

Physics 151 Class Exercise: Sound - KEY

1. Twenty violins playing simultaneously with the same intensity combine to give an intensity level of 82.5 dB. **(a)** What is the intensity level of each violin? **(b)** If the number of violins is increased to 40, will the combined intensity level be more than, less than, or equal to 165 dB? Explain.

(a) Approach 1—calculate the intensity of 20 violins, divide by 20 to get the intensity of 1 violin, and then calculate the intensity level of one violin.

$$\beta = 10 \log \left(\frac{I_{20}}{I_0} \right)$$

$$\frac{\beta}{10} = \log \left(\frac{I_{20}}{I_0} \right) = \log I_{20} - \log I_0$$

$$\frac{\beta}{10} = \log I_{20} + 12$$

$$\frac{\beta}{10} - 12 = \log I_{20}$$

$$I_{20} = 10^{\frac{\beta}{10} - 12} = 10^{\frac{82.5}{10} - 12} = 10^{-3.75} = 1.78 \times 10^{-4} \frac{W}{m^2}$$

$$I_1 = \frac{I_{20}}{20} = \frac{1.78 \times 10^{-4} \frac{W}{m^2}}{20} = 8.89 \times 10^{-6} \frac{W}{m^2}$$

$$\beta = 10 \log \left(\frac{I_1}{I_0} \right) = 10 \log \left(\frac{8.89 \times 10^{-6} \frac{W}{m^2}}{1.00 \times 10^{-12} \frac{W}{m^2}} \right) = 69.5 \text{ dB}$$

Approach 2 – Compare the two intensities directly.

$$\beta_{20} - \beta_1 = 10 \log \left(\frac{20I_1}{I_1} \right) = 10 \log(20)$$

$$\beta_1 = \beta_{20} - 10 \log(20) = 82.5 - 10(1.30) = 69.5 \text{ dB}$$

Approach 3 – Use our general rules that a change in intensity by a factor of 10 (or 2) causes a change in intensity levels by an amount of 10 (or 3). Twenty violins represent a 10-fold increase from one violin, followed by a 2-fold increase, or an increase of 10.0 dB + 3.0 dB = 13.0 dB. So one violin has an intensity of 82.5 dB – 13.0 dB = 69.5 dB.

(b) Doubling the number of violins doubles the intensity, producing an intensity level increase of 3 dB. The intensity level will be 85.5 dB, which is much less than 165 dB. One can double intensities

$$\beta = 10 \log \left(\frac{2I}{I_0} \right)$$

$$= 10 \log \left(\frac{(2) \left(1.78 \times 10^{-4} \frac{W}{m^2} \right)}{1.00 \times 10^{-12} \frac{W}{m^2}} \right) = 85.5 \text{ dB}$$

2. Residents of Hawaii are warned of the approach of a tsunami (tidal wave) by sirens mounted on the top of towers. Suppose a siren produces a sound that has an intensity level of 120 dB at a distance of 2.0 m. Treating the siren as a point source of sound, and ignoring reflections and absorption, find the intensity level heard by an observer at a distance of **(a)** 12 m and **(b)** 21 m from the siren. **(c)** How far away can the siren be heard?

$$I = \frac{P}{4\pi r^2}$$

$$I r^2 = \frac{P}{4\pi}$$

$$I_1 r_1^2 = I_2 r_2^2$$

$$I_2 = \left(\frac{r_1}{r_2}\right)^2 I_1$$

$$\text{(a)} \quad I_2 = \left(\frac{2.0 \text{ m}}{12 \text{ m}}\right)^2 I_1$$

$$\begin{aligned} 10 \text{ Log} \left(\frac{I_2}{I_0}\right) &= 10 \text{ Log} \left[\left(\frac{2.0 \text{ m}}{12 \text{ m}}\right)^2 \left(\frac{I_1}{I_0}\right)\right] \\ &= 10 \text{ Log} \left(\frac{I_1}{I_0}\right) + 10 \text{ Log} \left[\left(\frac{2.0 \text{ m}}{12 \text{ m}}\right)^2\right] \\ &= 120 + (-15.6) \\ &= \boxed{104 \text{ dB}} \end{aligned}$$

$$\text{(b)} \quad 10 \text{ Log} \left(\frac{I_2}{I_0}\right) = 10 \text{ Log} \left(\frac{I_1}{I_0}\right) + 10 \text{ Log} \left[\left(\frac{2.0 \text{ m}}{21 \text{ m}}\right)^2\right] = 120 + (-20.4) = \boxed{99.6 \text{ dB}}$$

$$\text{(c)} \quad \text{At maximum distance, } I_2 = I_0 \text{ and } 10 \text{ Log} \left(\frac{I_2}{I_0}\right) = 0.$$

$$\begin{aligned} 10 \text{ Log} \left[\left(\frac{2.0 \text{ m}}{r_2}\right)^2\right] &= -120 \\ \left(\frac{2.0 \text{ m}}{r_2}\right)^2 &= 10^{-12} \\ r_2 &= \frac{2.0 \text{ m}}{10^{-6}} \\ &= \boxed{2.0 \times 10^6 \text{ m}} \end{aligned}$$

This is a theoretical limit that could be realized in an ideal case. In a more realistic scenario, ambient noise, as well as energy losses when the sound waves are reflected or absorbed by surfaces, would prevent us from hearing the sound 2000 km away. Sometimes, the real-world factors we ignore make a huge difference.