Magnetic substitutional alloys are often found to excel in applications [1,2], because alloying broadens the parameter space for tuning the desired properties. However, wide tunability, combined with the need to target certain operating temperature ranges, presents a challenge for empirical materials design. Competing magnetic interactions in alloys can produce complicated magnetic phase diagrams (MPD) with multiple magnetic phases [3−6]. Understanding of the $c − T$ MPDs is therefore essential for the development of advanced magnetic materials.

Some MPDs can be computed using the Heisenberg model with empirical or calculated exchange parameters combined with the mean-field approximation (MFA) [7,8], Monte Carlo simulations [9−12], or spin-fluctuation theory [13]. However, many systems are not adequately described by the Heisenberg model. In metallic alloys the interaction parameters are sensitive to the electronic structure and population and thereby to the content of the alloy [8,9,11] and to the degree of spin disorder [14]. To avoid the limitations of the Heisenberg model, one can use first-principles spin-dynamics simulations [15] or construct a generalized spin Hamiltonian to map the adiabatic energy surface [16,17] for use in thermodynamic calculations. The energies of disordered spin configurations can also be obtained using the disordered local moment (DLM) model [18,19], where the spin-rotational averaging is done in the coherent potential approximation (CPA). While all these approaches fail in strongly itinerant magnets, they are applicable when the spin moments do not vary by more than 10%−20% in different spin configurations. We restrict ourselves to such systems here.

First-principles spin dynamics and the construction of a microscopic generalized spin Hamiltonian are computationally demanding and unfeasible for most systems of practical interest. We have developed [20] an alternative approach, in which self-consistent total energy calculations based on the coherent potential approximation for partially ordered and noncollinear magnetic states and is able to account for competing interactions and multiple magnetic phases. Application to the Fe$_{1−x}$Mn$_x$Pt “magnetic chameleon” system yields the sequence of magnetic phases at $T = 0$ and the $c − T$ magnetic phase diagram in good agreement with experiment, and a new low-temperature phase is predicted at the Mn-rich end. The importance of non-Heisenberg interactions for the description of the magnetic phase diagram is demonstrated.

A first-principles approach to the construction of concentration-temperature magnetic phase diagrams of metallic alloys is presented. The method employs self-consistent total energy calculations based on the coherent potential approximation for partially ordered and noncollinear magnetic states and is able to account for competing interactions and multiple magnetic phases. Application to the Fe$_{1−x}$Mn$_x$Pt “magnetic chameleon” system yields the sequence of magnetic phases at $T = 0$ and the $c − T$ magnetic phase diagram in good agreement with experiment, and a new low-temperature phase is predicted at the Mn-rich end. The importance of non-Heisenberg interactions for the description of the magnetic phase diagram is demonstrated.

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practical simplification we assumed perfect \( L_1 \) ordering and complete disorder of Fe and Mn atoms within their own sublattice. The method can also be applied to alloys with partial chemical ordering and in principle allows one to study the coupling between magnetic and chemical order parameters [25].

To construct the energy functional, we have extended our CPA implementation [26,27] based on the tight-binding linear muffin-tin orbital formalism [28] by special features designed to describe complicated magnetic states. First, we implemented the vector DLM (VDLM) model, in which partially ordered magnetic states are specified by the Curie-Weiss distribution functions \( p_{\mu i}(\theta) \propto \exp(\alpha_{\mu i} \cos \theta) \), where \( i \) is the lattice site and \( \mu \) the component index, and \( \alpha_{\mu i} \) are regarded as variational parameters. This formulation is suitable for systems with axial spin symmetry, i.e., those with collinear magnetic order. Different spin moment orientations are treated as different alloy components in the CPA formalism. The integral over the azimuthal angle in the CPA equations is taken analytically, while the \( \theta \) dependence is discretized using the 16-point Gauss-Legendre quadrature. The potentials for all atoms depend strongly on the concentration, which reflects the effect of band filling on the exchange interaction in metallic systems.

The second feature extends CPA to the noncollinear case, in which the orientations of the spin moments of different components on the same lattice site can be different. This method is suitable, in particular, for the \( 2Q \) structures appearing in the Fe\(_{1-x}\)Mn\(_x\)Pt system. Self-consistent constraining fields are also used in these calculations. Both VDLM and noncollinear CPA calculations yield the density-functional total energy.

Let us first examine the magnetic interactions in the vicinity of the paramagnetic state. We set up a unit cell for each of the three magnetic orderings and calculate the total energy for about 70 partially ordered VDLM states with \( |\alpha_{Fe}| \) and \( |\alpha_{Mn}| \) ranging from 0 to 3. Experimental lattice constants are used at the concentrations reported in Ref. [23]. At each concentration the calculated total magnetic energy \( E_{mag} \) (per Fe/Mn atom, referenced from the paramagnetic state) is fitted to even sixth-order polynomials in the reduced magnetizations \( m_\mu = \langle \cos \theta_\mu \rangle \), which are expressed through the fields \( \alpha_\mu \) by the Langevin function. The quadratic part of these polynomials, \( E_Q = \frac{1}{2}J_{FeFe}(Q)m_{Fe}^2 + \frac{1}{2}J_{MnMn}(Q)m_{Mn}^2 + J_{FeMn}(Q)m_{Fe}m_{Mn} \), defines the component-resolved effective exchange interactions \( J_\mu(Q) \) for the three orderings. Since the total energies are calculated with constraining fields, these exchange parameters are free from the errors associated with the long-wave approximation [31]. The results are shown in Fig. 1.

The concentration dependence of the parameters \( J_\mu(Q) \) shows that, in agreement with experiment, the \( F-, G-, \) and \( C- \)-type orderings are favored at the Fe-rich end, in the middle, and at the Mn-rich end, respectively. Further insight can be obtained from the reduced exchange parameters that are normalized by the concentrations, \( J_{\mu}(Q) = J_{\mu}(Q)/\langle c_\mu c_\nu \rangle \), shown in Fig. 2. These reduced quantities would be concentration independent in a Heisenberg system with pair exchange parameters \( J_{\mu}(R) \) depending only on the distance and the identity of the atoms in a pair. We see that the parameters \( J_{\mu}(Q) \) for like components (i.e., Fe-Fe and Mn-Mn) are almost constant for all ordering vectors, as is the reduced Fe-Mn coupling for the \( F \) ordering. However, the reduced Fe-Mn couplings at the \( G- \) and \( C- \) ordering vectors depend strongly on the concentration, which reflects the effect of band filling on the exchange interaction in metallic systems.

The local spin moments of Fe and Mn increase by about 10% as \( x \) goes from 0 to 1 (for example, from 2.87 to 3.11 \( \mu_B \) for Fe and from 3.35 to 3.74 \( \mu_B \) for Mn in the paramagnetic phase; similar variations are observed for different phases and spin directions). The adiabatic approach [19] is thus well suited for this system. We also repeated some calculations without the constraining magnetic fields, which is equivalent to making the long-wave approximation [31], and found that the resulting errors in \( J_{\mu} \) for all phases do not exceed 5%–7%.

FIG. 1 (color online). Magnetic exchange parameters \( J_{\mu} \) in the paramagnetic state corresponding to \( (F) \) ferromagnetic ordering, \( Q_F = 0; (G) \) \( G- \) type ordering, \( Q_G = (1,0,1/2); (C) \) \( C- \) type ordering, \( Q_C = (1,0,0) \). Red circles, \( J_{FeFe}(Q) \); blue diamonds, \( J_{MnMn}(Q) \); black squares, \( J_{FeMn}(Q) \).
To further extend the mapping of the magnetic configuration space, we performed noncollinear CPA calculations for the $F/G$, $G/C$, and $F/C$ noncollinear $2Q$ phases in the relevant concentration ranges. These calculations are needed to reveal the possible interaction between orderings at different $Q$ in the $2Q$ structures, which can appear in quartic and higher-order interaction terms. A $2Q$ structure is parametrized by two angles, $\theta_{Fe}$ and $\theta_{Mn}$, that the spin moments of Fe and Mn atoms make with the $z$ axis. Using the symmetries, the accessible space of $(\theta_{Fe}, \theta_{Mn})$ is reduced to the range $0 \leq \theta_{Fe} \leq \pi/2$, $-\pi \leq \theta_{Mn} \leq \pi$ with additional $\theta_{Mn} \rightarrow \pi - \theta_{Mn}$ symmetry at $\theta_{Fe} = \pi/2$ and $\theta_{Mn} \rightarrow -\theta_{Mn}$ symmetry at $\theta_{Fe} = 0$. This irreducible domain is covered by a uniform mesh of 38 points.

For a particular $2Q$ phase (say, $F/G$), we then combine the data from the separate VDLM calculations for the $F$ and $G$ phases with those from the noncollinear CPA results for the $2Q$ phase and fit the magnetic energy at the given concentration to a polynomial in the order parameters $m_{Fe,F}$, $m_{Fe,G}$, $m_{Mn,F}$, and $m_{Mn,G}$ (see Supplemental Material [24] for details). The high accuracy of the fits is illustrated in Fig. 3(a) for the $F/G$ phase at $x = 0.26$; all other fits are of similar accuracy. Figure 3(b) shows the magnetic energy as a function of $\theta_{Fe}$, $\theta_{Mn}$ in this phase at $x = 0.26$. There are two minima: the global one at $(\pi/2, \pi/2)$ corresponding to the collinear $G$-type phase, and a local one near $(\pi/6, \pi/2)$ corresponding to the $2Q$ phase. At a lower concentration there is a first-order transition where the $2Q$ minimum goes below the $G$-type minimum.

Using the combined fits for the magnetic energy, we now determine the ground states for all concentrations by choosing the lowest energy of all the competing phases. The results are shown in Fig. 4. We find that the $F$ phase is stable at $x < 0.11$ and the $G$ phase at $0.23 < x < 0.66$. There is a $2Q$ $F/G$ phase at $0.11 < x < 0.23$ separated from $F$ by a second-order and from $G$ by a first-order transition. The $G/C$ phase is stable at $0.66 < x < 0.85$.

Surprisingly, at $x = 0.85$ we find a first-order transition from the $G/C$ to the $F/C$ phase. The existence of this first-order transition is in excellent agreement with the observed abrupt drop in the mean magnetic moment at this concentration [23]. However, since the $F$ component in the low-temperature Mn-rich phase has not been previously identified, the existence of the $F/C$ phase is a prediction that needs to be verified experimentally. The $F/C$ and $G/C$ phases at the Mn-rich end differ essentially in the Fe ordering alone, as the ordering of the Mn spins is almost purely $C$ type. The energy difference between the $F/C$ and $G/C$ phases reaches about 20 meV per Fe atom near $x = 0.95$; it is barely visible in Fig. 4 because of the small Fe concentration.

The first-order transition from $F/G$ to $G$ at $x \approx 0.23$ is also in excellent agreement with experiment, while the
transition from $G$ to $G/C$ occurs at a larger $x$ compared to experiment, where it is close to $x = 0.5$. Note, however, that our calculations are for systems with perfect $3d/5d$ ordering, while the order parameter in the experimental samples for $x = 0.5$ and $x = 0.6$ was 0.79 [23].

Figure 5 shows the angles $\theta_{\text{Fe}}, \theta_{\text{Mn}}$ in the ground states as a function of $x$. (By convention, $F$ and $C$ amplitudes lie along the $z$ and $G$ along the $x$ axis, except for the $F/C$ phase, where $F$ is along $x$.) The spin moments of Fe and Mn are antiparallel in the $F$ phase and parallel in the $G$ phase. First-order phase transitions appear as discontinuous jumps of the angles. Note that in a wide concentration range at the Mn-rich end the Fe spin moments are almost perpendicular to the MnPt host in the ground state, while Mn ordering is almost pure $C$ type. (A full set of first-principles calculations with interpolated lattice constants was performed with a small 0.01 step in $0.94 < x < 1$ range to confirm this.) This feature highlights the importance of non-Heisenberg terms in the magnetic energy. This is also reflected in the fact that Fe aligns antiparallel to Mn in the $C$ phase, even though the corresponding Heisenberg Fe-Mn exchange is negative in the paramagnetic phase (Fig. 2).

We now turn to the full concentration-temperature phase diagram. We have the fits for the magnetic energy $E_{\text{mag}}(m_{\text{Fe}}, m_{\text{Mn}})$, where the $x$ and $z$ components of $m_{\text{Fe}}$ and $m_{\text{Mn}}$ correspond to the two wave vectors; in a collinear phase one of these components vanishes. $E_{\text{mag}}$ is available for discrete concentrations, and we use linear interpolation between them. The entropy is approximated as $S = (1 - x)S(m_{\text{Fe}}) + xS(m_{\text{Mn}})$, where $S(m)$ is the entropy of a classical spin in a Weiss field of such a magnitude that the magnetization is $m$. This corresponds to the mean-field-like distribution function $p_n(\theta') \propto \exp(\alpha_n \cos \theta')$, where $\theta'$ is the angle with respect to the direction of $m_{\text{v}}$. Given that $m_{\text{Fe,F}} = m_{\text{Fe}} \cos \theta_{\text{Fe}}$ and $m_{\text{Fe,G}} = m_{\text{Fe}} \sin \theta_{\text{Fe}}$ in the $F/G$ phase, etc., we minimize the free energy for each phase with respect to four parameters, $\theta_{\text{Fe}}, \theta_{\text{Mn}}, \alpha_{\text{Fe}}$, and $\alpha_{\text{Mn}}$.

For a system with purely Heisenberg interaction, this scheme is identical to MFA applied to $2Q$ phases [7]. Therefore, both second-order transitions to the paramagnetic phase (where non-Heisenberg terms have no effect in MFA) and ground-state properties (where only energy is important) are correctly described by this approach. In the intermediate temperature range our scheme can therefore be treated as a thermodynamic interpolation. At such intermediate temperatures we are essentially assuming that the presence of non-Heisenberg terms does not strongly change the distribution functions and that our magnetic energy fits remain valid for partially disordered $2Q$ phases. In most cases, these assumptions are likely consistent with the accuracy of the MFA itself. The thermodynamic description could, in principle, be improved by mapping the total energies to a microscopic spin Hamiltonian, but this expensive procedure is beyond the scope of this Letter.

Figure 6 shows the MPD obtained both from the full magnetic energy fits and from the same fits truncated at the quadratic (Heisenberg) terms. First-order transitions are shown by dashed lines. The overall structure of the phase diagram agrees well with experiment. The first-order transition from the $F/G$ to the $G$ phase may help explain the peculiarities of the observed temperature-dependent magnetization peaks [23]. Indeed, there is a concentration range from $x = 0.23$ to $x \approx 0.28$ where the ground-state $G$-type ordering turns into $F/G$ and then to $F$ on heating. The physics is complicated by configurational disorder, which may lead to the formation of small Fe-rich clusters at elevated temperatures [23].

If only Heisenberg terms are kept in the magnetic energy, the transitions into the paramagnetic phase remain

![FIG. 5 (color online). Angles $\theta_{\text{Fe}}, \theta_{\text{Mn}}$ made by the spin moments of Fe and Mn with the $z$ axis at zero temperature. By convention, the $G$ order parameter, as well as the $F$ order parameter in the $F/C$ phase, are assumed to be orthogonal to the $z$ axis; all others are parallel to $z$.](image-url)

![FIG. 6 (color online). Magnetic phase diagram of Fe$_{1-x}$Mn$_x$Pt. Temperatures are rescaled by the ratio $T_{\text{expt}}(x)/T_{\text{theor}}(x)$, where $T(x) = (1 - x)T_C + xT_N$; $T_C$ and $T_N$ are the ordering temperatures of FePt and MnPt from Ref. [23] (expt) or theory (theor). (MFA gives $T_C = 924$ K and $T_N = 1670$ K.) Thick (blue) lines, full fit for $E_{\text{mag}}$. Thin (red) lines, same fit but with non-Heisenberg terms set to zero. Solid (dashed) lines, second-first-order phase transitions. Symbols, experimental data [23].](image-url)
unchanged. However, all first-order transitions turn into second order; the $F/G$ to $G$ transition is shifted to much larger concentrations, and the $F/C$ phase disappears completely. Thus, in order to describe the observed first-order transitions at $x = 0.23$ and $x = 0.85$, it is important to take the non-Heisenberg interaction terms into account. The nonzero magnetization in the $F/C$ phase should facilitate an easy experimental verification of its existence.

To conclude, we developed a computational tool based on a combination of first-principles calculations that is capable of describing a complicated $c - T$ magnetic phase diagram of a metallic alloy with competing interactions. Its application to the Fe$_{1-x}$Mn$_x$Pt system produced a detailed interpretation of the experimental phase diagram and also predicted a previously unknown low-temperature magnetic phase on the Mn-rich end. The correct first-order transitions and the Mn-rich phase are only captured if non-Heisenberg terms are included in the magnetic energy, showing the limitations of the conventional approach based on the Heisenberg-model Hamiltonian. The wide applicability and predictive power of this approach make it useful for the design of magnetic materials with desired properties.

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