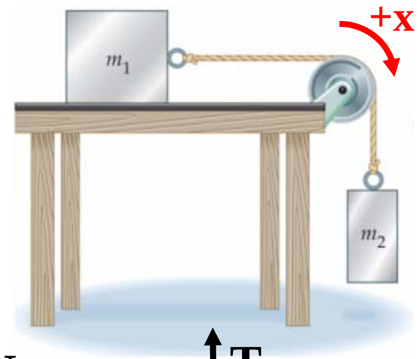
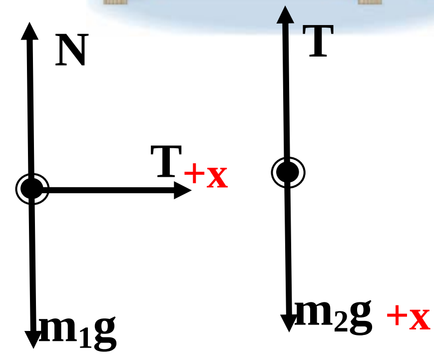
Physics 151 Class Exercise – Connected Objects w/Friction



1. Two masses are connected by a rope as shown in the figure to the right. The table is sufficiently smooth that friction can be ignored in this problem.

a) Indicate a coordinate system on the diagram. It should "follow the motion" of the string so that both masses accelerate in the positive direction with accelerations of equal magnitude.

b) Draw FBDs for each mass



c) Write the relevant summation of force equations for both masses.

$$\Sigma F_{x-m_1} = T = m_1 a$$

$$\Sigma F_{x-m_2} = m_2 g - T = m_2 a$$

d) Combine the equations to determine an algebraic expression for the acceleration of the masses. **Add the two equations together (which will eliminate T) to get**

$$m_2 g = m_1 a + m_2 a = (m_1 + m_2) a$$

$$a = \frac{m_2 g}{(m_1 + m_2)}$$

e) Determine an algebraic expression for the tension in the rope? $T = m_1 \frac{m_2 g}{(m_1 + m_2)}$

f) Is the tension greater than, equal to, or **less than** m_2g? Justify your answer with a common sense explanation and an algebraic argument.

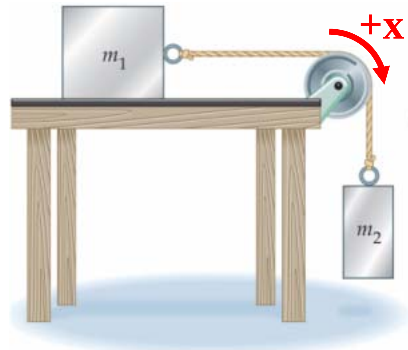
The tension would be m_2g if m_2 were suspended and not accelerating. Since it is moving downward the tension must be less. From the algebraic equation above: $\frac{m_1}{(m_1 + m_2)} < 1$

g) Determine the acceleration and tension if m_1 = 7 kg and m_2 = 4 kg.

$$a = \frac{m_2 g}{(m_1 + m_2)} = \frac{(4kg)\left(9.81\frac{m}{s^2}\right)}{\left[(7kg)+(4kg)\right]} = 3.57\frac{m}{s^2}$$

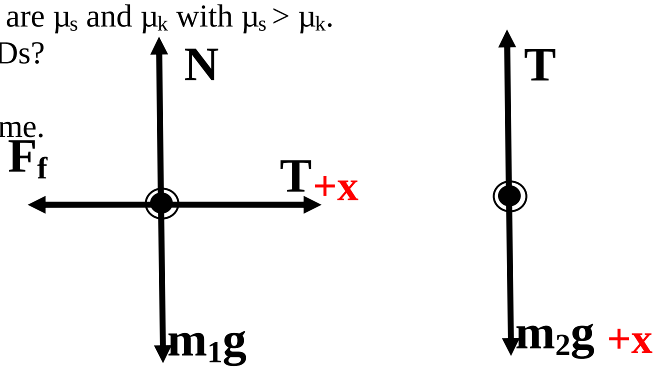
$$T = m_1 \frac{m_2 g}{(m_1 + m_2)} = (7kg)\frac{(4kg)\left(9.81\frac{m}{s^2}\right)}{\left[(7kg)+(4kg)\right]} = 25.0N$$

2. . Two masses are connected by a rope as shown in the figure to the right. The table is rough and the coefficients of friction for the block/table interface are μ_s and μ_k with $\mu_s > \mu_k$.



a) Draw the new FBDs?

FBD for m_2 is the same.



b) Write the relevant summation of force equations for both masses (assuming they are accelerating).

$$\Sigma F_{y-m_1} = N - mg = 0$$
$$N = mg$$
$$\Sigma F_{x-m_1} = T - F_f = m_1 a$$
$$= T - \mu_k m_1 g = m_1 a$$
$$\Sigma F_{x-m_2} = m_2 g - T = m_2 a$$

c) Formulate a condition that will determine whether or not the masses accelerate.

$$m_2 g > \mu_s m_1 g$$

d) Assuming that they do move, derive an algebraic expression for the acceleration. **Add the two equations together (which will eliminate T) to get**

$$m_2 g - \mu_k m_1 g = m_1 a + m_2 a = (m_1 + m_2)a$$
$$a = \frac{m_2 g - \mu_k m_1 g}{(m_1 + m_2)}$$

e) Determine the acceleration if m_1 = 7 kg, m_2 = 4 kg, and μ_k = 0.20.

$$a = \frac{m_2 g - \mu_k m_1 g}{(m_1 + m_2)} = \frac{(4kg)\left(9.81\frac{m}{s^2}\right) - (0.20)(7kg)\left(9.81\frac{m}{s^2}\right)}{\left[(7kg) + (4kg)\right]} = 2.32\frac{m}{s^2}$$