analyzing motion easier, we establish a **reference frame**. You've all probably done some graphing on a Cartesian coordinate system. The usual convention is that, on the x-axis, anything to the right of the origin is considered positive and anything to the left is considered negative. The y-axis works in a similar manner: anything above the origin is positive and below the origin is negative.



You can work your problems according to any convention you choose; however, you must make it very clear to anyone reading your work which convention it is you've chosen. It's a good idea to draw a picture of your coordinate system at the start of every problem you do.

For right now, we're limiting our motion to one direction - the x-direction, so our coordinate system will look like:



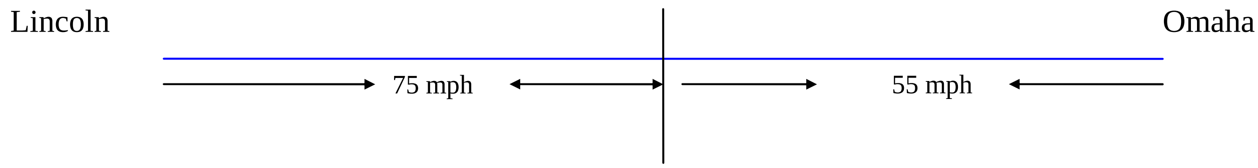
Now we have a slight problem. If you tell me how fast you're traveling, I can't tell from that number alone in which direction you are going.

Example: Let's use a car moving to introduce some of these ideas.

During the weekend, you drive from Lincoln to Omaha, which is about 60 miles. It takes you one hour to make the trip.

When you arrive in Omaha, your friend asks you whether you were speeding. Thinking about your trip, you figure that you went 60 miles in 1 hour, so you were traveling at a rate of 60 miles/hour.

Your friend says, "But I thought the speed limit was up to 75 miles per hour." You realize that you did indeed go 75 miles per hour during part of your trip, but there was some construction going on, and when you reach the city limits, you had to slow down to 55 miles per hour. The number that you reported to your friend is a quantity that we call the **average speed**



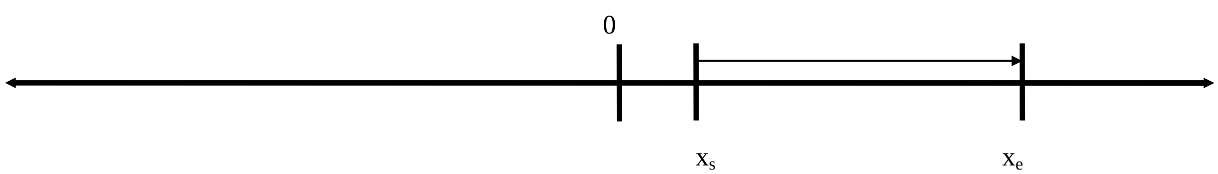
Average Speed

Average speed ignores the details of your trip – it only depends on how far an object travels in a set amount of time.

$$\text{average speed} = \frac{\text{distance traveled}}{\text{time needed to travel that distance}}\left[\frac{m}{s}\right]$$

The definition of speed is easy to remember because the units tell you what the quantity is: distance, in (m), divided by time (in s). Although the average speed does give you a general idea of how fast you were traveling, it doesn't tell you any details at all.

We define **displacement** to be a quantity that gives you both how far you went and in which direction. We represent this term by $\Delta\vec{x}$. The Greek letter Δ is usually used to indicate a change in a quantity. In this case, it's a change in x. Let's say you start off at a position that we'll call x_s (for start)



Some time later, you stop at position x_e. The displacement, Δx, is given by the distance between the two points:

$$\Delta\vec{x} = x_{end} - x_{start}$$

Example 3-2: Let's specify some values. If x_1 is at 20 meters and x_2 is at 40 meters

$$\Delta\vec{x} = 40\text{m} - 20\text{m} = +20\text{m}$$

Your displacement would be +20 meters. However, I said that the displacement also had to include the direction. In this example, you walked from a position at 20 meters to a position at 40 meters, so you'll notice that the sign on the displacement is positive.

What if you now turned around and walked back. Now your starting position is at 40 meters and your ending position is at 20 meters. Your displacement is:

$\Delta\vec{x} = 20\text{m} - 40\text{m} = -20\text{m}$

On the coordinate system, the negative sign simply means that you traveled left and the positive sign that you traveled right. The distance is the same for both trips, but the displacement is different.

Distance - how far - Δx
Displacement - how far, and in what direction, $\Delta\vec{x}$

The difference between these two quantities is very important. A quantity that simply tells you the magnitude of something is called a **scalar**. A scalar is just a number. A quantity like displacement, however, gives you both a number and a direction. We call these quantities **vectors**. This distinction is very important, so you'll notice that whenever I write a vector, I put an arrow over it to remind myself that it must have both magnitude and direction. In the book, the author writes all vectors in bold print.

We defined average speed as the distance divided by the time taken to go that distance. We can also define an average *velocity*.

$$\text{average velocity} = \frac{\text{displacement}}{\text{time elapsed}}$$

$$\bar{v} = \frac{\Delta\vec{x}}{\Delta t}$$

Note that the bar over the v indicates that it is an average velocity.

Speed and velocity are similar to distance and displacement in that speed is a scalar and velocity is a vector.

Speed = how fast?
Velocity = how fast and in which direction?

Note that speed is just the magnitude of the velocity. We write the magnitude of a vector without the arrow.

Scalar		Vector	
Distance	Δx	Displacement	$\Delta\vec{x}$
Speed	v	(Instantaneous) velocity	$\vec{v}$

Mathematically, we indicate direction by the sign. All vector quantities have a sign associated with them - that gives you a check on your answer.

In our above example, let's say that the first part of your trip takes you five minutes. We know that the displacement during the first part of the trip is $\Delta x = +20m$. The average velocity is thus:

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{+20.0\,\text{m}}{5.0\,\text{min}}\left(\frac{1\,\text{min}}{60\,\text{s}}\right) = \frac{1}{5\times 3}\left[\frac{m}{s}\right] = 6.7x10^{-2}\,\frac{m}{s}$$

Note that the average velocity in this case is positive. What about the return trip, if it takes the same amount of time?

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{-20.0\,\text{m}}{5.0\,\text{min}}\left(\frac{1\,\text{min}}{60\,\text{s}}\right) = \frac{-1}{5\times 3}\left[\frac{m}{s}\right] = -6.7x10^{-2}\,\frac{m}{s}$$

Because the average velocity on the return trip is to the left, it will be negative.

We like to write the definite of average velocity using symbols. Assume that you start at position x_1 at time t_1 and end at position x_2 at time t_2. The general definition of average velocity is thus

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}$$

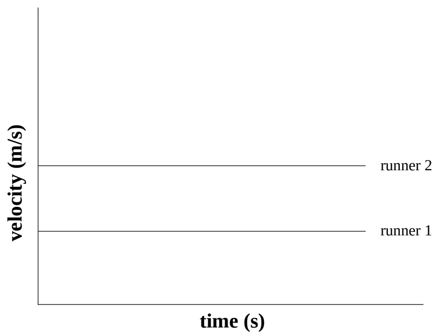
Remember that you're always subtracting the ending position from the beginning position.

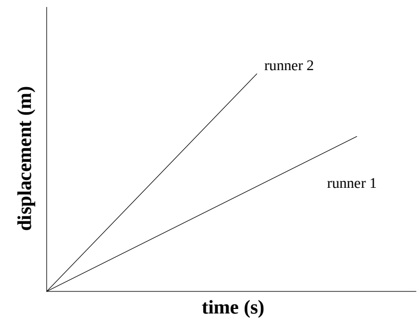
Suppose you combined the first part of the trip and the return trip and made calculations assuming it was just one longer round trip journey.

- What would the distance traveled be?
- What would the average speed be?
- What would the displacement be?
- What would the average velocity be?

Getting Displacement from Velocity-Time Graphs

Graphs can be very useful in understanding the motion of an object. We have already established that the graph of the displacement of a person travelling at a constant velocity vs. time is a straight line. Let's say that we have two people traveling at constant velocity, but one is traveling faster than the other. What would the graph of the displacement as a function of time be?



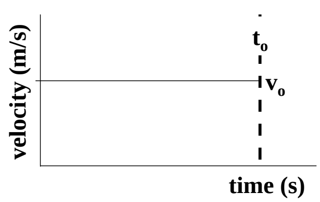


ANSWER: The velocity is constant, so they are both horizontal lines. Since runner 2 has a larger displacement than runner 1 at similar times, runner 2 must have a larger velocity (slope).

We know from before that the average velocity is just the displacement divided by the time it takes to move through that displacement. This means that

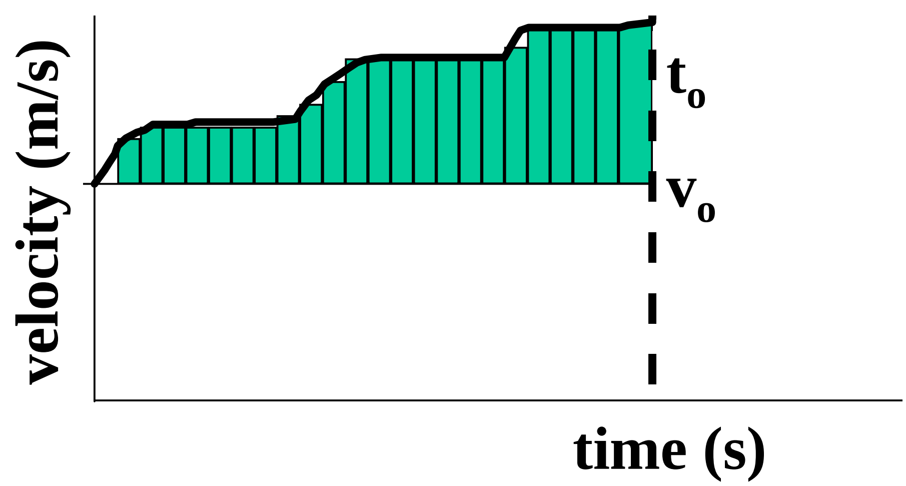
$$\Delta\vec{x} = \bar{\vec{v}}\Delta t$$

If I travel at a fixed velocity, v_o for a time t_o, a plot of velocity vs. time would look like this:



We know from the equations that the displacement is $\Delta\vec{x} = v_o t_o$. But note that the area underneath the v vs. t curve is also = $v_o t_o$. In general, the displacement occurring in a given time is equal to the area under the velocity-time curve.

For any arbitrary shape, I can make small rectangles and add up all of the areas to get an approximation of the displacement. For the curve at the right, the total area will be the sum of the area of each of the smaller rectangles, with each rectangle having a width Δt.



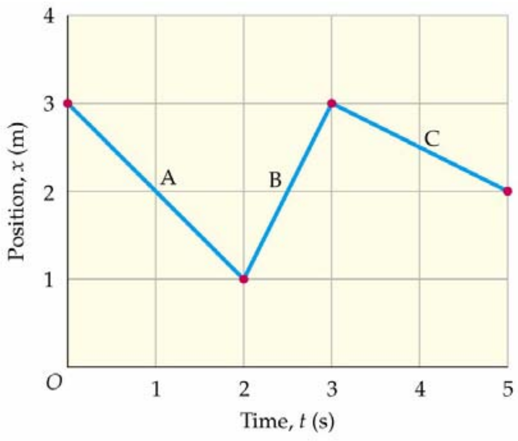
Instantaneous Velocity

Mathematically, we write

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta\vec{x}}{\Delta t}$$

The instantaneous velocity is written as 'v', with no bar. You can assume that, if I don't explicitly say average velocity, the word 'velocity' = instantaneous velocity. Velocity is the rate at which the displacement changes.

EXAMPLE: A child rides her tricycle back and forth along the sidewalk producing the position-versus-time graph shown below. For which segments of the graph A, B, or C does the child have the greatest speed? Perform calculations to verify this.



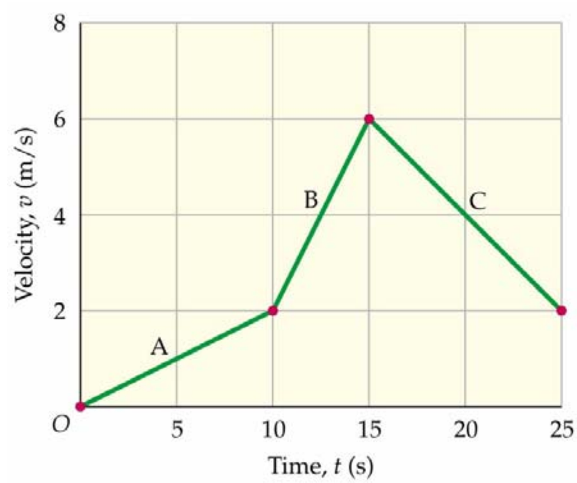
$$s_A = \left|\frac{1\text{ m} - 3\text{ m}}{2\text{ s} - 0}\right| = \left|\frac{-2\text{ m}}{2\text{ s}}\right| = \boxed{1\text{ m/s}}$$

$$s_B = \left|\frac{3\text{ m} - 1\text{ m}}{3\text{ s} - 2\text{ s}}\right| = \left|\frac{2\text{ m}}{1\text{ s}}\right| = \boxed{2\text{ m/s}}$$

$$s_C = \left|\frac{2\text{ m} - 3\text{ m}}{5\text{ s} - 3\text{ s}}\right| = \left|\frac{-1\text{ m}}{2\text{ s}}\right| = \boxed{0.5\text{ m/s}}$$

$$s_B > s_A > s_C$$

EXAMPLE: A person on horseback moves according to the velocity-versus-time graph shown below. Find the displacement of the person for each of the segments A, B, & C.



Segment A:
Area = 0.5(2 m/s)(10 s) = 10 m

Segment B:
Area = (2 m/s)(5 s) + 0.5(4 m/s)(5 s)
= 20 m

Segment C
Area = (2 m/s)(10 s)
+ 0.5 (4 m/s)(10s) = 40 m

or using average velocities

Segment A:
Area = 0.5(2 m/s)(10 s) = 10 m

Segment B:
Area = (2 m/s)(5 s) + 0.5(4 m/s)(5 s)
= 20 m

Segment C
Area = (2 m/s)(10 s)
+ 0.5 (4 m/s)(10s) = 40 m

EXAMPLE: A 9.0-h trip is made at an average speed of 50 km/h. If the first half of the distance is covered at an average speed of 45 km/h, what is the average speed for the second half of the trip?

Before beginning any calculation, estimate the answer.

The total distance of the trip must be given by distance = speed * time = 50 km/h * 9.0 h = 450 km.

The time for the first half is $time = \frac{distance}{speed} = \frac{225km}{45\frac{km}{hr}} = 5hr$

Thus, the speed of the second half is described by $speed = \frac{distance}{time} = \frac{225km}{4hr} = 56.25\frac{km}{hr}$

EXAMPLE: Phileas Fogg travels "Around the World in 80 Days" in the Jules Verne book. Assuming that he traveled along the equator, what was his average speed in m/s.

The radius of the earth is 6.38 x 10^6 m, thus the circumference is $2\pi R$ = 4.01 x 10^7 m

$$speed = \frac{distance}{time} = \frac{4.10\times10^7 m}{80days}\left(\frac{1days}{24hr}\right)\left(\frac{1hr}{3600\sec}\right) = 5.93\frac{m}{s}$$