Physics 151 Class Exercise: Centripetal Acceleration - KEY

For each of the following situations, draw a free body diagram showing all of the forces acting on the object. Then write an equation expressing the sum of forces in the radial direction – from the center of the circle to the object and write an algebraic expression for the centripetal acceleration $a_c$ in terms of the given variables.

Situation #1 – A boy ties a ball to the end of a string (length L) and then swings the ball in a vertical circle. Consider the ball when it is at the lowest point in its swing, the tension in the string is $T$ at that moment. In this situation all forces act along one direction y and centripetal acceleration points up, so that $ma = T - mg$ or

$$a_c = \frac{T}{m} - g$$

Situation #2 – The boy swings the ball (mass $m$) in a horizontal circle (radius $R$) about his head. (Hint: How does gravity affect the direction of the tension in the string?)

The direction of tension makes angle $\theta$ with the horizontal. The centripetal acceleration points along axes x to the center of the circle, because of that one has

- $x: T \cos \theta = ma$  \hspace{1cm} ma = T \cos \theta$
- $y: T \sin \theta - mg = 0$  \hspace{1cm} mg = T \sin \theta$, dividing second equation by the first gives \( \frac{g}{a} = \tan \theta \), $a = g / \tan \theta$, the angle $\theta$ itself can be found as $\theta = \arccos \left( \frac{R}{L} \right)$, so finally $a_c = \frac{g}{\tan \left( \arccos \left( \frac{R}{L} \right) \right)}$.

Situation #3 – A bug sits on the edge of a 45 record being played on a turntable, the coefficient of the static friction between the bug and a record is $\mu$.

The force, which provides the centripetal acceleration along axes x is the friction force $F_{fr}$, then

- $x: ma = F_{fr}$, since $F_{fr} = \mu N$, one has $ma = \mu N$
- $y: N - mg = 0$ \hspace{1cm} mg = N$ or $ma = \mu mg$, $a_c = \mu g$
Situation #4 -- A car moves on a circular exit ramp banked at angle $\alpha$ to the horizontal. Neglect friction. (Hint: Draw the car so that you are looking at the back or the front.)

In this situation centripetal acceleration will be along the x-axis as shown in the picture, so that

\[ x: ma = N \sin \alpha \quad \text{or} \quad ma = N \sin \alpha \]
\[ y: N \cos \alpha - mg = 0 \quad \text{or} \quad mg = N \cos \alpha \]

dividing the first equation by the second one can find
\[ \frac{a}{g} = \tan \alpha, \quad a_c = g \tan \alpha \]

Situation #5 – A “corkscrew” roller coaster does a loopty-loop. Consider the situation of a man of mass m riding in the roller coaster when he is upside-down, the normal force acting on the man is N.

In this situation all the forces and centripetal acceleration point down along axes y, so that

\[ ma = N + mg \quad \text{or} \quad a_c = \frac{N}{m} + g \]

Situation #6 - A daredevil stunt involves riding a motorcycle around the vertical inside wall of a cylindrical structure. The coefficient of friction between motorcycle and a wall is $\mu$ (Hint: How big does the normal force have to be?)

In this situation the normal force $N$ provides the centripetal acceleration in positive x direction, so

\[ x: ma = N \]
\[ y: F_{fr} - mg = 0, \quad \text{since} \quad F_{fr} = \mu N, \quad \text{one has} \]
\[ ma = N \]
\[ \mu N = mg \quad \text{or} \quad \mu ma = mg, \quad a_c = g \frac{1}{\mu} \]